



## The Formal-Structural View of Logical Consequence

Gila Sher

*The Philosophical Review*, Vol. 110, No. 2. (Apr., 2001), pp. 241-261.

Stable URL:

<http://links.jstor.org/sici?sici=0031-8108%28200104%29110%3A2%3C241%3ATFVOLC%3E2.0.CO%3B2-G>

*The Philosophical Review* is currently published by Cornell University.

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/sageschool.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## DISCUSSION

### The Formal-Structural View Of Logical Consequence

Gila Sher

In a recent paper, "The Concept of Logical Consequence," W. H. Hanson (1997) criticizes a *formal-structural* characterization of logical consequence in Tarski (1966) and Sher (1991, 1996a).<sup>1</sup> Hanson accepts many principles of the formal-structural view. Relating to Sher 1991 and 1996a, he says:

Sher's fundamental principles are sound. She holds that in deciding how the logical consequence relation shall be delineated we must take into account both theoretical and practical matters, and she suggests that to do so properly it is crucial to be clear about the role logic is expected to play (1991, 7, 36). I agree completely, and I have tried to use these broad principles to guide the present inquiry. She also sees herself as working squarely in the semantic tradition begun by Tarski, she wholeheartedly approves of his insistence that logical consequence be both a necessary and formal relation, and she accepts his characterization of formality as freedom from influence by empirical knowledge. Again I concur with her general approach. (Hanson 1997, 392–93)

But, according to Hanson, there is a flaw in the formal-structural view: it is neutral with regard to the apriority of logic. This neutrality,

---

I would like to thank a reviewer for the *Philosophical Review* for detailed comments. Parts of this paper were included in lectures on logic I gave at the Hebrew University in Jerusalem and a talk presented at the 1999 ASL meeting. I would like to thank the audiences in both places for their comments, and Carl Posy, Mark Steiner, Yemima Ben Menachem, Itamar Pitowski, Sol Feferman, Charles Parsons, Wilfried Sieg, and Harvey Friedman for their conversations following these talks. Finally, I would like to thank Harvey Friedman for his correspondence and Oron Shagrir and Peter Sher for their comments on earlier versions of this paper.

<sup>1</sup>I am using the name 'formal-structural view' for the view expounded in these works to indicate that its notion of formality is not the usual syntactic notion but a semantic notion, akin in certain respects to the notion 'structural' used in the philosophy of mathematics. I would like to thank Geoffrey Hellman for suggesting that I distinguish my use of 'formal' from other uses. Although Tarski's 1966 account is restricted to logical constants, its extension to logical consequence is straightforward. The founders of the formal-structural approach to logical constants are Mostowski (1957), Lindström (1966), and Tarski 1966.

he claims, (a) prevents it from capturing the intuitive concept of logical consequence, (b) renders it inconsistent (by conflicting with one of its own principles, namely, the indifference of logic to empirical knowledge of individuals in a given universe of discourse), and (c) leads to a criterion for logical constanthood (or, more simply, for logical constants) that classifies bizarre constants as logical and, when combined with the standard definition of logical consequence, yields bizarre empirical logical consequences.

In this paper I will offer a rebuttal of Hanson's criticisms. I will explain why the formal-structural view is neutral with regard to the apriority of logic, and I will show that this neutrality neither prevents it from capturing the intuitive notion of logical consequence nor renders it inconsistent. I will further demonstrate that Hanson's criticism of the formal-structural criterion for logical constants could be directed at almost any systematic criterion for logical constants, including the truth-functional criterion for logical connectives (a paradigm of a successful criterion of logicity), and I will argue that for the same reason that Hanson's criticism does not undermine (or even weaken) the latter, it does not undermine (or weaken) the former.

Although Hanson includes Tarski in his criticism, his discussion centers on Sher 1991, 1996a. In rebutting Hanson's criticisms I will, therefore, speak primarily for myself; but my defense of the formal-structural criterion for logical constants will pertain to Tarski (and other adherents of this criterion) as well.

## 1. The Formal-Structural Characterization of Logical Consequence

Our starting point is a common, presystematic conception of logical consequence:

Consider any class  $K$  of sentences and a sentence  $X$  which follows from the sentences of this class. From an intuitive standpoint it can never happen that both the class  $K$  consists only of true sentences and the sentence  $X$  is false. (Tarski 1936, 414)

[R]easoning is an argument in which, certain things being laid down, something ... necessarily comes about through them. (Aristotle 100a25–27)

In logic ... we deal throughout with completely general and purely formal implications. (Russell 1914, 54)

[L]ogic is concerned with the validity of arguments ... irrespective ... of their subject-matter— ... logic is, as Ryle ... puts it, 'topic-neutral'. (Haack 1978, 5)

[Logical consequence] cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence[s involved] refer. (Tarski 1936, 414–15)

In systematizing this conception, the formal-structural approach centers on one of the presystematic properties of logical consequence, *formality*. The reasons for characterizing logical consequence in terms of formality are both philosophical and methodological. Methodologically, (i) formality lends itself to a clear and precise characterization, (ii) the formality of logic entails the other properties attributed to it by the initial, presystematic conception, (iii) an account in terms of formality provides us with tools for constructing a precise criterion for logical constants and a precise demarcation of logic, and (iv) such an account is rich in mathematical and linguistic results. Philosophically, this account is embedded in a semantic analysis of the way logical consequence preserves *truth* and an epistemic analysis of the role logic plays in the enterprise of knowledge. I will not offer a comprehensive discussion of the formal-structural view here (for such discussions see Sher 1991, 1996a, 1996b, 1999a), but briefly, the main ideas are these:

*Underlying Semantic Analysis.* Consequence relations in general are relations of preservation of *truth*: the sentence  $\sigma$  is a consequence of the set of sentences  $\Gamma$  iff (if and only if)  $\sigma$  preserves the truth of the sentences in  $\Gamma$  (assuming the latter are all true). Truth, in turn, depends on whether things in the world are as a given sentence says: a sentence is true iff the objects it refers to have the properties it attributes to them. (In the case of atomic sentences—for example, sentences of the form ‘ $Pa$ ’—this is the usual correspondence principle; in the case of complex sentences, there is a natural way to extend this principle.)<sup>2</sup> Now, if truth is a matter of things having the properties attributed to them by a given sentence, then preservation of truth is a matter of a *connection* between things having the properties attributed to them by one sentence and things having the properties attributed to them by another. To see this more clearly, take any relation  $R$  between sets of sentences and sentences of a given language,  $L$ , and any pair,  $\langle \Gamma, \sigma \rangle$ , standing in this relation. Suppose things are as the sentences in  $\Gamma$  say but not

---

<sup>2</sup>For example, ‘ $P_1 b \ \& \ P_2 b$ ’ is true iff  $b$  (the individual referred to by ‘ $b$ ’) is in the intersection of  $P_1$  and  $P_2$  (the properties referred to by ‘ $P_1$ ’ and ‘ $P_2$ ’), ‘ $Pb \vee \sim Pb$ ’ is true iff  $b$  is in the union of  $P$  and its complement (in a given universe), ‘ $(\exists x)Px$ ’ is true iff  $P$  is not empty, etc.

as  $\sigma$  says. Then  $R$  is not a relation of truth-preservation (over  $L$ ).<sup>3</sup>

The idea arises of characterizing different consequence relations by specifying the properties of objects they take into account. It is clear that physical consequences (for example, ‘The net force exerted on body  $b$  is  $\alpha$ ; therefore, the rate of change in the momentum of  $b$  is  $\alpha'$ ’) take into account different properties of objects from those taken into account by logical consequences. The formal-structural view says that logical consequences take into account *formal* properties of objects. To see what this means, let us first examine a few paradigm examples. (Note: My use of ‘property’ in this paper is generic: ‘property’ stands for ‘property, relation, and /or function’.)

Consider the following example of a logically valid inference,

- (1) Some red roses are fragrant. Therefore: Some roses are fragrant,

understood as ‘Something is a rose and is red and is fragrant; therefore, something is a rose and is fragrant’.<sup>4</sup> In virtue of what is (1) logically valid? (I will formulate the formal-structural answer to this question in set-theoretical terms although, as I explained in Sher 1996a, this is not essential.) (1) is logically valid because (i) its premise says that the intersection of three sets (or properties) of objects is not empty, (ii) its conclusion says that the intersection of two of these sets (properties) is not empty, and (iii) it is a law governing intersections that if the intersection of three sets (properties) of objects is not empty, then the intersection of any two of them is not empty—that is, it is a law of intersections that the non-empty intersection of any three sets of objects is included in the non-empty intersection of any two of these sets.

The same kind of explanation applies to other paradigms of logical validity, for example,

- (2) All As are Bs; All Bs are Cs. Therefore: All As are Cs.  
 (3) For every  $a$ ,  $b$ , and  $c$ : if  $aRb$  and  $bRc$ , then  $aRc$ ; For no  $a$ :  $aRa$ .  
 Therefore: For every  $a$  and  $b$ : if  $aRb$ , then not  $bRa$ .  
 (4) There are at least two things. Therefore: There is at least one thing.

---

<sup>3</sup>This analysis can be viewed as an interpretation of Quine (1970, 97): “Logical truth ... is ... world-oriented rather than language-oriented; and the truth predicate makes it so.”

<sup>4</sup>Note that not every argument of the same *linguistic* form as (1) can be understood in this way. For example, ‘Some fake roses are fragrant; therefore, some roses are fragrant’ cannot. (Thanks to Graham Priest for this point.)

(2) is valid due to the transitivity of inclusion, (3) is valid due to interrelations between transitivity, reflexivity, and symmetry, and (4) is valid due to interrelations between cardinalities.

Properties (functions) such as intersection, inclusion, non-emptiness, universality (in a given domain), and so forth I call *formal properties*, and the operators representing them *formal operators* (see definition below). The formal-structural view says that *logical consequence is grounded in laws governing formal operators*. I call these *formal laws*.

As for nonlogical consequences, the formal-structural view explains their nonlogicality as due to their *not* being grounded in formal laws. Consider, for example:

- (5)  $a$  is (all) red. Therefore:  $a$  is not yellow.
- (6) Something is water. Therefore: Something is  $H_2O$ .
- (7) For every  $a$ ,  $b$ , and  $c$  if  $a$  is greater than  $b$  and  $b$  is greater than  $c$ , then  $a$  is greater than  $c$ ; Everything is greater than something. Therefore: Something is greater than itself.
- (8) There are at least two things. Therefore: There are at least three things.<sup>5</sup>

These examples fall under two categories: (5) and (6) are correct consequences and their truth-preservation is due to laws governing objects, but these laws are physical rather than formal. (7) and (8) are incorrect consequences. Although they do involve formal properties (transitivity, reflexivity, cardinality, etc.), they are not based on laws: it is not a law that a transitive relation with a universal domain is at least partially reflexive or that a universe with at least two objects is a universe with at least three objects.

*Underlying Epistemic Analysis.* Thinking of the project of knowledge as targeting objects and systems of objects in the world, we note that objects and systems of objects have, in addition to physical, biological, sociological, and many other kinds of properties, also formal or structural properties. Now, one central task of theories of the world is to enable us to pass from existent knowledge of the world to new knowledge. For example, if we know that water is the chemical compound  $H_2O$ , we can use our knowledge of chemical compounds to expand our knowledge of phenomena involving water. But while chemical, biological, sociological, and other kinds of laws contribute to our understanding of a limited array of phenomena, formal laws con-

---

<sup>5</sup>(6)–(8) are taken from Hanson 1997, 368.

tribute to our understanding of almost every facet of the world. In every area of the world, individuals are *identical* to (*different* from) individuals; objects lie in the *intersection*, *complement*, and *union* of properties; properties are *included* in properties; relations are (or are not) *transitive*, *reflexive*, *well-ordered*; properties and relations are *non-empty* or *universal*, *finite* or *infinite*, hold of  $n$  elements (tuples of elements) or  $m$  elements, and so on. Given the prevalence of formal properties and relations, the idea of developing a wholesale method for extending our knowledge based on formal properties of objects naturally arises. Such a method would enable us to move from existent beliefs to new beliefs without loss of truth and regardless of field of interest. The task of logic, on this approach, is to construct a theory of the transmission (preservation) of truth based on formal or structural grounds, and this involves (i) selecting constants referring to formal operators (operators corresponding to formal properties) as *logical constants*, and (ii) developing a method for identifying logical consequences based on the laws governing formal operators.

*Logical Constants.* The standard account of logical constants is hybrid: on the one hand it contains a highly informative, precise, and systematic criterion for logical connectives, namely, the Boolean or truth-functional criterion; on the other hand it contains an altogether uninformative and unsystematic definition of logical constants other than connectives, namely, a definition by enumeration— $C$  is a logical constant (other than connective) iff:  $C$  is ' $\forall$ ' or  $C$  is ' $\exists$ ' or  $C$  is '=' (or  $C$  is definable from constants on this list and/or logical connectives). The formal-structural approach seeks to do for the logical constants (other than connectives) what the standard approach does for the logical connectives: namely, identify a distinctive property of such constants (analogous to truth-functionality in the case of the logical connectives), and construct a precise, systematic and informative criterion for constants possessing this property.

Now, the standard account of the logical connectives centers on nonlinguistic entities (operators) rather than on linguistic entities (connectives).<sup>6</sup> It says that a connective is logical iff it refers to (stands for, represents) a truth-functional operator and it gives a precise mathematical definition of such operators. (An operator is truth-functional iff it is a function from  $n$ -tuples of truth values, T and/or F, to

---

<sup>6</sup>For the significance of giving priority to operators over constants, see McGee 1966.

a truth value, T or F.) The formal-structural account takes a similar approach. Using 'constant' for 'nonsentential constant', it says that a constant (linguistic entity) is logical iff it refers (in a way to be specified below) to a *formal operator* and it gives a precise mathematical criterion for formal operators. For detailed accounts and explanations see Tarski 1966, Lindström 1966, Sher 1991, 1996a, and 1996b, and others.<sup>7</sup> But very briefly, the account is this: An operator assigns to each universe  $A$  (a non-empty set of objects treated as individuals) a function from its arguments in  $A$  ( $n$ -tuples of members, subsets, relations, etc. of/on  $A$ ) to truth values,  $T$  and  $F$ . For example, the Identity operator assigns to a universe  $A$  a function,  $I_A$ , that gives a pair of individuals in  $A$  the value  $T$  iff they are identical; the Non-Emptiness operator (the operator referred to by ' $\exists$ ') assigns to  $A$  a function,  $\exists_A$ , that gives a subset of  $A$  the value  $T$  iff it is not empty; etc. The distinctive characteristic of *formal operators* is that they do not distinguish between individuals either within or across universes. Mathematically, formal operators do not distinguish between *isomorphic* argument-structures. (An argument-structure consists of a universe and an argument of a given operator in that universe.)

*Criterion for formal operators*

An operator is formal iff it is indifferent to all 1-1 replacements of individuals, both within and across universes. (Mathematically:  $O$  is a formal operator iff  $O$  is invariant under all isomorphisms of argument-structures.)<sup>8</sup>

All the standard (non-truth-functional) logical operators—the non-emptiness operator, the universality operator (referred to by ' $\forall$ ') and the identity operator—satisfy this criterion. To see how the crite-

<sup>6</sup>For the significance of giving priority to operators over constants, see McGee 1966.

<sup>7</sup>The accounts by different authors differ in certain ways. In my own writings the emphasis on operators is implicit rather than explicit.

<sup>8</sup>Explanation:

(i) An operator  $O$  assigns to each universe  $A$  a unary function,  $O_A$ . If  $O$  is an  $n$ -ary operator, the arguments of  $O_A$  are  $n$ -tuples. An operator is either relational or functional. If  $O$  is a relational operator and  $\beta$  is an argument of  $O_A$ , then  $O_A(\beta) \in \{T, F\}$ ; if  $O$  is a functional operator and  $\beta$  is an argument of  $O_A$ , then  $O_A(\beta) \in A$  or  $O_A(\beta) \in \mathcal{P}(A^m)$  for some  $m > 0$ .

(ii)  $\mathcal{I}$  is an isomorphism of argument-structures  $\langle A, \beta \rangle$  and  $\langle A', \beta' \rangle$  iff  $\mathcal{I}$  is a bijection from  $A$  to  $A'$  such that  $\beta'$  is the image of  $\beta$  under  $\mathcal{I}$ . When such an  $\mathcal{I}$  exists, we write:  $\langle A, \beta \rangle \cong \langle A', \beta' \rangle$ .

tion works, consider the non-emptiness operator,  $O_{\exists}$ . Take any universe  $A$  and a subset,  $B$ , of  $A$  (an argument of  $O_{\exists}$  in  $A$ ) and permute the elements of  $A$ . Call the image of  $B$  under this permutation  $B'$ . Then  $O_{\exists}$  gives  $B$  the value T (in  $A$ ) iff  $O_{\exists}$  gives  $B'$  the value T (in  $A$ ). That is,  $O_{\exists}$  does not distinguish between  $B$  and its image under this permutation. Now replace the elements of  $A$  by any objects in a 1-1 manner, obtaining a new universe,  $A'$ , and call the image of  $B$  under this replacement  $B''$ . Again,  $O_{\exists}$  gives  $B$  the value T (in  $A$ ) iff  $O_{\exists}$  gives  $B''$  the value T (in  $A'$ ).  $O_{\exists}$  does not distinguish between individuals either within or across universes.

The idea that logic does not differentiate between individuals has a long history:

[General logic] treats of understanding without any regard to difference in the objects to which the understanding may be directed. (Kant 1781/1787, A52/B76)

Pure logic ... disregard[s] the particular characteristics of objects. (Frege 1879, 5)

[The relation of logical consequence between a sentence  $X$  and a class of sentences  $K$ ] cannot be influenced in any way by ... knowledge of the objects to which the sentence  $X$  or the sentences of the class  $K$  refer. The consequence relation cannot be affected by replacing the designations of the objects referred to in these sentences by the designations of any other objects. (Tarski 1936, 414–15)

[Logical] quantifiers should not allow us to distinguish between different elements of [the universe]. (Mostowski 1957, 13)

This idea also reflects the present conception of logic as formal, since formal properties (and the laws governing them) are naturally characterized as invariant under isomorphisms of structures. (See Sher 1991, 1996a, 1999a). We commonly call the operators corresponding to logical connectives, that is, truth-functional operators, “logical”; in the same vein I will call formal operators (the operators corresponding to logical constants other than connectives) “logical.”

The formal-structural criterion, like the truth-functional criterion, leads to classifying many operators that traditionally were not considered logical as logical. These include Finitely and Infinitely Many (referred to in ‘There are finitely/infinitely many Bs’), Most<sup>1</sup> and Most<sup>2</sup> (referred to

---

(iii)  $O$  is invariant under isomorphisms of structures iff for any argument-structures  $\langle A, \beta \rangle$  and  $\langle A', \beta' \rangle$  of  $O$  such that  $\langle A, \beta \rangle \cong \langle A', \beta' \rangle$ :  $O_A(\beta) = O_{A'}(\beta')$  if  $O$  is a relational operator, and  $\langle A, O_A(\beta) \rangle \cong \langle A', O_{A'}(\beta') \rangle$  if  $O$  is a functional operator.

in ‘Most things are Bs’ and ‘Most As are Bs’, respectively), the Well-Ordering operator, and others.<sup>9</sup> The operators Is Red, Is H<sub>2</sub>O, Is Taller Than, Is a Property of Humans, etc. are not logical.<sup>10</sup> Note that in open formulas logical connectives refer to formal operators (Complement, Union, Intersection, etc.) rather than to truth-functional operators.

*Criterion for logical constants*

A constant is logical iff it rigidly refers to a formal operator. (A sentential connective is logical iff it rigidly refers to a truth-functional operator.)

For an explanation of the rigidity requirement, see Sher 1991, chap. 3.

*Definition of Logical Consequence.* The formal-structural view adopts the standard, model-theoretic definition of logical consequence, which it interprets according to its own principles. On this interpretation, a regularity across all models (for example, the regularity that a non-empty intersection of three subsets of a universe is included in a non-empty intersection two of these subsets) represents a formal law, and an argument-schema preserves truth in all models iff it is grounded in such a law. (For further discussion see Sher 1991, 1996a, 1999a.)<sup>11</sup>

## 2. Hanson’s Criticisms

Hanson criticizes the formal-structural view on three counts:<sup>12</sup>

### 2.1. Alleged Inadequacy of the Criterion for Logical Constants

Hanson says:

[G]iven the structural nature of [Sher’s] account of logical terms, it is the cardinality of the domain of a model that is crucial in determining how

---

<sup>9</sup>These operators are quantificational. We symbolize formulas governed by the corresponding quantifiers by: ‘(F<sub>x</sub>)φ<sub>x</sub>’, ‘(Inf x)φ<sub>x</sub>’, ‘(M<sup>1</sup>x)φ<sub>x</sub>’, ‘(M<sup>2</sup>x)(φ<sub>x</sub>,ψ<sub>x</sub>)’, and ‘(W<sub>xy</sub>)φ<sub>xy</sub>’. Note that not all these operators are cardinality operators: Well-Ordering (and many other logical operators) are not. Warmbröd (1999, 508) claims that the formal-structural approach identifies logicity with cardinality. This is obviously incorrect.

<sup>10</sup>The operator Is Red assigns to an individual in a universe A the value T iff it is red; Is a Property of Humans assigns to a given subset of A the value T iff its members are all human; etc.

<sup>11</sup>I hope it is clear from this brief account of the formal-structural approach that (contrary to Warmbröd 1999, 505-11) its criterion for logical constants is philosophically motivated.

<sup>12</sup>Hanson (403–5) has a fourth criticism. I anticipated and responded to this criticism in Sher (1996a). Hanson is aware of this response, but appears to have not fully understood it.

such a term behaves in that model. Thus it is possible that a logical term will behave quite differently in models with different-sized domains. As an example, let  $n$  be the least number of whole seconds (that is, the least number of seconds, disregarding fractions of a second) in which, up through the end of the twenty-first century, a human being runs a mile. Now consider a quantifier that behaves exactly like the universal quantifier (over individuals) in models with domains of cardinality  $\geq n$ , but like the existential quantifier in models with domains of cardinality  $< n$ . Call this quantifier ' $Q^*$ '. ' $Q^*$ ' is a logical term on Sher's account because it satisfies her semantic isomorphism conditions, although it seems bizarre to treat it as one. To see just how bizarre this is, consider the following argument:

$$\begin{aligned} [(9)] \quad & (Q^*x)(\text{Dog}(x) \rightarrow \text{Black}(x)) \\ & (Q^*x)\text{Dog}(x) \\ & \therefore (Q^*x)\text{Black}(x) \end{aligned}$$

As long as  $n \geq 3$ , argument [(9)] has countermodels. So we know that [(9)] is invalid, since we know that no one will run a mile in less than three seconds before the end of the next century (or ever, for that matter). Yet we don't and can't know this a priori. (Hanson 1997, 391–92)

In selecting ' $Q^*$ ' as an example of a bizarre logical constant, Hanson selects a constant that has two distinctive characteristics: (i) its semantic behavior is *irregular* or *unnatural*: it behaves like one familiar logical constant in some universes, like another in others; (ii) it is an *empirical* logical constant: its correct use requires some piece(s) of empirical knowledge. For the sake of analysis, I will distinguish these two characteristics. Hanson claims that the formal-structural criterion licenses bizarre (that is, unnatural and empirical) logical constants, and as such should be rejected. The blame, he implies, lies with a distinctive characteristic of this criterion, namely, constants satisfying it are sensitive to the cardinality of universes of models. My rebuttal will proceed in two steps. In step 1 I will show that: (a) the formal-structural approach has at best a limited commitment to ' $Q^*$ ' being a logical constant; (b) there is no connection between being an unnatural/empirical logical constant and taking the cardinality of the universe into account; and (c) inasmuch as the formal-structural criterion licenses unnatural and empirical logical constants, so do other criteria, for example, the truth-functional criterion for logical connectives. I will conclude that if Hanson's criticism provides a sound reason for rejecting the formal-structural criterion, it provides a sound reason for rejecting the truth-functional criterion (which many, including Hanson, consider a paradigm of a successful criterion of logicity). In step 2 I will argue that Hanson's criticism does not pro-

vide a sound reason for rejecting either criterion.

Notation: henceforth, a double-bar below a letter will indicate that it stands for an operator; otherwise it stands for a constant. Thus,  $Q^*$ -constant;  $Q^*$ -the operator it refers to.

STEP 1

(a) *Hanson treats  $Q^*$  as a bona-fide logical constant of the formal-structural approach. But the formal-structural approach has at most a limited commitment to  $Q^*$  being a logical constant.* First, it is not clear that  $Q^*$  satisfies the formal-structural criterion: if  $Q^*$  is not a rigid designator, it does not. Second, even if  $Q^*$  is a rigid designator, at the present (that is, before the end of the twenty-first century), the referent of  $Q^*$  is undetermined. That is, it is not determined *which* logical operator  $Q^*$  refers to. Therefore, at the present,  $Q^*$  cannot be used as a logical constant. Third, if and when  $Q^*$  can be used as a logical constant, it can be replaced by a non-empirical constant. That is, there is a numeral 'k' such that we can define  $\underline{\underline{Q}}^*$  using 'k' instead of Hanson's  $n$ . The result would be a non-empirical constant,  $\underline{\underline{Q}}$ , defined by: given a universe  $A$  and a subset  $B$  of  $A$ ,  $\underline{\underline{Q}}_A(B)=T$  iff: either  $|A|\geq k$  and  $\forall_A(B)=T$ , or  $|A|<k$  and  $\exists_A(B)=T$ .<sup>13</sup>

In spite of the limited commitment of the formal-structural approach to  $Q^*$ , I will assume, for the sake of argument, that  $Q^*$ , and other constants like it, are bona fide logical constants.

*Remarks:* Hanson treats the sensitivity of logical constants to the cardinality of the universe as distinctive of the formal-structural approach. It is not. Consider the quantifier  $Q^{**}$ , which behaves like  $\exists$  in universes with less than 100 elements and like  $\forall$  in others.  $Q^{**}$  is a standard logical constant (since it is definable in terms of the standard logical constants); yet it is no less sensitive to the cardinality of the universe than Hanson's  $Q^*$ . Indeed, even  $\forall$  takes the cardinality of the universe into account. Suppose a set  $B$  satisfies  $\forall$  in a model with a universe of cardinality  $\alpha$ . Then  $B$  does not satisfy  $\forall$  in any model with a universe of cardinality  $> \alpha$ . That is, whether  $B$  satisfies  $\forall$  depends (among other things) on the cardinality of the given universe.<sup>14</sup>

(b) *Hanson implies that there is an inherent connection between a constant taking into account the cardinality of the universe and its being unnatural and/or empirical. There is no such connection.*

(i) *Not all logical constants that take the cardinality of the universe into*

---

<sup>13</sup>' $\underline{\underline{Q}}_A(B)=T$ ' stands for: 'the value that  $\underline{\underline{Q}}$  assigns to  $B$  in  $A$  is  $T$ '.

*account are unnatural.* Consider  $\forall$  once again.  $\forall$ , as we have seen above, takes the cardinality of the universe into account, yet  $\forall$  is a paradigm of a natural logical constant. Other constants sensitive to the cardinality of the universe are also considered natural by many philosophers. Hanson himself (1997, 394) says that it would be reasonable to classify the 1-place Most and As-Many-As-Not as logical quantifiers.<sup>15</sup>

(ii) *Not all unnatural logical constants take the cardinality of the universe into account.* Consider the quantifier  $Q^{***}$ , which, in any given universe, behaves like the quantifier Even (as in '(An Even Number of  $x$ )  $\phi x$ ') when the extension of its argument is of cardinality  $<100$ , like the quantifier Odd when the extension of its argument is of cardinality  $\geq 100$  yet finite, and like Uncountably-Many otherwise.  $Q^{***}$  is an unnatural logical quantifier but it does not take the cardinality of universes into account (any more than  $\exists$ , say, does).

(iii) *Not all logical constants that take the cardinality of the universe into account are empirical.* The three logical constants mentioned in (i) above prove this point.

(iv) *Not all empirical logical constants take the cardinality of the universe into account.* Let  $n$  be as in Hanson's example, that is,  $n$  is defined as the least number of whole seconds in which, up through the end of the twenty-first century, a human being runs a mile, and consider the quantifier At-Least- $n$ . This quantifier is empirical, but it is indifferent to the cardinality of the universe.

(v) *Not all logical constants that are both unnatural and empirical take into account the cardinality of the universe.* Let  $Q^{****}$  be obtained from  $Q^{***}$  by replacing '100' by Hanson's  $n$ .  $Q^{****}$  is both unnatural and empirical, yet it is indifferent to the cardinality of the universe.

*Note:* Hanson's choice of  $Q^*$  as an alleged counterexample suggests a connection between a logical constant being unnatural and its being empirical. There is no such connection. Not all empirical logical constants are unnatural: the quantifier At-Least- $n$  described above

---

<sup>14</sup>The formal-structural approach explains why logical constants take into account the cardinality of universes: logical constants take into account the formal properties of their arguments, and in the case of quantifiers, the cardinality of the complement of a given argument is such a property. Cardinality of a complement is dependent on the cardinality of the underlying universe; hence, the cardinality of universes is taken into account by (some) logical constants. Note that the only formal property of a universe is its cardinality (Mostowski 1957).

<sup>15</sup>Let  $A$  be a universe,  $B \subseteq A$ .  $\text{Most}_A(B)=T$  iff  $|B|>|A-B|$ ;  $\text{As-Many-As-Not}_A(B)=T$  iff  $|B|\geq|A-B|$ .

is empirical but not unnatural. Not all unnatural logical constants are empirical:  $Q^{***}$  is unnatural but not empirical.

(c) *Hanson views unnatural and empirical logical constants as unique to the formal-structural approach. They are not. Even the truth-functional criterion for logical connectives does not rule out such logical constants.*

(i) *Any account of logic that affirms the truth-functional criterion for logical connectives affirms unnatural logical constants.* Consider the 3-place connective  $C^*$ , which behaves like the 3-place Minority connective when its arguments include an *even* number of Ts<sup>16</sup> and like the 3-place Majority connective when its arguments include an odd number of Ts. (The Minority connective refers to the truth-function MIN, where MIN( $X_1, X_2, X_3$ ) = T iff only a minority of  $X_1, X_2, X_3$  are T; the Majority connective refers to MAJ, where MAJ( $X_1, X_2, X_3$ ) = T iff a majority of  $X_1, X_2, X_3$  are T.)  $C^*$  is both truth-functional and unnatural: It is truth-functional since it does not distinguish between arguments with the same truth values (that is, if  $\langle v(p_1), v(p_2), v(p_3) \rangle = \langle v(p_4), v(p_5), v(p_6) \rangle$ , where  $v(p)$  is the truth-value of  $p$ , then  $C^*(p_1, p_2, p_3) = C^*(p_4, p_5, p_6)$ ). It is unnatural, since it sometimes behaves like one familiar logical connective, sometimes like another.

(ii) *Any account of logic that affirms the truth-functional criterion for logical connectives affirms empirical logical constants.* Let  $n$  be as in Hanson's example, and consider the 3-place connective  $C^{**}$ , which behaves like Conjunction if  $n < 3$  and like Disjunction if  $n \geq 3$ .  $C^{**}$  is both truth-functional (since it does not distinguish between sentential arguments with the same truth value) and empirical (since it is introduced under a description that involves essential reference to empirical events).

(iii) *Any account of logic that affirms the truth-functional criterion for logical connectives affirms logical constants that are both unnatural and empirical.* Let  $n$  be again as in Hanson's example, and consider the 3-place connective  $C^{***}$ , which behaves like the Minority connective when the number of Ts among its arguments is  $\geq n$  and like the Majority connective when the number of Ts among its arguments is  $< n$ .  $C^{***}$  is an unnatural connective;  $C^{***}$  is an empirical connective; yet  $C^{***}$  is a bona fide truth-functional connective and as such is affirmed by any theory that accepts the standard criterion for logical connectives.<sup>17</sup>

<sup>16</sup>Zero is an even number.

<sup>17</sup>Since empirically we know that  $n$  is larger than 3, one may argue that  $C^{***}$  is *in fact* not an unnatural connective. It is, however, easy to change  $C^{***}$  so that this objection does not apply: let  $C^{***}$  be a 1000-place connective, or let  $n$  be a number about which we only know that it is larger than 1 and this knowledge is empirical.

*Conclusion:* If admission of unnatural and empirical logical constants is a reason for rejecting a given criterion for logical constants, then even our paradigm of a successful criterion for such constants, namely, the standard, truth-functional criterion, should be rejected.

## STEP 2

*Should we reject the standard criterion for logical connectives?* The standard criterion for logical connectives sanctions the same kind of inferences as Hanson's (9). Consider:

- (10) C\*\*\*(Cerberus is a dog, Cerberus is a dog, Cerberus is black)  
 ~ Cerberus is black  
 ∴ ~ Cerberus is a dog,

where C\*\*\* is the truth-functional connective defined above. As long as  $n \geq 3$ , (10) has counter-models. So, we know that (10) is invalid, since we know that no one will run a mile in less than three seconds before the end of the next century. Yet we don't and can't know this a priori. Should we, then, reject the truth-functional criterion for logical connectives on the ground that it is incompatible with our intuitive conception of logical consequence as knowable a priori?

In answering this question, it is important to distinguish between the fact that the truth-functional criterion permits *unnatural* logical connectives and the fact that it permits *empirical* logical connectives. As we shall presently see, the exclusion of unnatural logical connectives would undermine the truth-functional criterion, but the exclusion of empirical logical connectives would not. Similarly, the exclusion of unnatural logical quantifiers would be a serious matter for the formal-structural criterion, whereas the exclusion of empirical logical quantifiers would not.

(a) *Should we exclude unnatural logical connectives? Should we reject the truth-functional criterion of logicity on the ground that it gives rise to such connectives?* Most logicians would answer no, and for good reasons:

(i) Excluding unnatural logical connectives does not amount to excluding one or two or even finitely many logical connectives. There are at least as many unnatural truth-functional connectives as natural truth-functional connectives, and therefore excluding unnatural logical connectives is tantamount to giving up the category of truth-functional connective altogether.

(ii) By excluding unnatural logical connectives we give up not only a criterion for logical *constants*, but also a criterion for logical *operators*. This is due to the fact that whether a logical connective is unnatural depends on the behavior of the operator (that is, truth-function) it refers to.

(iii) What is wrong with unnatural logical connectives? Nothing more than their unfamiliarity and the minor confusion their partial similarity to familiar connectives might cause. Surely, if we are to reject a criterion of logicity that is as general, precise, informative, and useful as the truth-functional criterion, we should do so on more substantial grounds. (It is a price often paid for a general, precise, and informative definition of a given concept that by accepting it we extend the range of objects falling under it in unfamiliar ways.)<sup>18</sup>

(iv) We cannot reject unnatural logical connectives without rejecting all reasonable collections of logical connectives. Reason: A reasonable collection is expressively complete, hence it allows us to define some (indeed, all) unnatural logical connectives.

(v) Logically valid inferences involving unnatural logical connectives are just as strong as logically valid inferences restricted to natural logical connectives: The former are just as necessary, just as indifferent to empirical properties of individuals, just as topic neutral, just as formal, etc. as the latter. Therefore, by excluding unnatural logical connectives we do not strengthen our notion of logical consequence. We do, however, make it less general, less coherent, less unified, less precise, and less intelligible.

(vi) The very idea of an unnatural logical connective is vague and subjective. Consider the 3-place Majority connective: Given an argument (a triple of truth values) with an even number of Ts it behaves like the 3-place Disjunction, while given an argument with an odd number of Ts it behaves like the 3-place Conjunction. Is it an unnatural connective? And what about the Biconditional? Given an argument with an even number of Ts it behaves like Material Conditional and given an argument with an odd number of Ts it behaves like Conjunction. Is *it* unnatural? Even Conjunction could be said to exhibit unnatural behavior: Given an argument with an even number of Ts it behaves like Disjunction, while given an argument with an odd num-

---

<sup>18</sup>Kronecker regarded reals that are not roots of algebraic equations as suspect (see Kline 1980, 232); the definition of '(continuous) curve' yields curves that lack tangents at all points and as such violate our intuitive conception of what a curve is (see Hahn 1933); etc.

ber of Ts it behaves like Biconditional. Should we reject Conjunction?

Essentially the same considerations (with obvious adjustments) apply to unnatural logical constants other than connectives.<sup>19</sup> If the formal-structural criterion for logical constants is to be rejected, it should be rejected for a more compelling reason than the “strange” or unfamiliar nature of some constants satisfying it or the operators they refer to.<sup>20</sup>

(b) *Should we exclude empirical truth-functional connectives? Should we reject the truth-functional criterion of logicity on the ground that it does not exclude such connectives?*

First, I would like to note that excluding empirical logical connectives would not affect our criterion for logical *operators*. The question raised by empirical logical constants is not, Should we reject the criterion for logical operators? but at most, Should we reject the criterion for logical constants?<sup>21</sup> Second, even with respect to the latter criterion, the question is not, Should we rescind any of its conditions? but merely, Should we add a new condition, restricting the range of expressions used to represent logical operators? Moreover, the criterion as it now stands already excludes many empirical constants, since many of these constants violate the rigidity requirement. So the question is a relatively minor one: Should we add to the present criterion another condition, excluding empirical yet rigid logical constants? My answer is, As you wish. From the point of view of the formal-structural approach, not much hangs on a positive or a negative answer to this question.<sup>22</sup>

*Neutrality towards Apriority.* Hanson says:

It should be noted that ‘Q\*’s violation of the apriority condition is not one that would bother Sher, for she explicitly denies that apriority is relevant to or important for logical consequence. (1997, 232)

Hanson connects the acceptance of constants such as Q\* with neutral-

---

<sup>19</sup>Although reasonable collections of logical constants (other than connectives) are not expressively complete, they do give rise to unnatural logical constants. (See, for example, Q\*\* above.)

<sup>20</sup>The present discussion of unnatural logical constants also responds to Machover 1994 and Feferman 1999.

<sup>21</sup>This point is eloquently made by McGee in his 1996 paper.

<sup>22</sup>Note that the “problem” of empirical expressions arises in all branches of mathematics. Arithmetic, for example, has no injunction against using empirical expressions to refer to numbers (e.g., using ‘the number of Martians’ to refer to 0). Does this render it empirical?

ity towards apriority. But the two are not connected. Let me clarify.

Why is the formal-structural approach neutral with regard to the apriority of logic? The reason is methodological: The formal-structural approach aims at critically developing and systematizing the common conception of logical consequence (reflected in the citations from Tarski, Aristotle, Russell, and Haack above). In so doing, it aims to accommodate the largest range of views compatible with the common conception without compromising its principles. Now, although traditionally philosophers regarded logical knowledge as a priori, more recently the a priori–a posteriori distinction has become a hotly debated issue (with Tarski, Quine, and others standing on one side of the controversy, and traditionally inclined philosophers as well as logical positivists on the other). The formal-structural approach seeks to avoid this controversy.<sup>23</sup> It regards an account of logic not committed to the a priori–a posteriori dichotomy as methodologically preferable to one committed to it.

Now, this consideration has nothing to do with the question of whether empirical expressions can serve as logical constants. What it does have to do with are the *grounds* of logical consequence. Logical consequence, on the formal-structural view, is grounded in formal laws: When  $\sigma$  is a logical consequence of  $\Gamma$ , this is due to a formal law connecting the situation described by the sentences in  $\Gamma$  to the situation described by  $\sigma$ . An apriorist (with regard to logic) will construe formal laws as a priori; a neutral person will leave their status (as a priori, a posteriori or neither) an open question.

I would like to emphasize again, however, that by not committing itself to the apriority of logic, the formal-structural approach does not commit itself to its aposteriority. The formal- structural approach is neutral with regard to apriority.

## 2.2 Alleged Inconsistency

Hanson claims that by not requiring logical consequence to be knowable a priori (when knowable at all), the formal-structural approach violates one of its own principles:

I believe that [Sher's] position [with regard to the apriority of logic] is untenable ... because of the example just considered [namely, the example of Q\* cited above] and ... because it is actually inconsistent with some of the fundamental principles on which her account is based. (1997, 232)

---

<sup>23</sup>The formal-structural approach also seeks to avoid philosophical controversies concerning necessity. (See Sher 1991 and 1996a.)

The principle Hanson refers to is the indifference of logical consequence to empirical properties of individuals in a given universe.

There is, however, no inconsistency here. The formal-structural approach is committed to the indifference of logic to empirical contingencies concerning individual objects,<sup>24</sup> but this commitment does not conflict, either logically or conceptually, with neutrality towards the *theoretical* distinction between a priori and a posteriori knowledge. A person may doubt the viability of a traditional philosophical category without doubting the common, *prephilosophical* conception of a subject matter placed under it (by traditional philosophers). The formal-structural approach accommodates such an attitude. It affirms the indifference of logic to empirical properties of individuals without affirming its apriority (in the philosophical sense).

It is worth noting that it is largely through its criterion for logical constants that the formal-structural approach guarantees the indifference of logical consequence to empirical properties of individuals: Let  $Q$  be a constant satisfying this criterion. For the sake of simplicity, suppose  $Q$  is a 1-place quantifier whose arguments are 1-place first-order predicates. (i) *Q does not distinguish the empirical properties of individuals within a given universe.* Let  $A$  be a universe and let  $a$  and  $a'$  be two individuals in  $A$  which differ in at least one empirical property, say  $a$  is red while  $a'$  is blue. Let  $B$  be a subset of  $A$  such that  $a \in B$  but  $a' \notin B$ , and let  $B'$  be obtained from  $B$  by replacing  $a$  by  $a'$ .  $Q$  rigidly refers to a formal operator,  $\underline{Q}$ . Hence,  $Q$  distinguishes between  $\langle A, B \rangle$  and  $\langle A, B' \rangle$  only if  $\underline{Q}$  does. But  $\underline{Q}$  is invariant under isomorphic argument-structures; hence,  $\underline{Q}$  does not distinguish between  $\langle A, B \rangle$  and  $\langle A, B' \rangle$ . (ii) *Q does not distinguish the empirical properties of individuals across universes.* Let  $A$  and  $A'$  be distinct universes that share the same formal properties (that is, the same cardinality (Mostowski 1957)), and let the individuals in  $A$  differ from the individuals in  $A'$  in some empirical property. Let  $B$  be a subset of  $A$  and let  $B''$  be obtained from  $B$  by replacing each member of  $B$  by a member of  $A'$  in a 1-1 manner. Then for the same reasons as above, neither  $\underline{Q}$  nor  $Q$  distinguishes between  $\langle A, B \rangle$  and  $\langle A', B'' \rangle$ . Note that these results hold regardless of whether  $Q$  is an empirical constant or not.

---

<sup>24</sup>And, indeed, to empirical necessities concerning individuals as well.

### 2.3 Alleged Failure to Capture the Common Conception of Logical Consequence

Hanson claims that by being neutral towards the apriority of logic, the formal-structural view fails to capture the common, presystematic conception of logical consequence.

Now, if we take the common, presystematic conception to be a *theoretical* conception, one of whose tenets is the apriority of logic (in the philosophical sense), then of course the formal-structural view fails to capture it. But if we take the common conception to be a *pretheoretical* conception, say, the conception expressed in the above citations, it does capture it.

*Formality:* The formality of logic is captured by the view that logical constants refer to formal operators and logical consequences are grounded in formal laws.

*Indifference to empirical properties of individuals within and across universes:* We have seen how the invariance criterion for logical operators guarantees their indifference to such properties.

*Generality:* In his 1966 lecture, Tarski pointed out that by varying the domain of invariance we arrive at more and less general notions. In geometry, for example, invariance under similarity transformations (transformations preserving the ratio of distances between points) yields the relatively narrow notions of Euclidean geometry, while invariance under bi-continuous transformations (transformations in which only closedness is preserved) yields the more general notions of topology. Similarly, notions invariant under transformations preserving the physical properties of objects are narrower than notions invariant under transformations preserving only their formal properties. The generality of formal notions is matched by the generality of formal laws: all structures of the special disciplines—physical structures, biological structures, psychological structures, etc.—are bound by formal laws, but formal structures are not bound by their laws.

*Topic neutrality:* Logic discerns the formal patterns of objects possessing properties and standing in relations, but not whether these objects/properties are physical, biological, psychological, etc. As a result, logic does not distinguish between physical, biological, psychological, and other types of discourse—that is, logic is topic-neutral.

*Necessity:* Logical consequence is grounded in formal laws, and formal laws are (intuitively) necessary. (For further elaboration of these points see Sher 1996a and 1999a.)

Note that although the formal-structural view does not portray logic as a priori, it does explain why logic is largely unaffected by empirical discoveries. Empirical discoveries commonly concern either properties of objects that are not preserved under isomorphisms (bijections) or regularities in the behavior of objects that do not constitute formal laws (or both). The former are not taken into account by logical operators, the latter are not preserved in all formal structures constituting logical models. Either way, logic is indifferent.<sup>25,26</sup>

*University of California, San Diego*

### References

- Aristotle. *Topics*, book 1. In *The Basic Works of Aristotle*, ed. R. McKeon, 188–206. New York: Random House, 1941.
- Ferferman, S. 1999. "Logic, Logics, and Logicism." *Notre Dame Journal of Formal Logic* 40:31–54.
- Frege, G. 1879. "Begriffsschrift, a Formula Language, Modeled upon That of Arithmetic, for Pure Thought." In *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*, ed. J. v. Heijenoort, 1–82. Cambridge: Harvard University Press, 1967.
- Haack, S. 1978. *Philosophy of Logics*. Cambridge: Cambridge University Press.
- Hahn, H. 1933. "The Crisis in Intuition." In *The World of Mathematics*, vol. 3, ed. J. -R. Newman, 1956–76. New York: Simon and Schuster.
- Hanson, W. H. 1997. "The Concept of Logical Consequence." *Philosophical*

---

<sup>25</sup>A possible exception to logic's indifference to empirical discoveries is that of especially general discoveries that affect our conception of formal laws and through this our conception of logic. Such discoveries were envisaged by Tarski (1944) and Quine (1951). (See the interpretation of Quine in Sher 1999b.)

<sup>26</sup>Hanson may claim that the a priori knowability of logical consequences belongs, along with formality, generality, topic neutrality, indifference to empirical properties, and necessity, on the list explicating the content of our prephilosophical conception of logical consequence. If in spite of the methodological advantages of an account that is not committed to the a priori–a posteriori distinction and notwithstanding the fact that the formal-structural account captures a pretheoretical, commonsense version of the apriority requirement (namely, indifference to the vast majority of empirical discoveries), Hanson demands a stronger apriority constraint, he has to justify his demand. His claim that such a constraint would weed out logical inferences involving unnatural and empirical logical constants like  $Q^*$  is, as we have seen above, based on a misunderstanding.

- Review 106:365-409.
- Kant, I. 1781/1787. *Critique of Pure Reason*. London: Macmillan, 1929.
- Kline, M. 1980. *Mathematics: The Loss of Certainty*. Oxford: Oxford University Press.
- Lindström, P. 1966. "First Order Predicate Logic with Generalized Quantifiers." *Theoria* 32:186-95.
- Machover M. 1994. "Review of Gila Sher, *The Bounds of Logic: A Generalized Viewpoint*." *British Journal for the Philosophy of Science* 45:1078-83.
- McGee, V. 1996. "Logical Operations." *Journal of Philosophical Logic* 26:567-80.
- Mostowski, A. 1957. "On a Generalization of Quantifiers." *Fundamenta Mathematicae* 44:12-36.
- Quine, W.V. 1951. "Two Dogmas of Empiricism." In *From a Logical Point of View*, 2d ed., 20-46. Cambridge: Harvard University Press.
- . 1970. *Philosophy of Logic*. Englewood Cliffs, N.J.: Prentice Hall.
- Russell, B. 1914. *Our Knowledge of the External World*. London: Routledge, 1993.
- Sher, G. 1991. *The Bounds of Logic: A Generalized Viewpoint*. Cambridge: MIT Press.
- . 1996a. "Did Tarski Commit 'Tarski's Fallacy'?" *Journal of Symbolic Logic* 61:653-86.
- . 1996b. "Semantics and Logic." *Handbook of Contemporary Semantic Theory*, ed. S. Lappin, 509-35. Oxford: Blackwell.
- . 1999a. "Is Logic a Theory of the Obvious?" In *The Nature of Logic, European Review of Philosophy* 4:207-38.
- . 1999b. "Is There a Place for Philosophy in Quine's Theory?" *Journal of Philosophy* 96:491-524.
- Tarski, A. 1936. "On the Concept of Logical Consequence." In *Logic, Semantics, Metamathematics*, 2d ed., ed. J. Corcoran, trans. J. H. Woodger, 409-20. Indianapolis: Hackett, 1983.
- . 1944. "A Philosophical Letter of Alfred Tarski," by M. White. *Journal of Philosophy* 84 (1987): 28-32.
- . 1966. "What Are Logical Notions?" Ed. J. Corcoran. *History and Philosophy of Logic* 7 (1986): 143-54.
- Warmbröd, K. 1999. "Logical Constants." *Mind* 108:503-36.

## LINKED CITATIONS

- Page 1 of 3 -



You have printed the following article:

### **The Formal-Structural View of Logical Consequence**

Gila Sher

*The Philosophical Review*, Vol. 110, No. 2. (Apr., 2001), pp. 241-261.

Stable URL:

<http://links.jstor.org/sici?sici=0031-8108%28200104%29110%3A2%3C241%3ATFVOLC%3E2.0.CO%3B2-G>

---

*This article references the following linked citations. If you are trying to access articles from an off-campus location, you may be required to first logon via your library web site to access JSTOR. Please visit your library's website or contact a librarian to learn about options for remote access to JSTOR.*

### **[Footnotes]**

#### <sup>5</sup> **The Concept of Logical Consequence**

William H. Hanson

*The Philosophical Review*, Vol. 106, No. 3. (Jul., 1997), pp. 365-409.

Stable URL:

<http://links.jstor.org/sici?sici=0031-8108%28199707%29106%3A3%3C365%3ATCOLC%3E2.0.CO%3B2-O>

#### <sup>9</sup> **Logical Constants**

Ken Warmbr#d

*Mind*, New Series, Vol. 108, No. 431. (Jul., 1999), pp. 503-538.

Stable URL:

<http://links.jstor.org/sici?sici=0026-4423%28199907%292%3A108%3A431%3C503%3ALC%3E2.0.CO%3B2-Z>

#### <sup>11</sup> **Logical Constants**

Ken Warmbr#d

*Mind*, New Series, Vol. 108, No. 431. (Jul., 1999), pp. 503-538.

Stable URL:

<http://links.jstor.org/sici?sici=0026-4423%28199907%292%3A108%3A431%3C503%3ALC%3E2.0.CO%3B2-Z>

#### <sup>12</sup> **The Concept of Logical Consequence**

William H. Hanson

*The Philosophical Review*, Vol. 106, No. 3. (Jul., 1997), pp. 365-409.

Stable URL:

<http://links.jstor.org/sici?sici=0031-8108%28199707%29106%3A3%3C365%3ATCOLC%3E2.0.CO%3B2-O>

**NOTE:** *The reference numbering from the original has been maintained in this citation list.*

## LINKED CITATIONS

- Page 2 of 3 -



### <sup>12</sup> **Did Tarski Commit "Tarski's Fallacy"?**

G. Y. Sher

*The Journal of Symbolic Logic*, Vol. 61, No. 2. (Jun., 1996), pp. 653-686.

Stable URL:

<http://links.jstor.org/sici?sici=0022-4812%28199606%2961%3A2%3C653%3ADTC%22F%3E2.0.CO%3B2-W>

### <sup>20</sup> **Review: [Untitled]**

Reviewed Work(s):

*The Bounds of Logic: A Generalized Viewpoint* by Gila Sher

Moshé Machover

*The British Journal for the Philosophy of Science*, Vol. 45, No. 4. (Dec., 1994), pp. 1078-1083.

Stable URL:

<http://links.jstor.org/sici?sici=0007-0882%28199412%2945%3A4%3C1078%3ATBOLAG%3E2.0.CO%3B2-Y>

## References

### **The Concept of Logical Consequence**

William H. Hanson

*The Philosophical Review*, Vol. 106, No. 3. (Jul., 1997), pp. 365-409.

Stable URL:

<http://links.jstor.org/sici?sici=0031-8108%28199707%29106%3A3%3C365%3ATCOLC%3E2.0.CO%3B2-O>

### **Review: [Untitled]**

Reviewed Work(s):

*The Bounds of Logic: A Generalized Viewpoint* by Gila Sher

Moshé Machover

*The British Journal for the Philosophy of Science*, Vol. 45, No. 4. (Dec., 1994), pp. 1078-1083.

Stable URL:

<http://links.jstor.org/sici?sici=0007-0882%28199412%2945%3A4%3C1078%3ATBOLAG%3E2.0.CO%3B2-Y>

### **Did Tarski Commit "Tarski's Fallacy"?**

G. Y. Sher

*The Journal of Symbolic Logic*, Vol. 61, No. 2. (Jun., 1996), pp. 653-686.

Stable URL:

<http://links.jstor.org/sici?sici=0022-4812%28199606%2961%3A2%3C653%3ADTC%22F%3E2.0.CO%3B2-W>

**NOTE:** *The reference numbering from the original has been maintained in this citation list.*

<http://www.jstor.org>

## LINKED CITATIONS

- Page 3 of 3 -



### **Logical Constants**

Ken Warmbr#d

*Mind*, New Series, Vol. 108, No. 431. (Jul., 1999), pp. 503-538.

Stable URL:

<http://links.jstor.org/sici?sici=0026-4423%28199907%292%3A108%3A431%3C503%3ALC%3E2.0.CO%3B2-Z>

**NOTE:** *The reference numbering from the original has been maintained in this citation list.*