FORMS OF CORRESPONDENCE: THE INTRICATE ROUTE FROM THOUGHT TO REALITY*

Gila Sher
UCSD

Abstract

The paper delineates a new approach to truth that falls under the category of "Pluralism within the bounds of correspondence", and illustrates it with respect to mathematical truth. Mathematical truth, like all other truths, is based on correspondence, but the route of mathematical correspondence differs from other routes of correspondence in (i) connecting mathematical truths to a special aspect of reality, namely, its formal aspect, and (ii) doing so in a complex, indirect way, rather than in a simple and direct way. The underlying idea is that an intricate mind is capable of creating intricate routes from language to reality, and this enables it to apply correspondence principles in areas for which correspondence is traditionally thought to be problematic.

Key Words: *truth, substantiveness, unity, diversity, pluralism, correspondence, indirect correspondence, mathematical truth, mixed inference.*

I. How to Maximize the Substantiveness of Truth without Minimizing Its Unity

My starting point is the observation that truth is a substantive and complex subject-matter, playing an important role in many areas of human life, and the object of multiple human interests. As such, it is unreasonable to expect that it could be adequately accounted for by a single and simple definition, definition schema, or necessary-and-sufficient condition. But many philosophers do equate the possibility of a substantive theory of truth with that of a substantive definition. It is not surprising, therefore, that the prevalent attitude toward truth is deflationist:

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Truth is not a substantive subject-matter since it resists a substantive definition. Truth, however, is not the only substantive subject-matter to resist a substantive definition, and the rational response, in most cases, is not to forego substantive theorizing. The theorists of truth, in my view, should learn from their colleagues in science and mathematics. Instead of insisting on one substantive comprehensive principle, they should be open to the possibility of a complex network of such principles. The task is to unravel the structure of this network, identify its general and special principles, and formulate a theory that fruitfully balances its attention to unity and diversity.[1]

Among the first to connect the substantiveness of truth with its plurality in a systematic manner was Crispin Wright in *Truth & Objectivity* (1992). Wright suggests that the substantiveness of truth lies in the variety of ways the (one) predicate of truth is instantiated in diverse areas. Accordingly, he divides the theory of truth into two parts: a part that deals with the common features of truths, and a part that deals with the specific principles underlying truth in particular areas. His underlying assumption is that the principles governing all truths are thin, obvious, and often trivial; hence, their account is minimalist. In contrast, the principles underlying the specific types of truth are thick, nonobvious, and nontrivial; hence they require substantive theorizing. The general principles of truth are captured by one-line *platitudes*:

> [T]o assert is to present as true;
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> [...] any truth-apt content has a significant negation which is likewise truth-apt;
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> [...] to be true is to correspond to the facts;
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> [...] a statement may be justified without being true, and vice versa;

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[1] This is a "substantivist" approach to truth, one that regards truth as a substantive subject-matter and requires the theory of truth to provide a substantive account of this subject-matter. By 'truth is a substantive subject-matter' I mean that truth has a rich, complex, and intricate nature or structure (of some kind). By 'the theory of truth is a substantive theory' I mean that it provides a theoretical, informative, systematic, and explanatory account of the nature and structure of truth.

[...]. [Wright 1992: 34]

In contrast, the special principles of truth in different areas of discourse are those elaborated by the different substantive theories of truth we are familiar with: correspondence, coherence, super-warrant, etc.[2]

Wright's approach has been further developed by Michael Lynch (2001a; 2004; 2005a; 2009). Lynch turns Wright's pluralist theory of truth into a *functionalist* theory, modeled after functionalist theories in the philosophy of mind. This approach enables him to sharpen the pluralist analysis of truth: Truth is a single, high-level concept, defined by its functional role.[3] This role may be fulfilled by different properties in different domains. Truth supervenes on these properties but is not reducible to them. We may say that truth is differently realized, or has different natures, in different domains. Following Wright, the general role of truth is accounted for by a list of platitudes, its specific realizations by the substantive yet radically diverse principles of correspondence, coherence, super-warrant, and so on.

The differences between the correspondence, coherence, and super-warrant conceptions of truth are, however, so radical as to result in a highly *disunified* theory — a theory in which what it is for, say, a physical statement to be true is altogether different from what it is for, say, a mathematical statement to be true. One manifestation of this disunity is the problem, raised by Christine Tappolet (1997), of mixed sentences and inferences: Consider a conjunction of, say, a correspondence truth and a coherence truth. Such a conjunction is true. But what kind of truth is it? Or consider a logically valid inference with true premises, some of the essential premises of which are merely super-warranted, and a conclusion whose truth is based on full-fledged

[2] As I understand him, Wright treats the platitude of correspondence as a nonsubstantive statement but theories of correspondence for specific fields (e.g., a theory of physical correspondence) as substantive theories.

[3] In his 2009 book, Lynch no longer characterizes truth as a high-level concept, but he continues to characterize it as a single concept defined by its functional role.

correspondence. How can such an inference guarantee the correspondence-truth of the conclusion based on the mere super-warrant of essential premises?

Another problem with Wright's and Lynch's pluralism concerns their claim that the universal principles of truth are platitudinous. Methodologically, the most problematic aspect of this claim is the presumption that with respect to these principles there is no room for further study, let alone a deep, thorough, and comprehensive investigation. But this presumption, as far as I can see, has never been justified. Recognizing the *plurality* of truth means recognizing the *partiality* of its commonalities, i.e., recognizing that the common principles of truth can provide only *partial* knowledge of truth. But partiality does not imply nonsubstantiveness: some partial principles are substantive (some substantive principles are partial).[4]

The solution to these problems lies, in my view, in the realization that on the one hand diversity comes in degrees, and on the other hand substantiveness is compatible with partiality. Recognizing the diversity of truth, therefore, need not involve commitment to a radically disunified theory of truth, and recognizing the partiality of the universal principles of truth need not involve commitment to their triviality. If, instead of viewing truth as based on correspondence principles in one area and on coherence principles in another, we view it as based on correspondence principles in all areas, yet some of these principles as possibly varying from area to area, we will achieve a better balance between unity and diversity in our conception of truth. And if, instead of viewing truth as based on *trivial* global principles and *substantive* local principles, we view it as based on a network of *substantive* principles, some more global, others more local, we will not sacrifice the substantiveness of truth by acknowledging its diversity. That is the solution I offer in this paper: truth is *correspondence* throughout, but correspondence is a *family* of substantive and interconnected principles rather than a single, monolithic, deflationary

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[4] For critical yet sympathetic discussions of current pluralist approaches to truth see Cory D. Wright (2005; 2006; 2010).

principle.[5]

What is meant by correspondence? By the view that truth is correspondence I understand something like the following:

(COR) *Truth is a matter of substantive and systematic connections between language and the world. These connections hold in a particular case if and only if (iff) that aspect of reality which a given sentence or theory is about is, directly or indirectly, and based on some pertinent principles (according to the type of case), as the sentence or theory says it is.*[6]

This is not intended as a *definition* of correspondence. Instead it is intended as an informal characterization, one that can be used as a guideline for an elaborate, substantive, and multi-faceted theory of truth as correspondence.

Why correspondence? In Sher (2004) I argued that truth emerges at the intersection of three features of, or conditions on, human thought: *immanence*, *transcendence*, and *normativity.* By 'immanence' I understand the cognitive stance we take when we think or speak from *within* a theory (in the sense of a body of knowledge), i.e., the stance of thinking directly about the world, about some facet of the world, or about something in the world. To be immanent, in this sense, is to be directed at something factual, usually external[7] — to place no barrier between us and our subject-matter, perceived as part of reality. Bearers of truth, according to this account, are *immanent* thoughts of some type (statements, beliefs, theories, or similar entities).

Immanent thoughts by themselves, however, are not sufficient for truth. To focus directly

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[5] For earlier renditions of this approach see Sher (1999a; 2004). To avoid repetition, I will not discuss here many of the issues raised in these papers.

[6] (i) 'World' and 'reality' are used as synonyms in this paper.
(ii) The idea of indirect correspondence was earlier suggested by Terence Horgan (2001) and Barnard & Horgan (2006). See also "The Synthetic Unity of Truth" (this volume). My view is similar to theirs in some ways, different in other. For example, my account of logical and mathematical truth is very different from Horgan's. (See below.)

[7] 'External' here implies '*significantly* independent of the mind', but not '*completely* independent of the mind'. The idea is that X can be significantly independent of Y in some pertinent respects and significantly dependent on it in others.) (See Section II below.)

on the world is not yet to see it through the prism of *truth.* Truth requires a *transcendent* perspective, a perspective from which we can view both our immanent thought and the world, or rather those facets of the world it is directed at. It is only at this level, a level in which we observe an immanent thought from outside it, that truth, as a property of, or more precisely a standard for, such a thought, can arise. It is important to note, however, that "transcendence", here, is not something mysterious or super-human. Rather, it is something quite simple and humanly commonplace, something on the order of *ascending to a meta-language* or *moving sideways to a background theory,* i.e., taking a perspective external to the immanent thought we are examining, a perspective from which we can ask questions and say things about various aspects of this immanent thought, including its object.

Immanence and transcendence by themselves, however, are still not sufficient for truth. By ascending to a higher level of discourse we can ask many kinds of questions about immanent thoughts, not just questions of truth. We can even ask many questions about their relation to the world that are not questions of truth. For example, we can ask whether a given sentence expressing an immanent thought names a certain object by a word whose sound imitates that object's sound (onomatopoeia), whether a given sentence describes a given situation briefly or at length, whether two sentences describing the same situation are synonymous, whether the objects a given theory is about are animate or inanimate, observable or unobservable, etc. *Truth* arises when we ask a *normative* question about immanent thoughts, and not just any normative question but a specific normative question, namely, the question whether things are, or the world is, as a given immanent thought says they are (it is). I.e., truth emerges when we ask whether a given immanent thought (statement, theory, etc.) "measures up" to reality. I will call this 'the question of *truth*'.[8] At issue is whether a given statement or theory is systematically connected to reality in a way that justifies a positive answer to the question of truth. Truth, on this conception, is a

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[8] I am talking about "the" question of truth here, but I could alternatively talk about a "cluster" of questions of truth, questions that raise, in one way or another, the issue noted above.

standard for a positive answer to this question, a standard satisfied by some immanent thoughts but not others. When a given immanent thought satisfies this standard we say that it is *true*, or that it has the property of truth.[9]

This brief account of the basic conditions for the emergence of truth is far from complete. Indeed, it is partly in elaborating and extending this account, attending to the issues it raises, and connecting it to parallel accounts of the emergence of language and knowledge, that the challenge (and opportunity) of a substantive account of truth, including a substantive theory of the universal principles of truth, lies. Now, it is clear that if this conception of the basic conditions of truth points in the right direction, truth is *correspondence with reality* (in the sense of COR above), rather than coherence or super-warrant. It is for that reason that my proposed solution to the tension between the unity and diversity of truth lies in *correspondence.* The solution lies in realizing, first, that truth is based on *correspondence* principles (*unity*), and second, that it is based on a *network* of correspondence principles (*diversity*).

II. Pluralism within the Bounds of Correspondence

One remarkable thing about truth is its enormous scope. Every declarative statement, it seems, is a candidate for a truth value, regardless of its content or the field of discourse it belongs to. The account of truth given in the last section explains why this is so. The question of truth, indeed correspondence truth, arises with respect to every immanent thought, and the domain of immanent thoughts encompasses all fields of knowledge and others besides. Given the immense scope of immanent thoughts and our unlimited ability to transcend them and raise the question of correspondence truth with respect to them, there is a sense in which we cannot get away from

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[9] This tripartite structure of truth is one of the distinctive characteristics of the present correspondence theory. Transcendence, for example, may not be required by other conceptions of correspondence. (It is required, however, by Alfred Tarski's 1933 theory, where truth is essentially a meta-theoretic notion.)

correspondence truth altogether no matter where we go.

Suppose you and I stand in meta-arithmetic, or in meta-meta-arithmetic, and you say: 'The truth of arithmetical statements is purely conventional'. I.e., you look at the totality of arithmetic statements and deny that their truth value is based on correspondence. Now, this itself is an immanent claim, a direct factual claim about some subject-matter, and I am free to ascend to the next level in the meta-theoretical hierarchy and raise the critical question: 'Is arithmetic truth in fact conventional?' with respect to it. Of course, you are free to refuse to answer my question (as you are free to refuse to answer any question), but the question all the same arises. Suppose you say: 'It is more convenient (simple, efficient, fruitful) to think of arithmetic as conventional than as true based on correspondence'. This gives rise to the truth question: 'Is it in fact simpler, more efficient, more fruitful, to think of arithmetic truth in this way?', and so on. So long as you make an immanent statement, the question of truth arises with respect to your statement. The ubiquity of truth is, thus, the result of the ubiquity of immanent thought together with our freedom to transcend any immanent thought and raise the critical question of whether it measures up to reality (or to that aspect of reality it is directed at). Transcendent claims, too, are for the most part immanent; in particular truth claims are immanent, and as such subject to the question of truth. We may say that whatever level of immanent thought we reside in at a given time, truth-as-correspondence arises at the next level of thought, a level that we are always free, and able, to ascend to.

This said, the question still arises whether the same correspondence principles are at work in all areas of truth. Are the standards of measuring up to reality the same for all truths? Are all true sentences connected to reality in the same way? Do they all correspond to the same "level" of reality? Are the routes or patterns of correspondence the same in all fields? At question is not whether all true sentences correspond to the same thing. Clearly, different sentences (up to synonymy) say different things about reality and therefore different things have to hold for different sentences to be true. That much is trivial. At issue is the nontrivial question whether the *principles* underlying correspondence in one area are systematically different from those in other areas. Suppose that in physics reference (an important constituent of correspondence) is largely based on a direct *causal* relation between physical expressions and objects in the world. Must reference in mathematics be based on a direct causal relation between mathematical language and the world? Suppose the existence of physical individuals is necessary for the truth of physical statements. Is the existence of mathematical individuals necessary for the truth of mathematical statements? These questions, I believe, cannot be answered by platitudes. We cannot decide in advance how truths in various areas are connected to reality, or what facets of reality they are connected to. Investigating these issues requires looking deeply, thoroughly, and critically into the matter. This investigation, however, is not a task for the theory of truth alone: to understand how truth is connected to reality in a particular area we need to understand the area in question. But it is not a matter for the specialist in this area to investigate alone either. Answering these questions requires an understanding both of truth and of the area we are interested in, and it is out of this joint understanding that the pattern of correspondence in that area will become known.

There is much more to say about the general conception of a theory of truth delineated above, but space constraints limit me in doing so. Therefore, I will limit myself to listing a few of the distinctive features of this conception:

1. It does not require either that truth be *exclusively* a matter of how the world is, or that the world's contribution to truth be *completely independent* of that of the mind. Instead, it requires that truth be *significantly* a matter of how the world is, and the world *significantly* independent of the mind.[10]

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[10] In this respect, the present theory, at least initially, is compatible with several views of "world", i.e., several metaphysical outlooks. My method is to start from more general and open-ended views and proceed to more definite views, so a reader can agree with the former but disagree with the latter. In this paper I will go as far as saying that there is just one reality and it has both physical and formal features, or more generally, both material and abstract features. This implies that neither extreme empiricism nor extreme Platonism (the view that there are two separate realities, material and abstract) is compatible with the present theory in its more definite form.

2. It does not determine in advance what the correspondence connection between truths and reality is, but takes this as an open question, the target of an open-ended investigation. In particular, it is not committed to any of the existent conceptions of truth as correspondence: the picture theory of correspondence, the museum metaphor of correspondence, the isomorphism view of correspondence, etc.

3. While it allows the correspondence connection between truths and reality to differ from one field to another, to be direct in some fields and indirect in others, to be affected more or less by context (purpose, interest, perspective, etc.), it leaves it an open question how it is in particular fields and particular contexts.

4. It admits units of truth of various sizes, from single statements to full theories and even clusters of theories.

5. It does not take a definite stand in the controversy on the bearers of truth (aside from the point indicated in (4)).

6. It is a holistic conception. It is holistic not in the sense of taking our language or body of knowledge as a whole to be the smallest unit of semantic or epistemic significance. It is holistic in acknowledging the existence, in principle, of a broad and diverse network of semantically and epistemically significant connections between language/mind/theory/ knowledge and reality (including connections involving nonvicious circularity). We may say that it is a *relational holistic* conception.[11]

7. It demands systematic connections between true statements (theories) and reality in all genuine fields of *knowledge.* This implies, among other things, that it rejects the traditional tie between holism and coherentism, and that it extends the correspondence approach to fields like logic and mathematics (to the extent that they are genuine fields of knowledge).

Elsewhere (Sher 1999a; 2004; 2008) I have investigated the applicability of correspondence to logic. My conclusion was that logical truth and consequence are grounded in the formal structure of reality, a structure studied in detail in mathematics. In Section IV I will investigate correspondence in mathematics; in the next section I will show how the pluralist approach developed in this paper avoids a challenge (involving logic) that threatens other pluralistic approaches.

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[11] For more on the holistic nature of this conception, see Section V below.

III. The Problem of Mixed Truths and Inferences.

In two articles, Tappolet (1997; 2000) raised two related challenges to Wright's version of pluralism about truth: the challenge of "mixed" truths, and the challenge of "mixed" inferences. Her analysis of the relevant features of Wright's theory proceeds as follows:

> Truth pluralism, as defended by Crispin Wright, is the view that there are different truth predicates corresponding to different sorts of sentences. Briefly, whereas descriptive sentences are claimed to be assessable in terms of 'heavyweight' truth, which involves realism about the corresponding entities, allegedly nondescriptive sentences, such as sentences about the moral or the comical, are supposed only to be assessable in terms of 'lightweight' truth, a kind of truth that does not involve realism. [Tappolet 2000: 382–383]

Her first challenge concerns "mixed" inferences:

> [T]here is a simple and [...] powerful objection to the claim that there is a plurality of truth predicates. Consider the following inference:
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> 1. Cruel cats are hungry.
> 2. This cat is cruel.
> 3. Therefore this cat is hungry.
>
> The validity of an inference requires that the truth of the premisses necessitates the truth of the conclusion. But how can this inference be valid if we are to suppose with Crispin Wright that two different kinds of truth predicates are involved in these premisses? For the conclusion to hold, some unique truth predicate must apply to all three sentences. But what truth predicate is that? And if there is such a truth predicate, why isn't it the only one we need? [Tappolet 1997: 209–210, 2000: 383][12]

Her second challenge concerns "mixed" sentences:

> There is a further problem for the claim that there is a plurality of truth predicates, thrown up by mixed sentences and, more particularly, mixed conjunctions. The sentence 'This cat is wet and it is funny' can obviously be true. But what sort of truth predicates would apply to it? This is a tough question for truth pluralism. On this view, the first conjunct is supposed to be T_1, if true at all, and the second T_2,

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[12] In this citation I replace Tappolet's original example of a mixed inference by a later example she gave, which is simpler to discuss since it contains truth-bearers of types that we are more familiar with than those appearing in the original example.

> if true at all. Given this, it would be extremely odd to say that the conjunction itself is assessable in terms of either T_1 or T_2. Suppose that T_1 is a matter of correspondence to natural facts, whereas T_2 is the result of a social agreement. The problem is that conjunctions involving the two kinds of truth predicates will be neither a matter of correspondence to natural facts nor a result of social agreement. [2000: 384]

Leaving it for Wright to defend his version of pluralism, let us see which parts of Tappolet's challenge apply to our theory. Clearly, there is no duality of "lightweight" and "heavyweight" truth-predicates in our theory, nor, indeed any plurality of truth-predicates. There is just one truth predicate: a "heavyweight", correspondence, truth predicate. This truth predicate has all truth-bearers in its scope, though to avoid paradox it may be technically construed as a hierarchy of truth predicates. This truth predicate attributes the same property — correspondence with reality — to every true truth-bearer, and it is this property that is transmitted from premises to conclusion in a valid argument. Still, Tappolet might argue, this single truth-predicate sets varied truth (correspondence) conditions on truth bearers in different fields. Does this not create a problem for mixed truths and inferences?

Before setting out to respond to this challenge, let me deepen and expand it. Tappolet presents the challenge as one of dealing with a specific "mixture" of truths, i.e., a mixture due to *logical* composition of sentences, and indeed to logical composition of *independent* sentences. But there is an important sense in which most sentences and inferences we use, including sentences that are logically simple (atomic) and sentences involving logical composition of proper parts of sentences rather than whole, independent sentences, are mixed in the sense of having constituents with different kinds of reference, satisfaction, and fulfilment conditions. Consider 'Causing pain is bad.' This sentence combines physical, mental, and moral expressions, and these may very well have different reference and satisfaction (hence truth) conditions. Lynch (2005b) regards mixed atomic sentences as belonging to a single domain (for example, the above sentence, according to him, belongs to the moral domain.) But the present theory offers a different approach, one that is far more general, works for sentences of any complexity, and does not depend on our ability to determine a single domain.

In a sense, the present theory approaches the challenge of mixed sentences in the same way that an engineer approaches a multi-faceted engineering challenge. Take, for example, the challenge of building a bridge over a large body of water in a big city during an economic downturn. In designing such a bridge the engineer has to combine principles belonging to multiple fields of knowledge: mathematics, various branches of physics, economics, sociology, aesthetics, and so on — fields governed by principles that are significantly different in kind from each other. There is no specific field of knowledge that consists of this particular combination of principles, but in each case the engineer creates a combination that fits the specific project he/she is engaged in. The situation faced by our correspondence truth theorist is similar. Having figured out (with the help of specialists) the general principles involved in physical, mathematical, psychological, biological, and moral truth, our truth theorist combines these principles together in determining the correspondence truth-conditions of each mixed sentence. When the sentence includes logical vocabulary, the correspondence theorist has to take into account the reference, satisfaction/extension, and fulfilment/value conditions of logical expressions as well. The fact that the truth and reference conditions of linguistic expressions with diverse components are often compositional is helpful in managing this task.

Dealing with sentences involving logical constants, as in Tappolet's example, let me first note without explanation that, on the present account, truth in logic is correspondence with the formal structure of reality, or more precisely, with certain laws governing its formal structure (laws studied in mathematics, as I mentioned above).[13]

Thus, take the mixed sentence:

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[13] This has been extensively discussed in Sher (*op. cit.*).

(1) The cat is wet and it is happy.[14]

Reading this sentence as a 1st-order sentence, it has the logical form:

(2) W(c) & H(c).

'&', on my correspondence account of truth in logic, is a denoting expression, whose denotation (in contexts of the above form) is *intersection*. Therefore, the truth-condition of (1) is captured by,

(3) 'The cat is wet and it is happy' is true
iff
The referent of 'the cat' is in the *intersection* the properties denoted (or indicated) by 'is wet' and 'is happy'.

On this account, the truth value of (1) depends (i) on the referents of 'the cat', 'is wet', 'is happy', and 'and', and (ii) on whether they are related in the requisite way in the world (i.e., on whether the referent of 'the cat' is in the *intersection* of those of 'is wet' and 'is happy')[15]. Now, the expressions 'the cat', 'is wet', 'is happy', and 'and' are biological, physical, mental, and logical, respectively, and this might affect their reference and satisfaction conditions on our account. But due to certain features of the referents involved, this is not problematic. Since a biological object (like a cat) can have properties of various kinds (including physical and mental properties), and since intersection, being a formal operation (see next section), can apply to properties of various kinds (including physical and mental properties), the truth of (1) is a mixed truth whose constituents mesh together seamlessly in spite of their diversity.

Proceeding to mixed logical inferences, consider Tappolet's

(4) (i) Cruel cats are hungry
(ii) This cat is cruel
(iii) This cat is hungry.

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[14] I prefer this variant of Tappolet's example since it is less controversial that there are facts concerning happiness than that there are facts concerning "funniness".

[15] More precisely, on whether it is in the intersection of the extensions of the properties denoted by 'is wet' and 'is happy'.

Since (4) is a *logical* inference, its validity depends only on the formal portion of the truth-conditions of its premises and conclusion. These are: (i) the intersection of 2 sets, call them 'A' and 'B', is included in a 3rd set, call it 'C'; (ii) an object, call it '*a*' is in the intersection of A and B; and (iii) *a* is in C.[16] The inference is logically valid in virtue of a certain formal law governing the world, namely: Whenever an object is in an intersection included in a given set, it is also in that set. Or, in terms of properties: Whenever an object has an intersective property included in another property, it has the latter property. This is a formal law governing the behavior of objects and properties in the world, and (4) is grounded in reality through this law.[17]

How, then, does (4) guarantee the unmixed (physical) truth of (iii) based on the mixed truth of (i) and (ii) (whose mixed nature is reflected in the mixed vocabularies of these sentences — physical, moral, and logical in the case of (i), and physical and moral in the case of (ii))? Our answer is that the truth conditions of all three sentences have a common element, namely, *formal* parameters, and the relations between their respective formal parameters are sufficient to guarantee the truth of the third sentence based on those of the other two. (Here is a partial analogy: How does the sale of John's home guarantee his ability to buy, say, a yacht, given that selling a home and buying a yacht are two different things? The answer is: Money. The fact that his *home's sale* gives John a large sum of *money* guarantees that he can do any number of things that have nothing to do with selling and homes, e.g., *buying* a *yacht*.) The law relating the formal constituents of the situations said to hold by the premises and conclusion of (4) is sufficient to guarantee that the conclusion-situation holds given that the premise-situations hold, in spite of their dissimilarities.

Since, as mentioned above, I have already explained the correspondence conditions of

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[16] The explanation could also be formulated in terms of having a property (instead of being in a set).

[17] This may involve some circularity, but circularity of this kind is permitted by our holistic methodology (see above).

logical truths and consequence in great detail elsewhere, I will not deal with them here. Instead, I will expand the earlier work by investigating the correspondence conditions of mathematical truths.

IV. Mathematical Correspondence[18]

In this section I will propose a tentative account of mathematical truth as based on *indirect correspondence.*[19] I will proceed in two steps: First, I will argue that reality has formal features and mathematics offers theories of the laws governing its formal features. Next, I will develop a tentative account of truth for mathematical theories of formal structure, based on

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[18] To avoid unnecessary confusion, let me explain my use of the terms 'property', 'feature', 'object', 'individual', 'formal', and 'mathematical' in this section:

(a) 'Property': Often, when I talk about properties I mean 'properties and/or relations', and sometimes 'properties, relations, and/or functions'. The reader will be able to figure out from the context which usage is appropriate.
(b) 'Feature': I use 'feature' as a general term. In the present context it usually means property (in the sense indicated in (a).). Often, 'feature' is used for properties of level 2 (i.e., properties of 1st-level properties), but more generally it is used for properties of any level, including 1st-level properties (i.e., properties of individuals).
(c) 'Object' vs. 'Individual': 'Object' is a general term, used for individuals, properties, relations, functions, etc. 'Individual' is used for an *atomic* object, an object of level 0, one that does not have (or is treated, in a given context, as not having) an internal structure (constituents, arguments, etc.).
(d) 'Formal' vs. 'Mathematical': 'Formal' is usually used to characterized a feature or law of objects, properties, or reality more generally; 'mathematical' is usually used to characterize linguistic expressions, theories, and laws on the linguistic level. These expressions, however, can be used interchangeably, since we can characterize a formal object as having the kind of properties that are indicated by mathematical expressions (or as being the kind of object that is denoted by mathematical expressions), and similarly we can characterize a mathematical expression as denoting (indicating) a formal object.

[19] The idea of indirect correspondence, as I have mentioned above, appears earlier in Horgan (2001) and Barnard & Horgan (2006). Horgan and Barnard, however, do not conceive of mathematical truth as based on indirect correspondence.

indirect correspondence.

Mathematics as a Theory of the Formal Structure of Reality. My starting point is the observation that objects in the world have, in addition to physical, biological, psychological, and other properties, also formal properties. Elsewhere[20] I have extensively discussed the notion of 'formality' and I will not repeat these discussions here. But in a nutshell, a *formal property* or relation is one that takes into account only the *pattern* delineated by its arguments in a given domain and *not the identity of the individuals* involved. Using model-theoretic terminology, we may say that a formal property *does not distinguish between isomorphic arguments* or argument-structures, or is *invariant under isomorphisms.*[21] Under this characterization, the 1st-level relation of identity is formal because it does not distinguish between isomorphic structures of the type $<A,b,c>$, where A is a non-empty set (domain or universe of individuals), and *b* and *b'* are members of A. That it to say, if $<A',b'c'>$ is the image of $<A,b,c>$ under some isomorphism, then $b=c$ iff $b'=c'$. In contrast, the 1st-level relation of having a greater mass than is not formal since it is not preserved under all isomorphisms: if $<A,b,c>$ is a structure of physical individuals

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[20] See Sher (1991; 1996; 1999b; 2001; 2002; 2008).

[21] (i) A *structure* $\mathfrak{S}$ is an n-tuple $<A, \beta_1,\ldots,\beta_n>$, where A is a nonempty domain (universe, set) of individuals, and for $1 \le i \le n$, β_i is a member of A, or a subset of A, or a relation on A, etc.

(ii) Structures $\mathfrak{S} = <A, \beta_1,\ldots,\beta_n>$ and $\mathfrak{S}' = <A', \beta'_1,\ldots,\beta'_n>$ are *isomorphic* iff there is a 1–1 and onto function *f* from A to A' such that for $1 \le i \le n$, β'_i is the image of β_i under *f*.

(iii) An *argument-structure* of a property P or a relation R is a structure representing an argument of P or R extensionally. An argument-structure for P is of the type $<A, \beta>$. If P is a 1st-level property, β is a member of A; if P is a 2nd-level property, β a subset of A. An argument-structure for an n-place R is of the type $<A, \beta_1,\ldots,\beta_n>$, where $<\beta_1,\ldots,\beta_n>$ represents an argument of R in A. An argument-structure of an individual *a* is of the same type as a structure for a 1st-level property P, namely, a structure $<A, \beta>$, where β is a member of A.

(iv) P is *invariant* under isomorphisms (does not distinguish between isomorphic argument-structures, or is preserved under isomorphisms) iff for all isomorphic argument-structures for P, $<A, \beta>$ and $<A, \beta'>$, β has the property P in A iff β' has the property P in A'. R is invariant under isomorphisms iff for all isomorphic argument-structures for R, $<A, \beta_1,\ldots,\beta_n>$ and $<A', \beta'_1,\ldots,\beta'_n>$, $\beta_1,\ldots,\beta_n$ (in that order) stand in the relation R in A iff $\beta'_1,\ldots,\beta'_n$ (in that order) stand in the relation R in A'. *a* is invariant under isomorphisms iff for all isomorphic argument-structures for *a*, $<A, \beta>$ and $<A', \beta'>$, $\beta=a$ iff $\beta'=a$.

such that *b* has a greater mass than *c*, there is an isomorphic structure <A',*b'c'*> in which this does not hold. (Let *b'* and *c'* be abstract individuals, e.g., numbers, thoughts, ideas, political institutions, etc.). Likewise, all the 2nd-level cardinality properties are formal, because they do not distinguish between argument-structures of the type <A,B>, where A is as above and B is a subset of A. But the 2nd-level property of being a property of humans is not formal, because whenever <A,B> is such that B is a set of humans (an extension in A of a 1st-level property that holds of some, and only, humans in A), there is an isomorphic structure <A',B'> for which B' is not a property of humans (i.e., not the extension of a property that holds of some and only humans in A').

Now, to see that objects in the world have formal properties, let us consider objects that are accepted both by nominalists and by nonnominalists; say, the students in my current graduate seminar ("Truth in Kant", UCSD, Fall 2010, 17 students). Clearly each student has the formal properties of being identical to himself/herself and being (numerically) different from me; the property of being a student in the class has various formal properties, e.g., the 2nd-level cardinality property we may call 'SEVENTEEN'; the properties of being a philosophy professor and being a woman can be combined by formal operations like union and intersection; students stand to other students in relations that have formal properties, e.g., the relation of studying in the same class as, which has the formal properties of being reflexive, symmetric, and nontransitive; and so on.

But if objects, properties, and relations in the world have formal or mathematical features, these features potentially exhibit regularities or are governed by laws. And these regularities or laws, like many other regularities and laws, have a certain modal force, a modal force that goes beyond their application to objects that actually exist (or are instantiated) in the world.

To see, next, that mathematics, through some of its theories, studies these formal laws, note how absurd or strange it would be if it did not. For example, it would be very strange if properties of things in the world had cardinality features, these features were governed by laws, mathematicians knew about these features and knew they were governed by laws, yet they studied the laws governing other, *unreal* (so-called) cardinalities, cardinalities governed by laws that had nothing to do with those governing the cardinalities of real properties of real objects in the world. It would not do to say that only *applied* mathematics has something to do with reality. To give a general and precise account of the *laws* governing formal features of objects in the world (e.g., the laws of cardinality) we need highly general and abstract theories of cardinality, i.e., something on the order of "pure" mathematical theories.

But if mathematical theories (or some mathematical theories) are theories of the laws governing formal features of objects in the world, then they are true or false in the sense of COR. If, and to the extent that, the laws of our current arithmetic theory do govern the relations between finite cardinalities in the world, there is a systematic connection between the laws described by arithmetic theorems and the laws governing finite cardinalities in the world. Our next task is to figure out what this systematic connection is like.

Mathematical Correspondence. In trying to figure out this connection, an apparent incongruity presents itself. Our analysis suggests that the level at which cardinalities arise in reality is the level of properties of properties, but modern arithmetic considers cardinalities to be individuals. This creates a puzzle: If cardinalities are in fact 2nd-level properties, how can 1st-order arithmetic get things right? How can an arithmetic theory which treats cardinalities as individuals be said to *correspond* to reality? From the opposite perspective the puzzle is this: If, in the world, there are no cardinal individuals but only 2nd-level cardinal properties, why do mathematicians construct their theories of cardinalities as theories of individuals? Why do they treat cardinalities as individuals if in fact they are properties of properties?[22]

To understand why mathematicians construe cardinalities as (numerical) individuals we

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[22] Note that even if there are mathematical individuals in the world, properties of individuals do often have cardinality features, so that cardinalities as 2nd-level properties still emerge in the world and must be taken into account in studying cardinalities.

have to take into account the fact that *mathematics is a discipline created by and for humans*. As such, it may take a form that is advantageous for humans even if circuitous from the point of view of correspondence. Thus, it is possible that the most natural or effective way for humans to make discoveries and/or develop theories of any subject-matter (or of formal subject-matters, or of certain formal subject-matters) is to do so on the 1st-level. I.e., humans may be better at discovering formal regularities and constructing a systematic theory of such regularities when they think of them as concerning individuals rather than higher-level properties. Their — our — cognitive resources may work better in a 1st-level setting than in a higher-level setting. And if reality does not supply such a setting, we create one for ourselves by constructing a 1st-level model of reality, or those parts/aspects of reality we wish to study. Arithmetic, in that case, gives an indirect account of some facets of reality. It describes the laws governing cardinalities by describing the laws governing their 1st-level numerical correlates (in a model constructed by and for humans). 1st-order arithmetic (if correct) thus corresponds to reality in an indirect manner, but that does not render its correspondence to reality insignificant or nonsystematic. 1st-order mathematical laws may not be true of reality in exactly the same way that laws of physics (the discipline) are true of reality, but they are true of reality just as much (in their own way). Rephrasing the title of Nancy Cartwright (1983), we may say that once you know how to read them, "the laws of arithmetic do not lie".[23]

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[23] (i) For my present purpose it does not matter whether mathematicians always prefer 1st-order theories to higher-order theories. It is sufficient that such a preference is possible. My task is to explain how this possibility is compatible with the correspondence approach. Similarly, for the present purpose there is no need to show that the conjectural explanation of people's preference for 1st-order theories offered above is empirically correct; it is sufficient to show that it is possible to explain this preference in a way that is compatible with our approach.

(ii) Someone might construe my view as saying that 2nd-order mathematics is "worldly mathematics" and 1st-order mathematics is "human mathematics". This might be useful in some contexts, but it would be misleading in others. The reason it would be misleading is that the very terminology of individuals, 1st- and 2nd-level properties, etc., is human terminology, and this means that worldly mathematics, on my view, is also human mathematics. Likewise, human mathematics has systematic connections with reality, and as such it is also worldly mathematics.

An Indirect Correspondence Relation. What form does the indirect correspondence of mathematics with reality take? Let us first compare two renditions of the truth conditions of the same mathematical sentence, the one direct, the other indirect. Consider

(5) $\alpha+\beta=\gamma$,

where 'α', 'β', and 'γ' are numerical singular-terms, '+' is a 1st-order function, and '=' is the 1st-order identity relation. A direct rendition of the truth (correspondence) condition of (5) will give us a *single-layer* condition:

(6) True '$\alpha+\beta=\gamma$' iff $n_\alpha +_n n_\beta \approx n_\gamma$,

where n_α, n_β, and n_γ are the individual numbers denoted by 'α', 'β', and 'γ', respectively, '$+_n$' is the 1st-level function denoted (or indicated) by '+', and '$\approx$' is the 1st-level identity relation denoted (or indicated) by '='. In contrast, an indirect rendition of the truth (correspondence) condition of (5) along the lines delineated above will give us a condition that, using familiar logical and set-theoretical terminology, can be formulated by:

(7) $(\forall P_1)(\forall P_2)[(\boldsymbol{\alpha}(P_1) \ \& \ \boldsymbol{\beta}(P_2) \ \& \ P_1 \cap P_2 = \varnothing) \supset \boldsymbol{\gamma}(P_1 \cup P_2)]$,

where $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ are the 2nd-level properties indirectly referred to by 'α', 'β', and 'γ', respectively.

This, of course, is not the traditional correspondence condition of (5). But traditional correspondence disregards the diversity of the patterns connecting immanent thoughts to reality, and therefore is limited to a single, monolithic pattern of correspondence.

To see how a complex pattern of correspondence might work and how it is related to the simple pattern, let us assume that everyday physical truths follow a simple route of correspondence, and let us compare physical and mathematical truths of the same syntactic form. First, consider the true (everyday) physical sentence

(8) Barack Obama is a male.

We can express its truth condition as:

(9) 'Barack Obama is a male' is true
iff
The individual denoted by the singular term 'Barack Obama' satisfies the 1st-level predicate 'is a male', i.e.,
iff
The individual Barack Obama has the 1st-level property of being a male.[24]

Skipping the intermediate condition, we have a *single-layered* definition of truth:

(10) 'Barack Obama is a male' is true
iff
The individual Barack Obama has the 1st-level property of being a male.

Next consider a mathematical sentence of the same syntactic form, say:

(11) 'Four is even'

In contrast to (8), the truth-condition of (11) is *two-layered*:

(12) 'Four is even' is true
iff
The individual four has the 1st-level property of being even,
iff
The 2nd-level property of holding of exactly four individuals has the 3rd-level property of being even.

Using the subscripts 0, 1, 2, and 3 to distinguish types of linguistic and ontological elements (0: individual terms/objects; 1–3: 1st-, 2nd, and 3rd-order/level predicates/properties), we can represent the difference between direct and indirect correspondence in terms of simple vs. complex or composite reference[25]:

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[24] To simplify the comparison, I chose a very simple, essentially deflationist, formulation of the truth condition of (8), rather than a more elaborate, substantive formulation. The same applies to the other examples in this paper. For examples of more substantive formulations of truth conditions see Sher (1999a; 2004).

[25] Terminology: I use 'refers' and 'denotes' (synonymously) for singular terms and 'refers', 'denotes', and 'indicates' (synonymously) for predicates. I talk about the 'satisfaction' and 'extension' of predicates (and by analogy, properties) and the 'fulfilment' and 'value' of

(13)

	Simple Reference			**Complex Reference**	
Language:	Sing.Terms$_0$	Predicates$_1$	**Language:**	Sing.Terms$_0$	Predicates$_1$
	↓	↓		⇩	⇩
World:	Individuals$_0$	Properties$_1$	**World:**	Properties$_2$	Properties$_3$

But this representation is still insufficient to *explain* the complexity of mathematical correspondence compared with (simple) physical correspondence. What the mathematician is actually doing is *positing* a new level of reality, a level containing mathematical individuals and 1st-level properties (relations, functions) of such individuals. These *represent* the 2nd- and 3rd-level properties that (1st-order) arithmetic objects are anchored in.

Figuratively, we can distinguish direct and indirect correspondence (in terms of reference) by a finer diagram:

(14)

	Simple Reference			**Composite Reference**	
Language:	Sing.Terms$_0$	Predicates$_1$	**Language:**	Sing.Terms$_0$	Predicates$_1$
				⇓	⇓
	↓	↓	**Posit:**	Individuals$_0$	Properties$_1$
				⬇	⬇
World:	Individuals$_0$	Properties$_1$	**World:**	Properties$_2$	Properties$_3$

It is important to note that the level of posits need not be *fully* reducible to the level of reality. In a sense, it has a life of its own. Once the adequacy of the 1st-level mathematical representation of the higher-level formal features of reality is established, we can proceed *as if* mathematical truth were based on straight (direct) correspondence. (Explaining the exact

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functional expressions (and functions).

standing of this posited level is another job for a substantive correspondence theory of truth.)

There are some similarities between this analysis of mathematical truth and the fictionalist's analysis (e.g., Hartry Field 1980; 1989), but there are also significant differences. For the fictionalist, (11) is false in the correspondence-with-reality sense; for the composite-correspondence theorist, (11) is true in that sense. For the fictionalist, physical applications of (11) are reducible to a *physical* statements; for the composite-correspondence theorist, they are reducible to a higher-order statements with both *formal* and *physical* constituents. For the fictionalist, reality has no irreducibly formal features; for the composite-correspondence theorist, it does. And so on. The fictionalist may be right in claiming that mathematical individuals are fictional posits, but he/she is wrong in thinking that mathematical theorems about these fictions are false, or that applied mathematical theorems are conservative extensions of physical truths.

V. An Intricate Mind — an Intricate Route from Thought to Reality.

Let us conclude with a few underlying principles and a few philosophical consequences of the present approach to truth.

Truth, on the present analysis, is a standard for immanent thoughts of all forms, in all areas, and of all sizes, from the atomic to the logically complex, from the physical to the moral and mathematical, and from the single statement to the single theory and up to our system of knowledge as a whole. It is a standard for a positive answer to the critical transcendent question: Is it so as a given statement, theory, or system of knowledge says it is? Are the things it talks about the way it says they are? Is the world as it says it is? Truth, on this conception, is a correspondence standard throughout, and when an immanent thought satisfies this standard it is said to be true, or have the property of truth.

Thoughts, however, are creations of our mind, and the more intricate our mind, the more intricate the route from thought to reality. In a sense, it is because of the mind's ability to create or engage in thoughts that go beyond direct perceptual (sensory and/or rational) contacts with the world that a standard of truth is so critical.[26] The mind's propensity to transcend direct perception is not an obstacle, however; it is our greatest asset in seeking knowledge of reality. Given the meagerness of the information provided by direct perception, we have no choice but to forge intricate, circuitous, and at times messy (though ideally systematic) routes to reality. The greater our desire to know the world, the greater our need to experiment with new cognitive routes; and the more we experiment with new routes to reality, the greater our need for a standard of truth. We thus vacillate between venturing further and further in exploring reality, and setting tighter and tighter restraints on our creative, adventurous mind.

In seeking to fathom reality, we use a wide variety of mental capacities. The availability of many of these capacities is, to a considerable degree, beyond our control. Partly, this is a matter of biology, partly - of culture, history, and chance. Either way, we have no choice but to cognize the world through the prism of our present cognitive machinery, some of which is less than ideal for the task. But our cognitive resources are not completely beyond our control. The development of new areas of knowledge, new theories, and new ideas is always accompanied by the development of new concepts, new methods of proof and experimentation, new perspectives - in short, new cognitive tools of a variety of kinds.[27] All this means that the route from thought to reality is dynamic, intricate, complex, and multi-faceted, a route that takes multiple forms and is constantly evolving.

In conceiving of truth as a standard for immanent thought having to do with its connection to reality, the present theory takes a holistic approach. By this we mean that it allows a multitude of patterns of such connections (including patterns involving nonvicious circularity), and in dealing with these connections it permits the (judicious) use of all our cognitive resources.

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[26] Direct rational contact with the world (rational perception or intuition) is advocated by, e.g., Kurt Gödel (1947/1964; 1953–1959).

[27] In the case of mathematics, the development of a 1st-order logical framework for the construction of theories is one example of a new cognitive tool.

This applies both to physical and to mathematical truth, and, indeed, knowledge. It means, among other things, that physical and mathematical knowledge may be attained by multiple combinations of (multiple) routes, spanning the whole spectrum of faculties, from sensory perception to conceptualization, categorization, abstraction, generalization, rational intuition, reflection, combinatorics, analysis, model building, experiment design, and others. And all these act in tandem to gain us access to different parts and aspects of reality: physical, formal, etc.[28]

This holistic approach to truth and knowledge has the philosophical advantage of avoiding the minefields of both Platonism and empiricism. In allowing rational reflection to play a crucial role in mathematical knowledge we eschew the problems of empiricism; in drawing a bridge between reason and experience we avoid the problems of Platonism. On the one hand, mathematics (or its veridical part) is genuinely grounded in the formal structure of reality; on the other hand, the reality in whose formal structure mathematics is grounded is the same reality that physics and other branches of science are grounded in. Mathematical truth is correspondence with the formal facets of reality, but these facets belong to the same reality that physical truth corresponds to (or to whose physical facets physical truth corresponds).

In eschewing mathematical Platonism we avoid some of the most pressing problems of contemporary philosophy of mathematics, including Paul Benacerraf's problems of *cognitive access* (1973) and *identity* (1965).

The problem of cognitive access is the problem of accessing those aspects of reality that are associated with the truth conditions of mathematical statements. The epistemology and semantics of mathematics must be closely connected, according to Benacerraf, so that (i) our knowledge of mathematical statements is knowledge of their truth, and (ii) the truth of

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[28] (i) Horgan (2001) gives good examples of indirect routes of empirical cognition.

(ii) This conception of cognitive access, I should emphasize, is holistic but not coherentist. Contact with reality is mandatory for truth, though it may take an abundance of routes and a great variety of interconnections.

mathematical statements is (in the case of knowable truths/falsehoods) a matter of conditions that are accessible to knowers. Now, Benacerraf rightly believes that neither empiricism nor Platonism can satisfy this requirement. But the outlook developed here has the potential of satisfying it. The problem of access can be solved by the combination of (i) a holistic methodology and (ii) the idea of indirect, composite, correspondence. This is achieved as follows: First, our outlook rejects the dualistic conception of reality characteristic of Platonism, so the problem of reaching mathematical reality from another, physical, reality, does not arise. Second, we do not require the existence of mathematical individuals, so the problem reduces to that of cognitive access to formal *features* of reality. Third, we show how the standard truth-conditions of 1st-order mathematics can be connected to higher-level formal facts. And fourth, we allow new cognitive routes to reality, e.g., routes generated by a combination of rational and sensory capacities, hence new avenues for accessing the formal features of reality. (An extended discussion and examples must be left for another essay.)

Benacerraf's identity problem is the problem of the identity of mathematical individuals. There are many distinct, yet isomorphic (standard) models of 1st-order arithmetic, including models in which the same numerals are assigned different referents. For example, in Zermelo's model the numeral 2 is assigned one object — the set $\{\{\varnothing\}\}$ — while in von Neumann's model it is assigned a different object — the set $\{\varnothing,\{\varnothing\}\}$.[29] Which object is the real number 2? Our (tentative) account of mathematical truth above shows how one can think of mathematical truth so this problem does not arise. Both Zermelo's and von Neumann's 2's are posits representing the 2nd-level property TWO, and since TWO is a formal property, all isomorphic systems of posits for it are equally good. I.e., Zermelo's 2 represents the 2nd-level TWO just as well as von Neumann's 2 (in their respective systems). In constructing, say, numerical posits, we have to give them a definite identity, but what identity we give them is immaterial, so long as the result is a

[29] Here '$\varnothing$' names the empty set, and '$\{x,y\}$' names the set of x and y.

systematic representation of cardinality properties.

Should we say, then, that the "real" 2 is the 2nd-level property TWO? From the point of view of our tentative account of mathematical truth above, the answer is 'Yes'. But the tentative status of our account suggests that this is still an open question. Another way to put this is that from an *immanent* perspective (the perspective of the proposed account) the answer is positive, but from a *transcendent* perspective, a perspective from which we acknowledge the possibility of alternative accounts of mathematical truth, the question is open.

Allowing a posited layer of mathematical individuals enables us to deal with another thorny ontological problem: the problem of the immense ontology of contemporary mathematics. So long as this "immense ontology" is a collection of posited objects, its size, by itself, poses no (genuine) ontological problem. If, and to the extent that, a large layer of posited entities is required, or even instrumental, for a precise, informative theory of the formal structure of reality (i.e., of the laws governing the formal features of objects in the world), then positing such a large layer is warranted.

Another advantage of the present approach is its ability to clarify the relation between the truth of statements and the truth of theories. One natural way to deal with the truth of theories is to say that a theory is true iff all its sentences are true. This view, however, is too simplistic. Suppose you have two theories, T1 and T2 such that for some sentence S, $S \in T1$ and $\sim S \in T2$, yet T1 and T2 both correspond to reality. For example, let T1 be Zermelo arithmetic, let T2 be von Neumann arithmetic, and let S be 'Successor $\{\varnothing\} = \{\{\varnothing\}\}$'. Then S is true in T1 and false in T2; hence, according to the above solution, T1 and T2 cannot be both true. Yet T1 and T2 are both accurate arithmetic theories, so if one of them is true, so must the other be. Our analysis offers a simple solution to this conundrum: T1 and T2 represent the system of laws governing the behavior of cardinalities in two different ways. This is not different from the decimal and binary systems representing the same mathematical operations in different ways. The fact that 'Successor $\{\varnothing\} = \{\{\varnothing\}\}$' is true in Zermelo arithmetic and false in von Neumann arithmetic is no more problematic than the fact that '10 +10=100 ' is true in binary arithmetic and false in decimal arithmetic. This phenomenon reminds us that truth, even correspondence truth, is a matter not just of the world but also of the mind.

Let me end with a general methodological point. It is important to realize that the task of a substantive correspondence theory of truth is not to give an algorithm for figuring out the route of correspondence in each and every case, or for each and every true sentence in our language. That task is not only unachievable but also pointless. We have already introduced the analogy between the task of figuring out the truth conditions of mixed sentences and the task of carrying out an engineering project based on a compendium of (pure) scientific principles. In the same way that it is absurd to demand from the scientist or even the engineer to develop a single algorithm describing all possible applications of all possible combinations of all "pure" scientific laws, so it is absurd to demand from the theorist of truth to develop such an algorithm for the truth (correspondence) conditions of all possible or even existent immanent thoughts. The task of a substantive correspondence theory of truth is a challenging task, but it is not an impossible task. The task is to identify and explain the central principles of correspondence, show how they are connected in principle, demonstrate their adequacy by well-chosen examples, and respond to pertinent objections. The task is not to construct an algorithm that tells what the (full) truth conditions of each and every sentence are and how all their particular elements intertwine.

References

Barnard, R. & Horgan, T. (2006). Truth as Mediated Correspondence. *Monist* 89: 31–50.

Benacerraf, P. (1965). What Numbers Could Not Be. *Philosophical Review* 74: 47–73.

----. (1973). Mathematical Truth. *Journal of Philosophy* 70: 661–680.

Cartwright, N. (1983). *How the Laws of Physics Lie.* Oxford: Oxford.

Field, H. (1980). *Science without Numbers: A Defense of Nominalism.* Princeton: Princeton.

----. (1989). *Realism, Mathematics & Modality.* Oxford: Basil Blackwell.

Gödel, K. (1947/1964). What is Cantor's Continuum Problem?. In Feferman *et al* (eds.), *Collected Works,* vol. 2 (254–270). New York. Oxford.

----. (1953–1959). Is Mathematics Syntax of Language?. In Feferman *et al* (eds.), *Collected Works,* vol. 2 (334–362). New York. Oxford.

Horgan, T. (2001). Contextual Semantics and Metaphysical Realism: Truth as Indirect Correspondence. In Lynch (2001b): 67–95.

Lynch, M.P. (2001a). A Functionalist Theory of Truth. In Lynch (2001b): 723–749.

----. (ed.) (2001b). *The Nature of Truth.* Cambridge: MIT.

----. (2004). *True to Life: Why Truth Matters.* Cambridge: MIT.

----. (2005a). Truth and Multiple Realizability. *Australasian Journal of Philosophy* 82: 384–408.

----. (2005b). Replies to Critics. *Philosophical Books* 46: 331–342.

----. (2009). *Truth as One and Many.* Oxford: Oxford.

Sher, G. (1991). *The Bounds of Logic: A Generalized Viewpoint.* Cambridge: MIT.

----. (1996). Did Tarski Commit "Tarski's Fallacy"? *Journal of Symbolic Logic* 61: 653–686.

----. (1999a). On the Possibility of a Substantive Theory of Truth. *Synthese* 117: 133–172.

----. 1999b. "Is Logic a Theory of the Obvious?". *European Review of Philosophy* 4: 207–238.

----. (2001). The Formal-Structural View of Logical Consequence. *Philosophical Review* 110: 241–261.

----. (2002). Logical Consequence: An Epistemic Outlook. *Monist* 85: 555–579.

----. (2004). In Search of a Substantive Theory of Truth. *Journal of Philosophy* 101:5–36.

----. (2008). Tarski's Thesis. In D. Patterson (ed.), *New Essays on Tarski and Philosophy* (300–339) Oxford: Oxford.

Tappolet, C. (1997). Mixed Inferences: A Problem for Pluralism about Truth Predicates. *Analysis* 57: 209–210.

----. (2000). Truth Pluralism & Many-Valued Logics: A Reply to Beall. *Philosophical Quarterly* 50: 382–385.

Tarski, A. (1933). The Concept of Truth in Formalized Languages. In *Logic, Semantics, Metamathematics*. 2nd ed. 1983. Indianapolis: Hackett. Pp. 152–278.

Wright, C. (1992). *Truth and Objectivity*. Cambridge: Harvard.

Wright, C.D. (2005). Remarks on the Functionalization of Pluralist Approaches to Truth. *Synthese* 145: 1–28.

----. (2006). *Truth & Cognition*. Doctoral dissertation, University of California, San Diego.

----. (2010). Truth, Ramsification, and the Pluralist's Revenge. *Australasian Journal of Philosophy* 88: 265–283.