A Beautiful Supertask

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Throughout this article I will consider elastic collisions between point particles which move in one and the same unidimensional space, the axis X. As we will see, this restriction is not essential; we could have taken particles of a finite size submitted to the restriction of moving in a straight line.

Let us take a reference frame in which the particle A, of mass m, moves rightward and approaches the point 0 of the axis X at a velocity v while particle B, also of mass m, moves to the left and also approaches 0 at velocity v. Let them collide at 0. Since the collision is elastic, the total kinetic energy of the particles will be conserved and, by symmetry, A will move away to the left from 0 at velocity v and B will move away to the right from 0 also at velocity v. Let us describe this collision in a frame of reference which moves to the right at velocity v with respect to the former. In it the initial situation corresponds to a particle A at rest which is approached by B (whose mass is identical to A) at velocity $2v$ and moving towards the left. After colliding, B remains at rest and A starts to move towards the left at the velocity of B before the collision, that is $2v$. This is a general well-known result in the classical mechanics of elastic collisions (valid even in relativistic mechanics) which we will make use of later. These preceding comments are intended to make this result more intuitively plausible.

Assume a reference frame in which there is an infinite number of particles P_i, each one of mass m, at rest in the points $X_i = \frac{1}{2^i}$ ($i \in \{1, 2, 3, 4, \dots\}$) while at the point $X = 1$, a particle P_0 of mass m and constant velocity v is moving towards P_1. After the collision, P_0 will remain at rest in $X_1 = \frac{1}{2}$, while P_1 will approach and collide with particle P_2 at velocity v, P_2 at rest in $X_2 = \frac{1}{4}$. After this second collision P_1 will remain at rest in X_2 while P_2 will approach P_3 at velocity v, and so on. In general, for any $i \in \{1, 2, 3, 4, \dots\}$, P_i will move during a certain interval of time at velocity v from $X_i = \frac{1}{2^i}$ to $X_{i+1} = \frac{1}{2^{i+1}}$, but it will remain at rest in $X_{i+1} = \frac{1}{2^{i+1}}$. The space covered by P_i is $X_i - X_{i+1} = \frac{1}{2^i} - \frac{1}{2^{i+1}} = \frac{1}{2^{i+1}}$ and the time taken to cover it is obviously $t_i = \frac{1}{v2^{i+1}}$. This is the time that transpires between the two collisions to which P_i is subjected ($i \in \{1, 2, 3, 4, \dots\}$). Consequently, once P_0 and P_1 collide, all successive collisions will have taken place after the period of time given by the sum of the series

(1) $\frac{1}{v2^2}, \frac{1}{v2^3}, \frac{1}{v2^4}, \ldots$.

It is known that the sum of the series $\frac{1}{2}$, $\frac{1}{2^2}$, $\frac{1}{2^3}$, $\frac{1}{2^4}$, ... is unity. So, the sum of $\frac{1}{2^2}$, $\frac{1}{2^3}$, $\frac{1}{2^4}$, ... is $\frac{1}{2}$ and, in consequence, the sum of (1) will be $\frac{1}{2v}$. Therefore after the collision between P_0 and P_1 and after any period of time greater than or equal to $\frac{1}{2v}$ (therefore finite) has elapsed, the situation will be as follows: the particle P_i will be at rest in the point $X_{i+1} = \frac{1}{2^{i+1}}$ ($i \in \{0, 1, 2, 3, 4, \ldots\}$), that is to say all the particles will be at rest.

This is the supertask, an illustration of how the total initial energy of the system of particles $\frac{1}{2}mv^2$ can disappear by means of an infinitely denumerable number of elastic collisions, in each one of which the energy is conserved. It will be clear too why the restriction to point particles is not essential. The discussion of the case would be formally analogous to the previous one if we suppose for example that each P_i is a sphere of radius $\frac{1}{2^{i+2}}$ ($i \in \{0, 1, 2, 3, 4, \ldots\}$).

Nevertheless, the point at which the above supertask acquires special theoretical importance appears in connection with the problem of determinism in Newtonian particle mechanics. In discussing this problem, John Earman (1986) mentions several examples of particle systems (pointwise or not, and in finite or infinitely denumerable number), the evolution of which implies that Newtonian particle mechanics is not completely deterministic. Clearly, he does not consider situations which involve non-deterministic collisions between particles (pointwise or not), since these would trivialize the problem. The examples cited by Earman are cases of indeterminism which involve the disappearance of particles in the spatial infinity and the apparition of particles coming from spatial infinity (obviously in finite time). So, all of these can be eliminated, as Earman himself recognises, imposing boundary conditions at infinity. The interest of the supertask presented in this article is to be found in its permitting us to find an example of indeterminism in Newtonian mechanics (which do not involve non-deterministic collisions) and for which the imposition of boundary conditions at infinity would not be effective.

Let us consider the temporal inversion of the supertask described above. This inversion corresponds to a physically possible process, given the temporal symmetry of the laws of Newtonian mechanics (Earman makes use of the temporal symmetry of the laws of mechanics in the same way in his examples). We take as base an initial configuration with point mass particles P_i, of mass m, at rest in points $X_i = \frac{1}{2^{i+1}}$ ($i \in \{0, 1, 2, 3, 4, \ldots\}$). The complete system may be spontaneously self-excited in such a way that, after any interval of time afterwards which is greater than $\frac{1}{2v}$, we will have a system of infinite point mass particles P_i, of mass m, at rest in points $X_i = \frac{1}{2^i}$ ($i \in \{1, 2, 3, 4, \ldots\}$) together with a particle P_0 of mass

m moving rightward from $X_1 = \frac{1}{2}$ and following the rising values of X at velocity v. This form of self-excitation in the system is unforeseeable from the point of view of classical mechanics; it can take place at any instant and it may in fact repeat itself in time any number of times. Besides, the particle P_0 which finally moves does so at a constant velocity v (which can take any value), for which reason it will not disappear in spatial infinity. So, the strategy of imposing boundary conditions at infinity will not at all prevent this form of indeterminism.

The example analyzed in this article is also of interest because of its mathematical simplicity. The examples cited by Earman pertain to the theory of dynamical systems and are sufficiently complex mathematically for him only to cite and explain them without proofs. This is not the case with the example studied here. This may help to make the problem of indeterminism in classical particle mechanics more accessible to philosophical analysis.

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