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Bolzano and Kant on Space and  
Outer Intuition\**Clinton Tolley***§1 Introduction: rethinking the relation between space  
and outer intuitions**

In his *Critique of Pure Reason*, Kant famously argues for what he calls the 'transcendental ideality' of space. A key step in Kant's argument is his attempted proof in the Transcendental Aesthetic that our most 'original' representation of space must be an *intuition* rather than a concept, and moreover, must be one that is *pure*, insofar as it must be in the mind *a priori*, prior to all actual 'empirical' (sensation-involving) intuitions of external objects, what Kant calls 'outer intuitions'. Kant thinks this intuition of space must be present (or 'occur') in the mind *a priori* since spatial representation is universally and necessarily involved in all of our outer intuitions. Kant then goes on to argue (briefly in the first *Critique* but then at length in the *Prolegomena*) that accepting his account of the pure intuition of space is also necessary in order to make sense of how it is possible that we could come to have the *a priori* cognition of space in pure geometry that Kant, along with most of his contemporaries, assumes that we possess.

Though a handful of Kant's most influential successors in the philosophy of mathematics have accepted the broad outlines of these claims about the role of pure intuition in geometry,<sup>1</sup> many of Kant's readers – even many of his most sympathetic ones – have been sharply critical of

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<sup>1</sup> Perhaps two of the most well-known are Frege and the early Carnap; for Frege, see his 1924/1925 'Erkenntnisquellen der Mathematik und der mathematischen Naturwissenschaften'; for Carnap, see his 1922 *Der Raum*.

this component of Kant's doctrine of space and spatial representation. Especially after Dedekind and Hilbert, it became common, even among self-styled neo-Kantians, to reject the idea that any appeal to intuition is necessary in order to account for the knowledge of space provided in pure geometry.<sup>2</sup>

As has now been increasingly appreciated, one of Kant's earliest critics on this point was Bernard Bolzano.<sup>3</sup> Challenges to Kant's account of geometry appear already in some of Bolzano's earliest publications (cf. Bolzano 1810), and are developed more sustainedly in his later discussions of Kant in the 1837 *Wissenschaftslehre* ('WL') and those recorded by Příhonský in the 1850 *New Anti-Kant* ('NAK'). Bolzano argues, against Kant, that it is possible to define the representation of space through mere concepts alone, without this definition including any representations whatsoever drawn from intuition (cf. WL §79.6, I.366; §79 Anm, I.369–370; NAK 74). In this respect, Bolzano thereby puts forward a form of geometrical 'logicism' *avant la lettre*.<sup>4</sup> In fact, Bolzano's criticisms go considerably further, insofar as he argues that the very idea of a pure intuition is essentially incoherent (as we will see below, cf. §§4–5).

Yet while existing treatments of Bolzano's criticism of Kant on space have focused primarily on Bolzano's contrasting account of knowledge in geometry and mathematics more broadly, much less attention has been paid to the consequences that Bolzano's rejection of pure intuition has for Bolzano's own account of our intuitions of external objects – representations that Bolzano himself also calls 'outer intuitions'.<sup>5</sup> This will be my focus in what follows.

What will emerge is that the position Bolzano is led to on the nature and structure of outer intuitions is considerably different from Kant's, from the ground up, as it were. Bolzano's rejection of a pure intuition of space turns out to be intimately connected with his denial that outer intuitions contain *any spatial representation whatsoever*. This is because Bolzano rejects the idea that the content of our outer intuitions has *any*

<sup>2</sup> For the rejection of pure intuition in geometry by the neo-Kantians, cf. Friedman 2000, 28, and Coffa 1991, 57f.

<sup>3</sup> See Coffa 1991; Laz 1993; Rusnock 2000, 45–50 and 131–140; and Sebestik 2003.

<sup>4</sup> Cf. Coffa 1991, 27f.; Sebestik 2003, 54f.; cf. Palagyi 1902, iii.

<sup>5</sup> An early start on this topic can be found in Palagyi 1902, chapter VI (esp. §18). Some more recent helpful treatments of related topics can be found in George 2003 and Rosenkoetter 2012. For a discussion of Bolzano's rejection of Kant's doctrine of the pure intuition of time that is in key ways complementary to what follows, see George 1987.

*universal or necessary 'form' whatsoever.* A fortiori, Bolzano also rejects the idea that such a form is provided by a representation of space. Rather, on Bolzano's account, the content of each outer intuition is each *essentially simple*, and so does not contain anything 'manifold' in itself (such as the manifold Kant thought was provided by sensation) that would need to be unified by such a form – and so they do not contain anything that would do such unifying either (such as the representation of space itself).

Bolzano will thus be seen to depart from Kant at a quite fundamental level concerning the nature of our sensible representations of external objects. As we will also see, however, Bolzano takes the grounds for his departure to lie in commitments that, at least as he reads him, *Kant himself* explicitly affirms. Especially important here, for Bolzano, are Kant's remarks that link representational unity to intellectual acts of synthesis and combination.

What is more, though one might suspect that Bolzano's rejection of pure intuition would be part and parcel of a rejection of idealism about space as well – given the role that the doctrine of the pure intuition of space plays in Kant's own argument for the ideality of space – Bolzano actually agrees with Kant (and Leibniz before him) that space itself is not an 'actual [*wirklich*]' object in its own right, and also agrees – more surprisingly – that spatial representation has 'ideal' contents, in something close to Kant's sense of the term. Or so I will argue.

In several respects, then, Bolzano's alternative account of outer intuitions can be seen to take shape as a kind of internal challenge to Kant's account.<sup>6</sup> In effect, Bolzano's alternative itself provides us with a competitor form of idealism developed from Kantian commitments.

My discussion will proceed as follows. I will begin in §2 by presenting the basics of Kant's account of space, spatial representation, and outer intuition, as it is developed in the Transcendental Aesthetic. In §3 I will then turn to Bolzano's account of intuition in general and outer intuition in particular, noting the extent to which he means for it to accord with Kant's own officially stated position on intuitions. In §4 I shift the focus to Bolzano's main departures from Kant on outer intuitions, departures made on the grounds that Kant's talk of intuitions containing a 'manifold' entails that synthetic intellectual activity (and

<sup>6</sup> In its focus on Kant's remarks on synthesis especially, Bolzano's criticisms of Kant's doctrine of intuition can be seen to directly anticipate points made by various 'conceptualist' revisions to Kant's views on intuitions, of both the neo-Kantian variety as well as contemporary philosophers inspired by Kant (cf. Tolley 2013).

hence, concepts) are involved in the constitution of intuitions – though at the same time I also show how Bolzano takes these to be grounds that Kant himself actually should accept. In §5 I then show how the foregoing parallels and divergences on outer intuition furnish Bolzano with the basic material for his criticisms of Kant’s account of pure intuition in particular, highlighting how Bolzano’s criticism here again actually draws upon an important shared commitment – this time concerning the ontological ideality of space. Perhaps more controversially, I also argue that Bolzano ultimately agrees with Kant on the more straightforwardly transcendental idealist thesis that the representation of space represents something which is broadly representation-dependent, even if it is not intuition-dependent. In the concluding section (§6), I will take up the question of whether a defender of Kant’s account might have any grounds for resisting Bolzano’s criticism of Kant on the nature of outer intuitions and the pure intuition of space, both in light of claims Kant makes elsewhere which Bolzano doesn’t consider, as well as in light of reflection on the psychology and phenomenology of such intuitions.

## §2 Kant’s account of space and outer intuition in the Transcendental Aesthetic

Let us begin by laying out Kant’s doctrine of space and outer intuition as it is found in the Transcendental Aesthetic. This section contains one of the most well-known and controversial conclusions Kant thinks he has established in the first *Critique* – namely, that space (the object) is something that ‘exists’ *only* ‘in the representation of it’ (A375n). Kant thinks he has demonstrated here that space ‘exists’ only as a ‘form’ of the contents of our sensible representations of objects which are ‘outside of’ or ‘external to [*ausser*]’ us (B42–43), rather than existing as something ‘actual [*wirklich*]’ in its own right, or existing as a determination of the way things are ‘in themselves’, independently of our sensibly representing them via intuitions (B37). As Kant ultimately puts this point, space is something that is ‘transcendentally *ideal*’ (B44). In support of this conclusion, Kant first sets out to demonstrate that the most fundamental, ‘originary [*ursprüngliche*]’ representation that we have of space is an kind of ‘*intuition*’ itself, rather than a concept (B39), and an intuition, moreover, that is ‘in’ us a priori, and is therefore ‘pure’ (B38–39). I will start with this preliminary argument.

### §2.1 The originary representation of space

Kant begins his argument for the ideality of space from the ‘exposition’, or ‘distinct representation’, of what ‘belongs to’ our ordinary concept

of space (B38). At the outset, Kant assumes that we understand space to be something that is related in some way to our sensible representations of objects which are ‘outside us’, objects which we represent by means of our ‘outer sense’ (B37). More specifically, we represent ‘all’ objects of outer sense ‘as *in* space’ (B37; my italics). Furthermore, Kant takes us to understand space as that in which the shape and magnitude of external objects, and their relations to one another (e.g. distance), are ‘determined’, or at least ‘determinable’ (B37). Finally, Kant takes space to be something that we don’t intuit ‘in’ us, in the sense that when we do represent our own mind and its states in intuition, these are not represented as ‘in’ space but as only in time (B37).

As a key step in his argument for the ideality of space so understood, Kant sets out to establish, first, that the ‘original [*ursprüngliche*]’ representation that we possess of space must be ‘in’ the mind *a priori*, prior to all actual ‘sensation [*Empfindung*]’, ‘intuition [*Anschauung*]’, and ‘experience [*Erfahrung*]’ of external objects, and so cannot be an ‘empirical’ representation, or one drawn from these experiences (B38–39). This representation must be present in the mind prior to all actual outer intuition because it contains the universal and necessary ‘form’ of the contents of all such intuitions, and so is what makes such intuitions possible in the first place (more on this in a moment). In Kant’s words, the representation of space must be the ‘ground’ of these outer intuitions and their contents (B38). And since experience arises out of the synthesis of intuitions via concepts in judgment (cf. *Prolegomena* §20, 4: 300f.), the representation of space must therefore lie at the ground of outer experiences as well.

Kant then sets out to establish, second, that this original representation of space must nevertheless also be a special kind of ‘intuition [*Anschauung*]’ itself, rather than a general, common, or discursive concept (B39–40). Kant’s arguments here depend on the consideration of the special nature of this universal and necessary form of the contents of outer intuition – most importantly, that this content represents an object that is ‘essentially unitary [*einig*]’ (B39), even though this content in some sense also ‘contains within itself’ an infinity of further representations (B40).

The former point about essential unitariness leads Kant to insist that all the further representations we form of space (e.g. of parts of space, spaces, points, shapes, distances) arise due to acts of abstraction from a more original representation which first gives this single object itself immediately as the essentially unitary whole that it is. This abstraction takes the form of an intellectual delimitation of what are essentially non-independent parts of space, by ‘thinking’ these parts ‘into’ the ‘single

all-encompassing space' (B39). Since Kant takes intuition to be both the only 'representation that can be given prior to all thinking' (B132), and also to be the only representation that can 'give' its object 'immediately' (B33), Kant concludes that this more original representation of space must itself be a kind of intuition.

Concerning the latter point about infinity: Kant takes it for granted that no concept could contain 'in itself' (in its content) the possibility for an infinity of additional representations (B40). Yet the original representation of space can and does serve as the ground for an infinity of further representations – namely, the representations of all of the different kinds of parts (delimitations, shapes) of space and relations in space (B40). Kant sees this point about content as confirming his previous conclusion that the original representation cannot be a concept, but must be an intuition.

Combining these two theses (a priority and intuitionality), Kant then concludes that the original representation of space must be what he has earlier called a *pure intuition*, one which would 'occur [*stattfindet a priori*]', and so be non-empirical, and which would give its individual object (space) 'immediately' (cf. B33 and B41), all at once, as the essentially unitary object that it is (B34–35).

## §2.2 Space as the form of outer appearances

Nevertheless, when viewed in relation to the 'outer' intuitions of external objects that it makes possible, Kant holds that this original representation of space must ultimately contain only the '*form*' of the content of these outer intuitions, without containing any of the 'matter' eventually provided through sensation (B34). This points up the fact that, for Kant, what is 'contained in' an outer intuition is a composite of form and matter. The whole content is what Kant calls an 'appearance [*Erscheinung*]', which serves as the immediate 'object' of an outer intuition (B34). Space is what provides the 'form of all appearances of outer sense' (B42). The 'matter' is provided by the contents of sensation, e.g. colour, impenetrability, hardness (B34–5). This is then what fills in space itself, a matter which is then 'ordered' according to spatial relations (of 'extension [*Ausdehnung*]', 'figure [*Gestalt*]', etc.) (B35). And since Kant takes the matter from sensation that is 'contained' in every intuition to be 'something manifold [*ein Mannigfaltiges*]', the order that space provides to this matter is what brings a kind of 'unity [*Einheit*]' to the content of outer intuitions (A99).

Finally, it is this whole outer appearance – the unity which arises from sensory contents being ordered in a certain spatial configuration – which itself represents the further thing which is responsible for

bringing about the outer intuition in the first place. This further thing is what Kant at times describes as ‘something = x’ (cf. A104, A250), since we don’t have any insight into what it is like, except through its effects on our sensibility – namely, the sensory contents which get ordered in a spatial form.

While this is how things stand with ‘empirical’ outer intuitions (ones which involve sensation), all that a *pure* outer intuition would contain, by contrast, is ‘that within which’ whatever ‘matter’ sensations will deliver ‘can be ordered in certain relations’ – i.e. the mere form of outer appearances (B34). Indeed, it is by containing *only* this form that such an intuition can occur *a priori*, ‘without any actual [*wirkliche*] object of the senses’ being yet encountered through it affecting our sensibility (B35). This is possible because Kant thinks that, in order to do such ordering of sensory material, the ‘form’ itself is something that ‘must lie ready for [the matter] in the mind *a priori*’ (B34).

### §2.3 The ideality of space itself

With all of this in place,<sup>7</sup> Kant then draws his famous conclusions concerning the ontological standing of space itself. Kant assumes that

<sup>7</sup> Because of my focus here (cf. §1), I have skipped over the part of the Transcendental Aesthetic where Kant then turns briefly to the relation between the foregoing and our knowledge in ‘pure’ (*a priori*) geometry. There Kant points out that the syntheticity and the apodicticity of such knowledge provides separate confirmation of the correctness of his claim that the original representation of space needs to be a pure intuition. Concerning apodicticity, Kant argues that it is only by accepting that we have a pure (*a priori*) representation of space that we can account for how we can know *a priori* that geometrical propositions will be universally and necessarily true of everything we encounter in space (B41). Yet since in geometrical reasoning, we use construction (drawing) to come to know that certain predicate-concepts are truly related to some subject-concepts, despite the fact that the contents of these predicate-concepts ‘go beyond’ what is contained in the subject-concepts (which makes the relevant judgments synthetic), Kant thinks that this pure representation cannot itself be merely another concept (B40–41).

The argument from geometry for the necessity of a pure intuition of space, as well as for the ideality of space, receives much fuller development in Kant’s *Prolegomena*, where Kant uses the ‘analytic’ method to demonstrate the truth of transcendental idealism, which proceeds from the accepted fact of *a priori* cognition in pure mathematics, to the ideality of space as a condition for the possibility of such cognition. In the first *Critique*, by contrast, Kant proceeds according to the ‘synthetic’ method, which moves from accepted facts about the elements of all our cognitions, to an inventory of what *a priori* cognition is possible given these elements. (Kant discusses the difference between these two methods at the outset of the *Prolegomena*, cf. 4: 263–264.)

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'neither absolute nor relative determinations [of things] can be intuited prior to the existence of the things to which they pertain, thus cannot be intuited a priori' (B42). And yet he also takes himself to have just shown that space itself *can and must* be intuited *a priori*. Kant therefore concludes:

Space represents no property at all of any things in themselves nor any relation of them to each other, i.e., no determination of them that attaches to objects themselves and that would remain even if one were to abstract from all subjective conditions of intuition. (B42)

Instead of being something 'actual', with a representation-independent existence 'in itself', space exists only 'in' our intuitions and in our experiences, and is therefore dependent for its existence on the 'subjective constitution' of our senses being the way it is:

We therefore assert... [space's] transcendental ideality, i.e., that it is nothing as soon as we leave aside the condition of the possibility of all experience, and take it as something that grounds the things in themselves. (B44)

[I]f we remove our own subject or even only the subjective constitution of the senses in general, then all constitution, all relations of objects in space and time, indeed space and time themselves would disappear, and as appearances they cannot exist in themselves, but only in us. (B49)

Note, finally, that Kant concludes as well that all relations of objects in space are also 'ideal'. This entails that outer appearances as a whole are themselves ideal as well, insofar as Kant believes that 'nothing is given to us through outer sense except mere representations of relation' (B67).

### §3 Kantian themes in Bolzano's account of outer intuitions

When we now turn to Bolzano's criticisms of Kant's account of space and spatial representation, we should first take care to determine how the terminology lines up between the two authors, especially concerning the term 'intuition' and 'outer' intuition in particular. On the one hand, Bolzano makes clear that he means to be taking over some of the key elements of Kant's analysis of 'intuition [*Anschauung*]'. Most importantly,



Bolzano accepts Kant's claims that outer intuitions are representations that 'give' their object *immediately*, involve *sensation*, and are essentially *singular*, in that they necessarily represent only one individual object. On the other hand, as we will see in the next section (cf. §4), Bolzano thinks that these two facts about intuitions entail that intuitions bear two further marks, at least the second of which Bolzano recognizes that Kant does *not* seem to accept: first, the object of an intuition must be something '*actual* [*wirklich*]'; second, intuitions must have a content that is *simple*.

### §3.1 The immediacy of intuition

Bolzano says he means to be using 'intuition' in a way that picks up on Kant's use (cf. NAK 15).<sup>8</sup> The first respect in which he means to be following Kant is that he, too, accepts that intuitions are 'immediate'. As Bolzano sees it, for a representation to be 'immediate' in the sense in which Kant has in mind – for it to 'give' its object to the mind (as Kant puts it; B33) – the occurrence of this sort of representation must entail the existence of an actual object to which it is related: '[Kant's] expression, intuition gives the object, seems to have the sense that we are justified in concluding, from the possession of an intuition, that there must exist an object which brought it about' (WL §77.2, I.346; cf. NAK 44). Understood this way, Bolzano thinks 'there seems to be something very true in this': 'We can indeed infer from the possession of a subjective intuition to the existence of an object corresponding to it, which brought it about through its influence upon our faculty of representation' (WL §77.2, I.347).

Though Bolzano accepts that the 'arising' of 'all other representations which appear in our consciousness' must also have 'an appropriate cause', Bolzano thinks that the case of an intuition is special: 'The difference is that from the presence of [an intuition], we can infer to a cause which is itself the one and the same actual [*wirkliche*] object *which we are representing*' (WL §77.2, I.347; my italics). Thus Bolzano identifies the object represented by an intuition with the cause of the intuition.

What is more, in order to preserve what is special about intuitions, Bolzano thinks we must restrict the application of the label 'intuition'

<sup>8</sup> This is so, even though Bolzano is much more explicit (and persistent) about the distinction between the act of intuiting and the content thereby intuited. Bolzano calls the former 'subjective intuitions', and calls (something closer to) what Kant means by the latter an intuition 'in itself [*an sich*]' (cf. WL §76.2, I.342).

*only* to those representations which do, in fact, come about due to the influence of an *actual* object which the representation thereby represents:

It does seem quite correct to me...to say that an intuition (that is, a subjective one) always concerns an actual [*wirkliches*], and indeed, if you will, a present [*gegenwärtiges*] (that is, acting on us at the time) individual thing [*Einzelding*]...and that the content of the intuition is not applied to anything other than this thing. (*WL* §77.8, I.352; my italics)

This feature is present in Bolzano's primary example of a representation that is 'immediate' in this sense of 'giving' an actual object – namely, the representation which occurs in our soul when we direct our attention to a 'change' that is also in our soul:

As soon as we direct the attention of our mind upon the change that is brought about in our soul by some external body that is brought before our senses, e.g., a rose, the next [*nächste*] and immediate [*unmittelbare*] effect of this attending is that a representation of this change arises in us. (*WL* §72, I.326; cf. *NAK* 16)

A few sections later, Bolzano makes explicit that he takes the inclusion of a relation to an actual object to be a feature not just of these examples but to characterize *all* human intuitions:

The intuitions that I gave in §72 as examples were all constituted in such a way that the objects which corresponded to them all belonged in the realm of actuality [*Wirklichkeit*], since these were throughout certain changes occurring in our soul. Now, I am of the opinion that this holds of all intuitions, at least those of which we humans are capable, i.e., I believe that the object of any humanly attainable (subjective) intuition must be an actual [*wirkliches*] thing. (*WL* §74.1, I.331)

One might wonder whether all of the foregoing entails that Bolzano thinks we can only have 'inner' intuitions ('inner' representations of changes among the representations 'in' our soul). There is, in fact, something to the thought that, for Bolzano, the immediate objects of all intuitions are 'inner'. Bolzano states explicitly that intuitions are distinguished from all other representations due to the fact that 'they appear

as the next and immediate *effect* of certain changes just having occurred *in us* (my italics), changes in us ‘which are therefore the object represented through them [*der durch sie vorgestellte Gegenstand*]’ (WL §286.1, III.84–85). And since we have seen that intuitions are special in that they always represent their causes, the represented objects of every (human) intuition must therefore be things ‘*in the soul*’.

### §3.2 The involvement of sensation

Even so, Bolzano means to follow Kant in accepting that there is reason to distinguish certain intuitions as especially ‘inner’ and others as ‘outer’. For Bolzano, the difference between inner and outer intuitions rests upon a difference in the kind of ‘change in us’ which is ‘immediately’ represented – namely, whether this change is *itself* already a representation or whether it is not:

We can, however, distinguish two kinds of intuition, depending on whether or not this change itself is already a representation (or even a judgment). Intuitions which have as their object another representation equally present in our mind may be called inner [*innere*] intuitions. Those, by contrast, which concern a change which is not itself a representation, may be called outer [*äußere*]. (WL §286.1, III.85)

Now, we might wonder what Bolzano is thinking of by referring to a change which is ‘in our soul’ and yet which is not itself a representation, but which nevertheless is to function as the object of the representation that he calls an ‘outer’ intuition (since, as we have just seen, *all* intuitions have ‘changes in us’ as their objects). A promising proposal here has been made by Rolf George, who has argued that Bolzano is thinking of mere ‘sensations [*Empfindungen*]’ as the objects of outer intuitions, as opposed to other ‘mental appearances [*Erscheinungen*]’ which are properly called ‘representations’ (cf. George 2003, 21f.). Now, like Kant, Bolzano’s examples of outer intuitions do, in fact, involve representations of sensory qualities, e.g. of red (cf. WL §286.1, III.85). What is more, Bolzano claims that it is outer intuitions that we ‘subsume’ under ‘the common representations of colors, sounds, odors, etc.’, which he identifies as the ‘common sensible [*sinnliche*] representations’ (WL §286.8, III.88). All of this suggests that sensations are what is represented by outer intuition.

Are sensations, however, not themselves a form of representation? We saw above that Kant takes them to provide contents which factor into

outer appearances and so serve to represent external objects.<sup>9</sup> In *WL* §143, however, when Bolzano is classifying ‘what occurs in our own inner sphere [was in *unserem eigenen Innern vorgehet*], he does in fact distinguish sensation from both representations and judgments, with sensation being said only to ‘accompany’ many of our representations (II.67). Elsewhere when Bolzano again explicitly distinguishes the sensations we have from both the representations that we have of them and the judgments we make about sensations, he also argues that we cannot make a judgment about a sensation until we have formed a representation of them:

We can only make judgments about sensations if we first represent them. The sensation itself is one thing, the representation of it is quite another, and the judgment about it (e.g., that it is pleasant or unpleasant, is constituted thus and so, etc.) is still something further. (*WL* §35.8, I.163)

This might be thought to give some evidence for the related, though somewhat indirect argument made by George. George points out that, given Bolzano’s explanation of the concept of a representation, Bolzano in principle restricts the term ‘representation’ to that which can function as a component in a proposition (content of a judgment) but which is not itself a proposition (cf. *WL* §52, I.228; *NAK* 8). Bolzano’s claim that we need to first form representations of sensations in order to make judgments about them might be taken to suggest that it is only representations of sensations, rather than the sensations themselves, that can function as components in the contents of judgments. But, then, if sensations themselves cannot become components of propositions (as contents of judgments), they would thereby be ruled out from being representations.<sup>10</sup>

<sup>9</sup> Kant seems to explicitly classify sensations as a kind of representation on the so-called ‘Stufenleiter’ (cf. B376–377). What is more, Kant also at times explicitly describes sensations as having an ‘objective’ representational function (as an ‘objective representation of the senses’; cf. *Critique of the Power of Judgment* §3). Both of these speak against George’s claim (in George 1981) that sensations for Kant do not represent anything.

<sup>10</sup> This is so, even if Bolzano of course allows that judgments (propositions) can be *about* sensations (cf. again *WL* §35.8, and *WL* §143.3, II.69). What functions, for example, as the predicate-representation in a proposition of the form ‘A has the sensation D’ just needs to be understood as a representation *of* the sensation D, not the sensation D itself. In *WL* §42, Bolzano identifies these components of judgments with intuitions of objects that ‘obtain outside of representations’ (I.181).

Such an argument is not conclusive, however, because here Bolzano is talking explicitly only about the possibility of judgments *about* sensations, not the possibility of judgments about other things *through* representational contents which might include sensations. For this same point holds for intuitions and concepts as well: we cannot make a judgment *about* an intuition or a concept until we first form a representation *of* the intuition or concept. Nevertheless, while Bolzano does regularly talk about intuitions and concepts both functioning as components in the contents of judgments, he nowhere (to my knowledge) talks about sensations functioning as possible contents (rather than objects) of judgments. For this reason, George's suggestion seems to be on the right track.

Yet if sensations are something 'inner', and if outer intuitions are immediate representations of sensations, and thereby have something inner as their immediate object, why should these representations of sensations still be called 'outer' intuitions?<sup>11</sup> Bolzano here points us to what is involved in the causal origin, or the bringing about, of such intuitions. Every intuition of a sensation is, Bolzano claims, 'a representation that has its existence immediately due to a change in our soul *produced by the action [Einwirkung] of an external [äußeren] object*' (WL §288.1, III.104; my italics). These representations are called 'outer', therefore, because the nature of their 'arising' is such that it 'leads [*leitet*] us immediately to the presupposition [*Voraussetzung*] of an external object that, through its action [*Einwirkung*], must have brought about this change in our soul' (WL §286.1, III.85).

Matters are complicated further, however, by the fact that Bolzano takes the *full* cause of the change in outer intuitions to also include other changes 'in me', rather than solely being ascribed to the action upon me of an external object:

For each change which precedes my intuitions which are related to external objects, I must presuppose as cause a change either in me, or in some external objects, indeed either in their inner properties or in their spatial relations to me, or (what is more correct) in *all of*

<sup>11</sup> To be fair, Bolzano very occasionally seems to suggest that the object of an outer intuition is actually a change that is 'external' to us – in the following passage, for example: '[E]very subjective intuition has its own object, namely, the change to be found *outside of* or in us which is the immediate cause of its arising' (WL §75.1, I.334; my italics). On balance, however, it would seem that the account given above is his considered view.

*these things at once*, only in varying degrees. Because my intuitions, when they *are related to* [*sich beziehen auf*] an external object, *represent* [*vorstellen*] changes which an external object has brought about in me through its action [*Einwirkung*], their nature [*Beschaffenheit*] is determined by my nature and the nature of the external object and through the spatial relation between us. (*WL* §303.21, III.151; my italics)

As is evident from this passage, then, Bolzano therefore actually means to distinguish (a) the total cause of the outer intuition, which involves not just the external object but factors ‘in me’, (b) the effects of this cause, i.e. the sensations which are ‘in me’, and which are actually the immediate objects that the intuition ‘represents [*vorstellt*]’, and (c) the external object to which the intuition ultimately ‘is related [*sich bezieht*]’. What the outer intuition *immediately* ‘represents’ are the sensations as changes ‘in me’ that are brought about by the action of an external object upon me – representing them in a way that is determined jointly by my own nature, the nature of the affecting object, and the spatial relations between us. What the outer intuition is *ultimately* ‘related to’, however, is the external affecting object alone, and is thereby taken to be an intuition ‘of’ that object.

This ‘relating’ to the external object, then, is something we associate with the intuition only secondarily, as a ‘presupposition’ to which we are ‘led’ by the presence of the intuition itself (cf. *WL* §286.1, III.85, quoted above). Indeed, later in the same passage Bolzano describes the secondary act of ‘relating’ an outer intuition to an external object as one that occurs through my ‘inferring [*schliessen*]’ (*WL* §303.21, III.151). For Bolzano, then, sensations are not directly or autonomously representational – at least not in the same way that other representations (intuitions, concepts) are – since our minds must judge or infer them to ‘be related to’ a further object, rather than simply being conscious of this further object (as it appears) through a grasping of the sensation itself. Hence, though Bolzano agrees with Kant on the involvement of sensations in outer intuition, sensation plays a quite different role in Bolzano’s account.

### §3.3 The singularity of intuition

Finally, Bolzano also takes himself to agree with Kant on the essential *singularity* of intuitions (cf. *NAK* 50). Though it is not stated explicitly in the discussion of space in the *Transcendental Aesthetic*, Kant later (and elsewhere) makes clear that he takes one of the marks of an intuition is

that it is a ‘singular [*einzelne*]’ representation, that it is a representation of an individual object (cf. B376–377, *Jäsche Logic* §1, 9: 91). One might also think that this is at least implicit in his argument that only an intuition could relate us to an ‘essentially unitary’ and hence individual object (one with no ‘real’ parts), in order for us then to be able to ‘think’ parts ‘into’ the object (cf. §2 above).

For his part, Bolzano arrives at the singularity of intuitions by asking us to reflect further on what occurs in our soul when we direct our attention to some change in us which is occurring due to the influence of an external object – e.g. to what occurs when our attention is directed to the change that takes place in our soul (i.e. the sensation) when a rose is brought before our senses (cf. *WL* §72, I.326). As we saw above, Bolzano thinks that the representation of the sensation which thereby arises in us is ‘immediate’ because it comes about due to an *actual* immediate effect ‘in us’ (namely, the sensation as change in our soul), an actual effect which it also thereby represents. In other words, we can always infer from the presence of the representation of the change in our soul to the existence of its object (the sensation itself). Yet Bolzano also takes it to be evident that the representation which arises as the ‘immediate effect’ of our ‘attending’ to a sensation is one which can have only one *single* object:

Now, this representation is an objectual [*gegenständliche*] one; its object is, namely, the change that has just occurred in our soul, and nothing else; therefore, a singular [*einzelner*] object. Thus, we can say that this representation is a singular representation [*Einzelvorstellung*]. (*WL* §72, I.326; cf. *NAK* 16)

As Bolzano also puts the point, the ‘content’ of such representations ‘is related to *nothing* distinct from this thing [*ihr Inhalt sich auf nichts von diesem Dinge Verschiedenes beziehe*]’ (*WL* §77.8, I.352; my italics).

What Bolzano has in mind would seem to be the following: if we are able to limit our attention solely to the ‘next and immediate’ change in our soul (rather than some indeterminate collection (series) of changes), then we can be sure that our attention (intuition) has one and only one object – namely, just that one change in our soul and nothing else. It is *this* representation-relation (intuition-to-sensation) that Bolzano thinks must be singular – even if the content of an outer intuition is *also* ‘related to’ something distinct from this sensation as a change ‘in us’, when it is related in judgment or inference to the single external object which is (partly) responsible for determining the nature of the intuition by

(partly) bringing about the relevant change. Still, it is only because the initial relation is indexed, as it were, to the particular change brought about – more specifically, to one particular sensation – that Bolzano takes the singularity of each outer intuition to be secured.<sup>12</sup>

#### §4 Bolzano's 'Kantian' departure from Kant on the simplicity of intuitions

On the points covered so far – with the notable exception of the precise role of sensations – Bolzano's account of outer intuitions is broadly in line with what Kant himself held about such intuitions. As noted above, however, Bolzano takes his account to involve two further commitments that he recognizes might not square directly with the letter of Kant's positions. First, whereas Kant seemed to hold that the 'content [*Inhalt*]' of an outer intuition involves both a matter, consisting in a (*complex*) manifold of sensation, and a form, consisting in space, Bolzano holds that the content of an outer intuition must be something entirely 'simple [*einfach*]'. Second, whereas Kant's account of the pure intuition of space forces him to accept the possibility of an outer intuition of an *ideal*, non-actual object, Bolzano insists, by contrast, that the object of every outer intuition must be something '*actual* [*wirklich*]'.<sup>12</sup>

Yet if Bolzano recognizes that he is here departing from Kant's official position, what is striking is that in both cases Bolzano takes his departure from Kant to be justifiable by reference to commitments that Bolzano thinks *Kant himself* actually held. In this way, these departures from Kant can be viewed as criticisms from 'within' a broadly Kantian framework.

##### §4.1 The simplicity of the content of intuitions

By the 'content [*Inhalt*]' of a representation, Bolzano means 'the *sum* of the components of which this representation consists', which Bolzano contrasts both with the *object* that the (whole) representation itself represents, but also with 'the *way* in which these parts are connected with one another' (*WL* §56, I.244), which he later associates with the '*form*' of the representation (*WL* §81.1, I.389).

<sup>12</sup> Bolzano's argument for the singularity of inner intuitions follows in a parallel fashion, since (subjective) representations themselves are also among the changes in our soul (along with sensations), and so can be attended to as 'next and immediate' (cf. *WL* §75, I.334–335).



As Bolzano sees it, given the nature of the arising of intuitions and their immediate objects, Bolzano thinks that we must conclude that the content of every intuition must be ‘*simple [einfach]*’ (WL §72, I.327), in the sense of having *no* parts. Bolzano’s argument for this claim runs as follows:

[I]f they were composed of parts, they would not be the next and immediate effect that arises from the observation of the change just having occurred in our soul; rather, the singular representations which would form the parts of any such complex representation would have been produced earlier and more immediately. (WL §72, I.327; cf. NAK 16)

Bolzano is here clearly assuming that no bringing-together or composition of representations into a complex one can occur *as immediately as* the representation produced by the observation or attention to a just-occurring change in our soul. Why can’t anything complex be the immediate effect of our attention or observation? Bolzano takes all such complexity to be the result of a further compositional or synthetic ‘activity’ of our soul, something over and above the mere directing of our attention toward something. Bolzano makes this further premise explicit later on in the *Wissenschaftslehre*, in a critical discussion of the alleged possibility – one with clear Kantian echoes – that outer intuitions are ‘infinitely complex’ representations:

Should one not already find something impossible in itself in such a representation composed out of infinitely many parts, it can in no way be assumed that such representations could occur merely due to the immediate action [*Einwirkung*] of an external object on our soul. Rather, it seems indisputable to me that any such composite representation requires a special activity on the side of our soul to bring it about out of the simple representations which are called its parts. Even supposing that the representations: ‘red, pleasantly fragrant, prickly’, etc., arise immediately through the action of an external object upon me, the representation which is supposed to be composed of all of them (the representation, perhaps, of an object which has in itself the collection of all of these properties) would not yet be present, but would rather require a special activity of my soul for its arising. (WL §287.3, III.94–95)

What is more, Bolzano thinks that, to actually form the representation: an object (being) which has in itself this collection of properties, this

'special activity' would need to avail itself of certain special connective representations which he also thinks cannot be viewed as 'immediately produced' by the object: '[Such composition] indeed would also require certain representations which most certainly were not immediately produced by that object, such as the concept of a being [*Wesen*], that of having [*Haben*] certain properties, of a collection, etc.' (*WL* §287.3, III.95; cf. *NAK* 51). In fact, Bolzano thinks that the representations – red, pleasantly fragrant, etc. – involved in such a complex representation are actually *general concepts* as well, for they are functioning as predicates under which we 'subsume' the immediate simple representation (*ibid.*).

For these reasons, Bolzano insists that the more accurate expression for the content of intuitions would be simply the demonstrative 'this', as comes out in his discussion in *WL* §42 of judgments which 'contain intuitions of certain objects that obtain outside of [our] representation'. Bolzano takes the canonical expression for this type of judgment to be: 'this (what I see here now) is red', such that 'the subject-representation of the proposition is a simple intuition (this) and the predicate-representation is a concept (red)', with the parenthetical expression 'what I see here now' therefore not being essential to the expression of the content of the subject-representation but 'redundant' (*WL* §42, I.181; cf. *NAK* 16–17). Later Bolzano writes similarly that when 'complete judgments are made...about the change itself that has just occurred in us', these can be expressed, for example, as: 'this (what I right now see) is the sensation or representation red; this (what I now smell) is a pleasant fragrance; this (what I just feel upon touching a thorn with the tips of my fingers) is a painful sensation, etc.' – with the word 'this' in each case serving to 'designate [*bezeichnen*]' a representation which is *simple* as to its content, one which relates immediately and directly to some presently noticed sensation as a 'next and immediate' change in us (*WL* §72, I.326).<sup>13</sup>

<sup>13</sup> Cf. *WL* §73.3, I.331 and §278, III.22. For more discussion, see George 2003 and Rosenkoetter 2012: §6.

To head off a possible misunderstanding, let me here note that Bolzano accepts that the ultimate *object* (a sensation) that an outer intuition represents, in virtue of having the content that it does, does not have to be simple – though it must be an individual: '[L]et us first note that the expression *individuum* may be misunderstood, since if no further explanation is forthcoming it can be interpreted as if the object of an intuition had to be simple, which is by no means the case' (*WL* §77.1, I.344).

#### §4.2 The Kantian motives behind Bolzano's simplicity condition

Now, Bolzano recognizes that, with the condition of simplicity of content, his account of intuition is departing not just from Kant but from many of his predecessors and contemporaries, however close it otherwise might be:

By the word *intuition* almost all modern logicians have in mind a representation that has only a *single* and *actual* [*wirklich*] object, and depart from me therefore only in that they ... do not require as firmly as I do that a [mere] intuition must be a thoroughly *simple* representation. (*WL* §76.2, I.342; my italics)

With respect to Kant in particular, however, Bolzano thinks that both of the two key points that he marshals in favour of this departure consist in aligning himself *with what should have been Kant's own considered position*. For Bolzano thinks he has fairly straightforward evidence from the *Critique* itself that Kant, too, holds, first, that all synthesis (combination) requires a special act of the mind (an act of understanding) over and above any act of mere sensibility, and holds, second, that all synthesis of the manifold given in intuition requires the use of concepts, and so cannot consist in mere intuitions.<sup>14</sup>

Concerning the former point, Kant claims explicitly that the synthesis (combination) of the manifold in intuition is something done by our understanding, not our sensibility:

all combination, whether we are conscious of it or not, whether it is a combination of the manifold of intuition or of several concepts, and in the first case either of sensible or non-sensible intuition, is an *action of the understanding*, which we would designate with the general title *synthesis* ... (B130; my italics)

Concerning the latter point, Kant again claims quite explicitly that the means by which our understanding achieves the synthesis of the manifold in intuition is not some further intuition but is a 'function' which

<sup>14</sup> In this respect Bolzano's criticism of Kant prefigures both neo-Kantian intellectualist revisions to Kant's doctrine of intuition (cf. Friedman 2000, 31f. and 89f.), as well as more recent 'conceptualist' interpretations of Kant's views on intuitions (cf. Tolley 2013).

he associates with a 'pure' concept or category: 'The same function that gives unity to the different representations in a judgment also gives unity to the mere synthesis of different representations in an intuition, which, expressed generally, is called the pure *concept* of understanding' (B104–105; my italics).<sup>15</sup> Finally, Kant even seems to explicitly and directly reject the idea that such combination can arise from intuition itself, or from what is 'given' in them directly from objects:

[T]he combination (*conjunctio*) of a manifold in general can never come to us through the senses, and therefore cannot already be contained in the pure form of sensible intuition... Among all representations combination is the only one that is not given through objects... (B129–130)

These passages (and others) suggest to Bolzano that Kant, too, should agree that anything which involves combination or synthesis through concepts cannot be the *most immediate* representation of what is present in our mind due to the affection of our sensibility by an external object. For in order for such combination to be possible, Kant too seems to hold that we must first have simple representations of the elements in this manifold itself, with these representations being there prior to any combinatory or synthetic act by the mind.

Despite Bolzano's sense of accord with Kant on the premises for his conclusion that intuitions per se (in Bolzano's words: 'pure' (i.e. mere) intuitions) must have simple contents, Bolzano acknowledges that neither Kant himself nor his followers anywhere explicitly accept this conclusion:

[In Kant's writings] there is not even the faintest trace to be found of the other mark that a pure intuition (one mixed with no concept) must be a simple representation, neither in his work nor in that of any of the later philosophers who have adopted from Kant, and maintained, the distinction between intuitions and concepts. (NAK 50; cf. NAK 219)

<sup>15</sup> Already in his initial discussion of synthesis, Kant appears to allude to the necessary involvement of 'concepts [*Begriffe*]': 'By synthesis in the most general sense, however, I understand the action [*Handlung*] of putting different representations together with each other and *comprehending* [*begreifen*] their manifoldness [*Mannigfaltigkeit*] in one cognition' (B103).

What is worse, as we have already anticipated, Bolzano thinks that Kant's official position is actually incompatible with this conclusion. This is due to the fact that, as we saw in §2, 'Kant speaks very often of the manifold which lies in intuition' (NAK 50). Yet based on the passages here just cited, Bolzano thinks Kant himself actually accepts that

a manifold can be found in a singular representation *only if* it was first brought about through a combination of several simple representations, a combination that can *only* come about through the mediation of some representations which are not intuitions but are rather concepts. (NAK 50–51; my italics)

From this, Bolzano concludes that Kant's account of intuition as immediate and yet a unity of a manifold is not fully consistent.

#### §4.3 From simplicity to the rejection of form

Now, as was also seen above, what Kant officially takes to 'unify' the manifold of an outer intuition is not an act of the understanding or any concepts, but is *space itself*. This is because space itself is to function as the 'form' of the content of an outer intuition (a form of our sensibility), as something that 'orders' the 'matter' provided by sensation into a single whole, according to how such matter fills out the places or locations in space. What is more, as we also saw above, Kant argues that we must possess apriori a 'pure' intuition of this order-giving form itself, since it must lie ready 'in the mind' prior to the reception of any matter from sensation.

As we might now suspect, Bolzano takes his argument for the simplicity of the content of intuitions in general, along with Kant's own claims about combination, to also demonstrate the untenability of these aspects of Kant's analysis of pure intuitions in particular.

[A] pure intuition would be accordingly something essentially complex, would contain a manifold which it would combine into a certain unity. Yet at the same time, it is certain, and Kant himself acknowledges elsewhere..., that combination is a feat [*Werk*] of the understanding, i.e. comes about only through concepts. In this way, intuition would then be a complex representation that comprises concepts in itself as components. (NAK 52)

Having shown, then, that Kant himself should conclude that whatever is responsible for such a unity is something that necessarily involves

*concepts* (e.g. the concept <and>), Bolzano then challenges Kant to explain why this representation should nevertheless be called an *intuition*: ‘But is there really a point in taking the concept of an intuition in such a broad, vague signification?’ (NAK 52–53).

#### §4.4 Rethinking outer intuitions

This departure has the following important consequences for how Bolzano himself views the nature of our outer intuitions. Since the content of every intuition is simple, Bolzano himself must conclude that what an intuition contains cannot have *any form* whatsoever, in the sense of something which would unify its parts (matter). Intuitions contain no parts, and hence contain nothing at all to unify. But then, a fortiori, the contents of outer intuitions cannot all share *space* as their *universal and necessary form*. Conversely, if the content of intuitions is in fact simple, then they cannot ‘contain’ a manifold of sensations as the *matter* (component parts) unified by space as a form. Indeed, as we saw above (cf. §3.2), on Bolzano’s view, a single sensation corresponds to each outer intuition as its immediate *object* rather than as a component of its content, as Kant would have it. The content itself is, again, the wholly ‘simple’ representational correlate of the demonstrative ‘this’.

In fact, Bolzano thinks that Kant’s willingness to apply the term ‘intuition’ to the representation which would perform such a unifying function just shows that Kant does not have a clear conception of what an intuition is in the first place:

Kant’s claim, that the pure form of sensibility (i.e., that in the appearance which makes it so that its manifold can be ordered in certain relations) can also be called pure intuition, shows especially how vacillating the concept which Kant himself connected to this word must have been. (NAK 52)

Since no intuition can contain anything that unifies *at all*, there can be no intuition which has a mere ‘form’ of unity as its content. But then there can be no pure intuition of space in particular as an intuition of the form that unifies each outer appearance.

The absence of form in each outer intuition forces Bolzano to conclude that no outer intuition contains *any* spatial representation or spatially extended content (such as distance) whatsoever:

[T]he distance between the tips my two fingers is and can be an object of my intuition just as little as... the distance between this fixed star

and the disc of the moon that floats above it. What we intuit are merely certain colors, sounds and tones etc. and we conclude from their presence, often with many steps in between...that certain objects are present that bring about these representations in us, and which stand in these or those spatial relations to each other. (NAK 95–96)

Here even more explicitly than before, we see Bolzano insisting that the spatial relation to an external object comes into our representations *only* with an act of *inference* (a ‘conclusion’), rather than being already present in the immediate sensible representation itself – i.e. in the intuition.

### §5 Bolzano against Kant on intuitions of ‘ideal’ (non-actual) objects

While this disagreement over the simplicity of intuitions surely provides *one* key platform for Bolzano’s criticism of Kant’s doctrine of the pure intuition of space, Bolzano takes himself to have a *second* platform from which to criticize Kant’s doctrine. What is more, just as in the previous case, Bolzano takes this second platform to be grounded in commitments that Kant himself ultimately shares. In fact, as we will see in this section, Bolzano actually thinks that he can marshal an argument against Kant’s doctrine of the pure intuition of space *on the basis of their shared commitment to the ideality of space itself*. For, as Bolzano sees it, Kant must not have thought through what his commitment to the ‘immediacy’ of intuitions actually requires, and concludes that Kant must have lost sight of this commitment when he tries to introduce the possibility of an intuition of *non-actual*, ‘ideal’ object, like space.

#### §5.1 The actuality of the object of every intuition

Given the conditions on being an intuition that we have set out above (that the representation give its object immediately, involve sensation, and have a simple content), Bolzano concludes that all human intuitions must have ‘*actual things*’ as their objects:

It seems to me that this already follows from the mere concept of an intuition alone, as a simple singular representation. For if a representation is to represent merely one object, despite all of its simplicity, then it must have something so peculiar (something exclusively relating to only this object) that its arising in our mind can hardly be explained in any other way than through the assumption that it

is related to this object as an effect is to its cause. From this it follows at once that this object must be something actual, since as a cause it should show itself to be efficacious [*wirksam*]. (WL §74, I.331)

And as we have also already anticipated, here again Bolzano appears to think that Kant implicitly accepts this condition, something Bolzano thinks is indicated by Kant's aforementioned talk of intuitions 'giving' their objects 'immediately':

[B]y intuitions, Kant has always thought of just those representations that refer to one single and indeed actual object. ... [H]e has always thought by an intuition a representation from whose presence we are at once justified in inferring the presence of an object which corresponds to it (which is represented through it). For it is obvious that this is just what he wants to tell us by the phrase: intuition gives us the object, gives it immediately, and indeed no object can be given to us in any other way (at least immediately). (NAK 48–49)

Bolzano is confused, then, as to why Kant would want to call a representation of a *non-actual* or *ideal* object like space an intuition, since by Kant's own lights – and, as we will see now, by Bolzano's, too – space is not an actual object that can ever actually be present to our senses.

### §5.2 The non-actuality (ideality) of space

Bolzano gives several arguments for why space is not something 'actual' [*wirklich*]. One key argument concludes that space (and time as well) is not something actual from the fact that space itself is not 'active' in the sense that it brings about no effects:

I ask anyone who knows what mathematicians understand by the words time and space, whether he must not concede that only the objects that are found in time and space are something actual, but not the times and spaces themselves. And if someone wanted to define time and space as something actual, then he would have to claim that they also have certain effects [*wirken*]. And what could these be? (WL §79.2, I.362)

In addition to this argument from the non-efficaciousness of space, Bolzano gives several further arguments for the ideality of space, many of which echo those Leibniz gives in his correspondence with Clarke.



One argues in the form of a dilemma about whether space is changeable or not:

If time and space were something actual, then their actuality would have to be one of the two, either unconditioned or conditioned [*unbedingt oder bedingt*]. In the first case they would be God, in the second, they would be created things that are subject to change. Now, nobody can really say either that space and time are God himself or that they are subject to change, since only the things that are in time and space change but not time and space themselves. (WL §79.2, I.363)

Another argues for space's ideality from the non-equality of every actual thing:

If time and space were something actual, then no two moments or durations, or two points or distances could be exactly equal [*gleich*] to one another, since among actual things there are not two that equal one another exactly. But this is quite contrary to the concepts that mathematicians have about these objects. (WL §79.2, I.363)

A third argues for space's ideality from the equality of all of its parts in conjunction with the principle of sufficient reason:

[I]f two moments or two points are exactly equal (as has been asserted by all mathematicians for eternity), and if time and space were something actual, then the existence of a thing at this determinate time and this determinate place would have to be something actual that has no ground. For there would be entirely no ground for why this thing should be in just this particular state at this particular time and place and no other, not just none for us humans to give, but there would be none available in itself, since these places and times are completely equal internally [*innerlich völlig gleich*]. (WL §79.2, I.363)

And Bolzano accepts, as does Kant, that, ontologically speaking, what is true of the parts of space is true of space as a whole:

If no individual moment and so also no individual point is, for itself, something actual, then neither can we hold the collection of all moments, i.e., the whole infinite time, nor the collection of all points, i.e., the whole infinite space, to be something actual. (WL §79.3, I.364)

### §5.3 The impossibility of intuiting space

Yet while Bolzano therefore shares Kant's views that space as a whole is not an 'actual [*wirklich*]' object, Bolzano takes this to entail that there can be *no intuition* of space, whether of its parts (places, distances) or as a whole. As we saw above, Bolzano holds that *all* intuitions must have something actual as their object – whether a sensation or another representation or judgment:

Recall that every (subjective) intuition that appears in our mind must have an existing object. This gives us a means for proving that the aforementioned representations [of space and time] are not intuitions, and indeed without having to decide anything about whether they are complex or simple, merely on the ground that the objects that are represented through them since the objects they represent are not at all something actual (something existing). (*WL* §79.2, I.362)

Note that this argument is supposed to proceed independently of whether the representation of space is something *simple* or complex.

Bolzano also takes his conclusion to be independent of the fact – which he too acknowledges, again, in agreement with Kant – that space itself, as a whole, is an individual object, and so its representation will be *singular*. For even though space is an individual, it is still not actual:

If no individual moment and so also no individual point is, for itself, something actual, then neither can we hold the collection of all moments, i.e., the whole infinite time, nor the collection of all points, i.e., the whole infinite space, to be something actual. And thus also neither of these two representations [of time and space] can be called intuitions, even though both of them have only one single object (since there is only one infinite time and only one absolutely infinite space). (*WL* §79.3, I.364)<sup>16</sup>

Since they are not intuitions, Bolzano concludes that these representations must be *concepts* (cf. *NAK* 58f.).<sup>17</sup> What is more, Bolzano thinks

<sup>16</sup> See *NAK* 59f. for a review of all of these arguments for the ideality of space.

<sup>17</sup> Bolzano adds the following further argument for the representation of space as a whole being a concept rather than an intuition, which picks up on the Leibnizian points made above: '[J]ust like *Leibniz*, I find a proof that space is not actual, and that its representation is therefore not an intuition, in the fact that we cannot determine a single point in space through mere concepts, since they are all completely equal to each other' (*WL* §79.6 Anm, I.375).

that this is a conclusion Kant himself can and should accept. To be sure, Bolzano recognizes that at times Kant says things that suggest he thinks that only intuitions can be ‘singular’ representations, in the sense of representing individual objects (cf. B376–377). Bolzano also thinks, however, that this cannot be Kant’s considered view, because Kant accepts (cf. B596, B603) that our representation of God is both a representation of an individual and (obviously) not an intuition (cf. *NAK* 65).

#### §5.4 Spatial representations as conceptual ‘determinations’ of actual objects

Because it represents something non-actual, Bolzano concludes that the basic representation of space must be a concept, rather than an intuition. Even so, Bolzano still retains a key part of the spirit of Kant’s account of the nature of space itself, insofar as he accepts that space is ideal rather than actual. This raises a question about whether Bolzano is also an idealist about space in the further sense that he thinks that space has no existence (or rather, subsistence) independently of the representations of it.

As I will now show, Bolzano does in fact seem to accept that the distinctive role spatial representations play within our representations of external objects is one that contrasts in a very important way with the role played by other conceptual representations, and that it contrasts in a way that does seem to share a core affinity with Kant’s thesis of the dependence of space upon spatial representation.

Bolzano takes the class of conceptual representations of objects to be one which includes both representations of actual ‘*properties* [*Beschaffenheiten*]’ of objects, but also other representations that function to direct our representations to objects but do not themselves represent properties of these objects. To count as a representation of an actual property of an object, Bolzano holds that the conceptual representation must be able to occur as the *predicate*-representation in a proposition:

The representation that appears in the place of b [in: A has b] (the predicate-representation) must, if the proposition is to be true, always be a genuine [*echte*] representation of a property; and conversely, every genuine representation of a property must be able to function as the predicate-representation in a true proposition. (*WL* §80.2, I.380)

Other conceptual representations, however, help to ‘determine [*bestimmen*]’ which object the proposition is about (i.e. the object of

the subject-representation) in a fundamentally different way – namely, by qualifying only the *subject*-representation in ways that do not ultimately track any real or actual properties of the object itself, and, in fact, by being representations that can never function as predicate-representations:

[T]here are representations that serve for the determination [*Bestimmung*] of an object without being properties of it. These representations have the peculiarity that they can never occur in the place of the predicate-representation (b) but only as parts of the subject-representation (A) itself. (*WL* §80.2, I.380–381)<sup>18</sup>

For our purposes, what is relevant about this distinction is that, immediately after introducing it, Bolzano goes on to state explicitly that the representations of space (and time) belong to the class of mere determinations *rather than* representations of properties:

Of this sort [i.e., mere determinations] are especially the spatial and temporal determinations of existing things. For the time in which an actual thing is to be found, during which a certain property can with truth be attributed to it, *is not a property of this thing*, and, for this reason, the representation of this time does not occur in the predicate-, but in the subject-representation of the proposition. The same thing holds also of the determinations of the places of things. (*WL* §80.2, I.381; my italics)

When viewed in light of Bolzano's commitment to the ideality of space (and time), it becomes clear why Bolzano wants to distinguish what function spatial representations have in a proposition about actual things from the function of other conceptual representations. Since

<sup>18</sup> Bolzano actually uses the term 'determination' to pick out the 'broader concept' which includes both the representation of a property of the object and 'mere' determinations: though all representations of properties also determine the objects of the subject-proposition (when the proposition is true), not all determinations of objects are representations of properties of the object (*WL* §80.2, I.380); some are mere determinations. For some scepticism about whether Bolzano is consistent throughout all of his works in his usage of 'determination' – see Schnieder 2009, 58f.; cf. Morscher 1973, 73f. I am assuming here only that he is consistent across these few sections of the *WL* (§§79–80), and that the same usage is in play in the *NAK*. For scepticism about whether the resulting view of temporal representations is consistent, see George 1987, 454f.



these representations do not represent anything actual, they cannot represent actual properties of the object in question.

Nevertheless, Bolzano does accept that using spatial (and temporal) determinations in the subject-representation is a necessary condition for counting as true the 'ascription' of certain predicates to the relevant actual object – i.e. a condition for asserting, thinking truly about actual objects. Here is Bolzano making this point about temporal determinations:

[W]e place everything that is actual, perhaps with the exception of the single being of divinity, in a certain time; and *if we want to ascribe [beilegen] a property with truth of something actual, then we must always add* a certain time at which this property is supposed to pertain to it. (WL §79.5, I.364–365; my italics)

If we examine the matter more closely, it becomes apparent, as I believe, that by the word *time* we think nothing other than that particular determination [*Bestimmung*] of something actual which is the condition which must take place *so that we can ascribe* a certain property in truth. (WL §79.5, I.365; my italics; cf. NAK 57)



And here is Bolzano making the same point concerning space:



As concerns the concept of space, it will be admitted first of all that by space in general we represent nothing other than the collection of all possible locations, and so the only question is what we think of as the locations of things... [L]ocations of (actual) things are those determinations [*Bestimmungen*] of these things that we must think in addition to their forces in order to comprehend [*begreifen*] the changes that they bring about in one another. (WL §79.6, I.365–366; cf. NAK 58)

What we find, then, is Bolzano ascribing a hybrid nature to spatial representation, one that brings them much closer to the role they have in Kant's system. Despite not being sensible representations (intuitions), Bolzano's spatial representations are nevertheless like Kant's in that they serve only to help pick out the subject of judgments without actually representing features that that object has 'in itself'. In this sense, for Bolzano as well, the representations of space and spatial relations have only an 'internal' role to play, laterally, within the context of other representations, rather than serving to represent anything in the actual



world as it is in itself. In this further sense, too, Bolzano would seem therefore to be an idealist about spatial representation.<sup>19</sup>

### §6 Conclusion: exploring replies on Kant's behalf

We have thus seen, first, that Bolzano means for his doctrine of *outer intuition* to agree with much of Kant's official published account of intuitions. Like Kant, Bolzano, too, takes intuitions to be singular representations that give their objects immediately, and are related to external objects that we represent through spatial representations. We have seen, second, that Bolzano joins Kant in taking *spatial representation* itself not to track any actual properties in the outer objects themselves.

Against Kant, however, Bolzano thinks all spatial representation is *conceptual*, because all intuitions have *actual* objects, and space is not something actual. Furthermore, Bolzano holds, against Kant, that all outer intuitions are *non-spatial*, since they have entirely *simple* contents, because they immediately represent inner sensations as objects, rather than being representations that 'contain' an infinitely divisible 'manifold' within themselves. All of these differences lead directly to Bolzano's rejection of Kant's account of the nature and structure of outer intuition, as well as the grounds for Kant's postulation of an allegedly pure (*a priori*) intuition of space itself, since Bolzano takes himself to have shown that the representation of space is actually *not* a condition for, or a constituent in, outer intuition (cf. §2.1).

Nevertheless, we also saw that these departures from Kant's views are departures that Bolzano takes to be required by commitments that he and Kant both share. What we should turn to now, in conclusion, is what Kant might say in response to such challenges, especially Bolzano's claim that they arise from within the Kantian framework itself.

#### §6.1 Kant on unity without synthesis

A key step in Bolzano's argument against intuitions containing complexity was his thesis that no representation which has a content that is composite could be an immediate representation, because its arising would depend on a further intellectual act of mind which would synthesize or combine the relevant manifold of parts into a whole. What is more, Bolzano takes Kant to likewise hold that all combination

<sup>19</sup> The possibility of this deeper parallel with Kant is touched upon by Palagyi 1902, 110f.; cf. as well Benoist 2003, 147–148.

or synthesis is the result of a further act of our understanding, and so is not present in what is given. And as we saw above, Bolzano is able to point to several texts which push in this direction.

What is not captured in Bolzano's reporting of Kant's views, however, is the fact that Kant appears to posit a *separate* kind of unity or belonging-together that he explicitly distinguishes from the kind of combination or synthesis that he assigns to the activity of understanding. This is the unity that accrues to the sensible manifold prior to any synthetic or combinatory activity of the understanding, a unity that consists in what Kant calls the '*synopsis of the manifold a priori through sense*' (B127; my italics). Though this synopsis accords a unity to intuition that 'corresponds to' the one which results from synthesis (combination) by the understanding (A97), it is present *prior to* such activity, as a kind of primitive and 'absolute unity' that every intuition has in the 'moment' it arises (A99). It is a primitive kind of seeing altogether (*syn-opsis*), all at once, though not in the way that the mind 'runs through and takes together' the various parts of this manifold and so becomes conscious of them as different (A99).<sup>20</sup>

If this is right, then Kant would simply seem to reject Bolzano's claim that every representation that contains parts (is composite) requires a separate act of understanding which unifies the relevant manifold of parts by means of concepts. With the synopsis of the sensible manifold, we have a mental content which contains parts (a manifold) and yet that arises immediately, in each moment, without any act of combination of synthesis by our understanding.

### §6.2 Kant on phenomenal presence without actual objects

It is even more clear that Kant cannot accept Bolzano's condition that all intuitions must have *actual* things as their objects. Indeed, this condition is so straightforwardly opposed to the possibility of the intuition of ideal things, it is no wonder that Bolzano cannot find a place where Kant states it explicitly. In fact, Bolzano himself (and Přihonský too) cites a place where Kant explicitly states the opposite view – namely, in Kant's initial exposition of the notion of a pure intuition in the Aesthetic itself: '[P]ure intuition ... occurs *a priori*, even without an actual [*wirklich*] object of the senses or sensation' (B35; my italics). What is especially bizarre is that, just one page after quoting the above passage from B35 (NAK 47), we are told that 'by *intuitions*, Kant has always

<sup>20</sup> For more discussion, see Tolley 2013; cf. Allison 2004, 14–15.

thought of just those representations that refer to *one single* and indeed *actual* object' (NAK 48).

Nor is this the only place where Kant makes such a claim. In fact, in *Prolegomena* §8, *Kant himself* takes up this worry about the possibility of an intuition without an actual object as a possible objection:

How is it possible to intuit something *a priori*? An intuition is a representation of the sort which would depend immediately on the presence [*Gegenwart*] of an object. It therefore seems impossible originally to intuit *a priori*, since then the intuition would have to occur without an object being present, either previously or now, to which it could refer, and so it could not be an intuition. (4: 281–282)

To this Kant gives the same reply in the next section (*Prolegomena* §9) that he gives in the Aesthetic:

There is therefore only one way possible for my intuition to precede the actuality [*Wirklichkeit*] of the object and occur as an *a priori* cognition, namely if it contains nothing else except the form of sensibility, which in me as subject precedes all actual [*wirkliche*] impressions through which I am affected by objects. (4: 282)

### §6.3 Kant on the phenomenology of our sensible representations of external objects

If this serves as a textual defence for Kant to be able to escape charges of incoherence, what should we say about the competing positions themselves? Which of the two accounts fits better with the phenomenology of our sensible representations of external objects? On Kant's account, we are immediately and sensibly aware of contents that have spatial dimensionality (extension) and are composed of a manifold of sensory qualities like colour, texture, and so on. These are the appearances of objects external to us. To this extent, the content of an outer intuition (an outer appearance) is, for Kant, something like a (partial) image of the 'something = x' which has brought about its intuiting.<sup>21</sup>

On Bolzano's account, by contrast, we are immediately and sensibly aware only of single sensations of colour, etc., through simple and

<sup>21</sup> Cf. George 2003, 26. We have to qualify the sense in which an appearance is an image, for Kant, due to Kant's own usage of the term 'image [*Bild*]' to pick out something formed out of appearances by the 'imagination [*Einbildungskraft*]'; cf. A120.



direct contents. They are not themselves image-like depictions, because strictly speaking the contents of outer intuitions have no form whatsoever, but are absolutely simple. Any consciousness of a composite of these contents, in an ostensibly spatial array, owes its spatial aspects not to something sensible but to concepts, with the resulting spatial array itself also not being something sensible in the strict sense but a unity of contents woven together conceptually or intellectually.

This also helps point up the difference in Bolzano's position on the representation-internal role that spatial representations are to play in our representations of outer objects. This is not a role that is played *within* the direct immediate sensible representations that are associated with external objects – i.e. *within* outer intuitions. For the representations in question are subject-concepts, and moreover seem to occur only within propositions, as contents of judgments (inferences).

What, then, is the truth about our most immediate sensible representations? Studies of so-called 'early vision', for example, suggest that it at least takes the form of a two-dimensional extended array, bracketing whether or not (as Berkeley had suggested many centuries prior) the representation of depth only comes at a later stage and incorporates conceptual representations, judgment, inference, and so on.<sup>22</sup> This might be taken as a first bit of partial evidence that our most primitive external sensible awareness is spatial.

There is a further question, however, of whether Bolzano's position is itself internally coherent. For even Bolzano's own account of the genesis of our concepts of space would seem to rely on *some* acceptance of the idea that intuitions have a common form in which certain material can vary according to different magnitudes. The variable magnitude that is most important for Bolzano pertains to the quality of the intuition, i.e. its 'liveliness': 'We admit that every *subjective intuition*, indeed every representation in general, has a magnitude in respect to its *duration* as well as its *liveliness*' (NAK 95). It is precisely from noticing the variations in this quality that Bolzano thinks we first form the concepts of something being present, absent, closer, and farther from our sense-organs, and so is that upon which he bases his own genetic account of the formation of our basic concepts of spatial representation (cf. *WL* §303.21, III.151f.). But for this to be true, Bolzano would seem to have

<sup>22</sup> See Grush 2007. See George 1987, 464f. for worries about Bolzano's parallel account of the formation of the representations of time by way of inference or judgment rather than by way of a primitive aesthetic awareness, as Kant suggests.

to posit something as a background against which such variation can be noticed – i.e. something quite close to a common form in which such variation in intensity occurs.

Whether there is any ultimate or inescapable inconsistency here would require further research into Bolzano's genetic account. And even if Bolzano's account were shown to be inconsistent, this would not by itself be a vindication of Kant's claim that there is a primitive, *sensible*, 'given' space of appearances. Even less would it speak for the plausibility of the remainder of Kant's views on space, especially those concerning our ability to immediately, sensibly, and yet 'purely' represent space per se, *a priori*, or those concerning the necessary role that this intuition is supposed to have to play within geometry.

Still, seeing what sort of picture emerges of ordinary outer intuitions, once pure intuition is rejected, might be enough to give one pause and explore once again a more Kantian alternative.

\* \* \*

In the foregoing I have aimed to bring into focus several dimensions of Bolzano's views on outer intuition that have not yet received sufficient attention, with the goal of coming to a better understanding of his own accounts of both spatial and sensible representations. What is more, the path we have followed gives us a new angle into the debate about the nature and origin of spatial representation, one that proceeds largely independently of Bolzano's views on the nature of geometrical knowledge – and, for that matter, independently of those of Kant's as well. It also allows us to see the extent to which the commitment to idealism about space (something Bolzano and Kant share) can swing free not just from one's commitments about geometrical knowledge, but also from one's views on the nature of outer intuition as well.

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