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Is Logic a Theory of the Obvious?

> [L]ogic is peculiar: every logical truth is obvious, actually or potentially. Each, that is to say, is either obvious as it stands or can be reached from obvious truths by a sequence of individually obvious steps. (Quine 1970, pp. 82–3)

> Consider ... the logical truth "Everything is self-identical", or "$(x)(x=x)$". We can say that it depends for its truth on traits of the language ... and not on traits of its subject matter; but we can also say, alternatively, that it depends on an obvious trait, viz., self-identity, of its subject matter, viz., everything. (Quine 1954, p. 113)

1. Introduction

The idea that logic is a theory of the obvious is a puzzling idea: Is logic a theory of all obvious truths? Of only obvious truths? Of truths not necessarily obvious themselves, but obtainable from obvious truths by finitely many obvious steps? From obvious truths of what kind? (Any kind? Some specifically logical kind?) By obvious steps of what type? By steps and truths obvious to whom? Obvious in virtue of what? To understand the view that logic is a theory of the obvious we have to understand what features make its truths and inferences obvious, what features distinguish them from other obvious truths and inferences, etc. In short, to understand the view that logic is a theory of the obvious we have to know what properties *other than obviousness* are characteristic of logical truths.

In spite of this air of question-begging—or, perhaps, just because of it—the view that logic is a theory of the obvious lies behind a popular trend in the philosophy of logic. The view is commonly traced to Quine, who introduced it not to say something illuminating about logic but to discard the claim of another theory, namely, Carnap's 'linguistic' theory, to say something illuminating about it. The gist of Quine's argument is that whatever phenomena are explained by the linguistic theory, these phenomena are explained just as well by saying that logic is obvious. Since the latter statement says very little about the nature of logic, the more elaborate linguistic account is redundant.[1]

Today the claim that logic is a theory of the obvious is 'in the air', so to speak, and its influence is deeper and more subtle than in Quine. This claim is implicit in a certain prevalent attitude in the philosophy of logic: an attitude of *skepticism* towards the feasibility of a philosophical explanation of logic, combined with an attitude of *contentment* with (or at least *acquiescence* to) the absence of such an explanation. A philosophical explanation of logic, according to this attitude, is not possible, but this is no cause for alarm: such an explanation would in any case be superfluous. This special combination of criticism and acquiescence is naturally explained by the obviousness of logic. If the logical truths and inferences are obvious, there is nothing to explain: the very idea of an explanation of logic is based on a misunderstanding. By definition, the obvious cannot, need not, and should not be explained.

In this paper I would like to challenge both the claim that an explanatory account of logic is impossible and the claim that such an account is not needed. In sections 2–3 I will discuss some of the difficulties involved in an explanatory account of logic and I will show how these difficulties are naturally linked to the view that logic is obvious. In sections 4–10 I will argue that an explanatory account of logic is highly important and I will demonstrate the feasibility of such an account by constructing an outline of a concrete example. In section 11 I will conclude by showing how the difficulties motivating the skeptical approach are dealt with by the proposed account.[2]

1. See Quine 1954, pp. 112f; 1970, pp. 97f.
2. There are of course many exceptions to the attitude I am criticizing in this paper, and the present volume (whose designated topic is "the nature of logic") is a testimony to their existence. Still, the number of serious attempts to explain the nature of logic is relatively small and the prevalent attitude among epistemologists, philosophers of science, philosophers of language, philosophers of mathematics and philosophers of logic proper is that an explanatory account of logic is neither possible nor needed. In discussing some of the reasons motivating this approach I will not direct myself to any particular philosopher or piece of philosophical writing. Rather, I will present a series of basic difficulties facing the theorist of logic, and I will examine the possibility of responding to these difficulties. The positive account I will outline in this paper incorporates elements from Sher 1989, 1991, 1996a, 1996b, and Forthcoming.

2. Why is Logic Resistant to Explanation?

What makes logic resistant to explanation is, in my view, not the identity of its properties, but the extreme degree to which it possesses its properties. Not the *generality* of logic, but its *unbounded generality*; not the '*basicness*' of logic, but its *ultimate basicness*; not the *normative force* of logic, but its *supreme normative force*; not its *certainty*, but its *absolute certainty*. It is the excesses of logic that make it resistant to explanation.

A. Unbounded generality. The generality of logic is unbounded: logic applies to every subject matter and no subject matter is conceivable beyond logic. Two problematic aspects of this unrestricted generality are (1) the threat of emptiness, and (2) the impossibility of an external viewpoint on logic. Both aspects were elaborated by Wittgenstein in his *Tractatus Logico-Philosophicus*.

1. The emptiness of logic. The argument for the emptiness of logic can be formulated as follows: Logic applies to every subject matter (logic is 'topic neutral'), therefore, logic has no subject-matter of its own; logic is subject-less (3.031).[3] Its constants do not represent any objects, properties or relations (4.0312, 5.4, 5.42), and its statements do not say any particular thing about any particular matter. It is, however, inherent in statements with content that they say something specific about some particular matter. Therefore the statements of logic are contentless: 'all the propositions of logic say the same thing, to wit nothing' (5.43); the statements of logic are tautologous, hence empty. '[T]he propositions of logic say nothing.' (6.1–11)

By claiming that logic says nothing, Wittgenstein does not mean to say that logic is not about the world (that logic is purely linguistic). On the contrary: logical statements do show something important about the world, but *showing* is contrasted with *saying*. Logic *shows* something, but *says* nothing: 'The fact that the propositions of logic are tautologies *shows* the formal—logical—[structure] of language and the world.' (6.12) But 'what *can* be shown, *cannot* be said.' (4.1212) Logic 'displays' the logical form of reality (4.121), but what it displays cannot be described in words: 'Logic is not a body of doctrine, but a mirror-image of the world.' (6.13) But if logical statements *say* nothing, there is nothing to explain: no logical content to be

3. References to the *Tractatus* are given by proposition number only.

elucidated, no logical objects to be identified, no logical properties and relations to be characterized. A substantive account of logic is impossible.

2. The impossibility of an external viewpoint on logic. The unbounded generality of logic means that the laws of logic are universally applicable: nothing is excluded from logic, nothing is beyond logic. Logic governs the world in its entirety: there is no realm of being beyond logic. In Wittgenstein's words:

> Thought can never be of anything illogical, since, if it were, we should have to think illogically. (3.03)
>
> It is impossible to represent in language anything that "contradicts logic" (3.032).
>
> In order to be able to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world. (4.12)
>
> [W]e cannot say in logic, "The world has this in it, and this, but not that." For that would appear to presuppose that we were excluding certain possibilities, and this cannot be the case, since it would require that logic should go beyond the limits of the world; for only in that way could it view those limits from the other side as well. We cannot think what we cannot think; so what we cannot think we cannot *say* either. (5.61)

Two related arguments can be extracted from these citations: (a) To say something substantive about logic is to exclude certain possibilities with regard to logic; but there is no realm of possibility beyond logic; hence we cannot speak substantively about logic. (b) To identify the distinctive characteristics of logic, we have to examine logic without using (or assuming) logic; but that would require stepping outside the boundaries of logic; since there is 'nowhere to stand' outside of logic, we cannot give an explanatory account of logic. There is simply no vantage point from which to explain logic.

B. Ultimate basicness. Logic lies at the foundation of all knowledge. Logic delineates the most basic forms of human thought and its expression (statements, theories), provides us with the most basic tools of valid inference, tells us what combinations of statements are permissible and what combinations are impermissible, etc. Logical form, logical inference, logical criteria of consistency, are ingredients that no system of knowledge can do without. Our system of knowledge can survive the removal of many a science, but not logic. Knowledge is not possible without logic. If we think of our total system of knowledge as a hierarchical structure in which theories higher in the structure are dependent on theories below them in the structure, then logic constitutes the base of the structure. Every science and theory is dependent on logic, but logic is dependent on none. Logic abstracts from the content of all sciences, but no science abstracts from logic. If no science is prior to logic, however, then an explanatory account of logic is impossible. If nothing is more basic than logic, then there are no concepts in terms of which to explain logic. Any attempt to provide an explanatory account of logic is bound to succumb either to the fallacy of infinite regress or to the fallacy of circularity. Even the view that logic is based on convention, Quine emphasizes, leads to a fallacy: 'if logic is to proceed mediately from conventions, logic is needed for inferring logic from the conventions.' (Quine 1935, p. 104)

C. Supreme normative force. The normative force of logic exceeds that of all other sciences. All systems of norms are bound by logic, but the logical norms themselves are bound by no other norms. Scientific methodology, ethics, jurisprudence, etc., are all subject to the authority of logic, but logic is not subject to theirs. However, if the logical norms are not subsumed under any other norms, how shall we explain their authority? By reference to what shall we explain the authority of the law of non-contradiction? If logic's authority is not based on any other authority, based on what can we explain it?

D. Absolute certainty. Similar problems arise with respect to the ultimate certainty of logic. To explain its certainty we have to appeal to something more certain than—or at least as certain as—logic; but nothing satisfies this condition. The certainty of logic is unequaled.

It is thus not simply its properties that render logic resistant to explanation. It is the *extreme* degree to which it possesses its properties—its *extreme* generality, *extreme* basicness, *extreme* certainty and *extreme* authority—that renders it inexplicable.

3. Logic as a Theory of the Obvious

The view that logic is a theory of the obvious is naturally arrived at in two ways: (A) The same features that render logic resistant to explanation also suggest the obviousness of logic. (B) The view that logic is a theory of the obvious offers a convenient solution to the problem of an explanatory account of logic.[4]

A1. The limits of imagination. The absolute generality of logic means, among other things, that the illogical is unimaginable. The law of non-contradiction, the law of self-identity, Modus Ponens and the laws of universal instantiation and existential generalization, are laws whose falsity, like the falsity of the basic arithmetical laws ('1+1=2', '2+3=3+2') is unimaginable to us. We cannot construct in our imagination a situation in which we eat one apple, then another, yet we do not eat two apples, or a situation in which we both eat and do not eat the same apple. But this is just what 'obvious' means: something is obvious if its negation is unimaginable.[5]

A2. The immediacy of seeing. Logic, according to Wittgenstein, is demonstrative: it *shows* the logical (formal) structure of the world but it does not describe it. 'The logic of the world ... is shown ... by the propositions of logic' (6.22); 'Logical ... propositions *shew* [the] logical properties of ... [the] Universe, but *say* nothing.' (*Notebooks*, p. 108) 'The logical form of [a given] situation ... cannot be described.' (*Ibid.*, p. 20) The logical form of a situation can, however, be seen, and seeing is naturally associated with *intuition*. (Mathematics, which is 'a method of logic' (6.234), is, according to Wittgenstein, based on intuition: 'The process of calculating serves to bring about [the] *intuition* ... needed for the solution of mathematical problems' (6.2331, 6.233. My italics).) Intuition, in turn, is commonly associated with *immediacy* (see, e.g., Kant: '*intuition* ... relates <u>immediately</u> to the object ...' (*Critique of Pure Reason* A320/B377, my underline); '*intuition* is that through which [a mode of knowledge] is in <u>immediate</u> relation to [objects]'

4. In this paper I assume that if T is a theory of obvious phenomena, then T is a collection of obvious truths and in this sense T itself can be said to be "obvious". This assumption involves some simplification. (It is possible to construct a very complicated theory of a very simple subject matter; a large collection of obvious truths may not be systematizable by a small number of obvious principles, etc.) But my basic observation is that obviousness is an epistemic property and an obvious phenomenon can, mutatis mutandis, be accounted for by a theory comprised of obvious truths.
5. There are, of course, well known challenges to the validity of the logical laws (or at least some of the logical laws). But adherents of the obviousness view are likely to reject them, explain them away, etc.

(A16/B33, my underline)).[6] And what is immediately intuited is, by definition, *obvious*: 'Obvious—easy to see or understand; plain; evident' (*Webster New World Dictionary*). Thus, a natural chain of associations leads from seeing to intuition, from intuition to immediacy, and from immediate seeing to obviousness.

A3. Vestiges of foundationalism. The view that the most basic, most general and most certain statements in our system of knowledge are *obvious* (self-evident) is typical of foundationalism. Different foundationalist epistemologies place different statements and theories at the foundation of knowledge, but either directly or indirectly, the view that basicness, generality, and certainty entail obviousness (or self-evidence) supports the claim that logic is obvious. A classical example of a foundationalist epistemology is Aristotle's theory. Our system of knowledge, according to Aristotle, is a hierarchical structure whose lowest rank consists of 'first principles'—so-called 'axioms' or 'common notions'—fundamental to all sciences. These first principles include the law of excluded middle, the law of non-contradiction, various laws of equality, and other laws that today we will classify as logical (*Metaphysics* 996^b 25–30). It is characteristic of axioms, according to Aristotle, that they are 'primary, immediate' (*Posterior analytics*, I, 71^b 20), 'self-evident' (*Metaphysics*, 1006^a 10), and apprehended by 'intuition' (*Posterior analytics* II, 100^b 5–15). Another example of a foundationalist epistemology is Descartes' theory of 'clear and distinct ideas'. The mark of absolute certainty, according to Descartes (*Meditations*), is intellectual clarity and distinctness: the more clearly and more distinctly an idea presents itself to us, the more basic and certain it is. Although (for reasons that I will not discuss here) Descartes himself did not apply this principle to logic, such an application naturally suggests itself. Russell's foundationalist epistemology does characterize logic as a theory of self-evident truths: '[T]here is general knowledge not derived from sense, and ... some of this knowledge ... is primitive. Such general knowledge is to be found in logic. ... [The proposition] "If anything has a certain property, and whatever has this

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6. (a) Kant, of course, did not regard logic as 'intuitive', but here we are interested merely in his view on the relationship between intuition and immediacy. (For more on this relation in Kant, see Parsons 1969, 1979-80). (b) It is not clear to me whether Wittgenstein himself viewed logic as obvious, given his negative remarks on self-evidence. (See 1921: 5.4731, 6.1271; 1914-16: 3.9.14 (last paragraph), 8.9.14.)

property has a certain other property, then the thing in question has the other property" ... is absolutely general: it applies to all things and all properties. And it is quite *self-evident*. Thus in such propositions of pure logic we have ... *self-evident* general propositions' (1914, p. 66, my italics).[7]

B. Solution to the skeptical problem. We have seen how extreme generality, basicness and certainty are naturally associated with obviousness. But the view that logic is obvious can also be arrived at as a means for disarming the skeptical argument. The view that logic is obvious takes the sting out of this argument. This view allows us to accept the skeptical conclusion without accepting the skeptical challenge. If logic is obvious, its inexplicability is innocuous. There is no need to examine logic from a vantage point beyond logic, no need to base logic on something more basic than logic, no need to justify the authority of logic by a higher authority than logic, no need to ground our certainty in logic in something more certain than logic. The skeptical conclusion does not threaten either the validity of the logical laws, or our reasons for believing these laws, or our ability to know these laws. The logical laws are sanctioned by their obviousness.

4. The Need for Explanation

But does the claim that logic is obvious obviate the need for an explanatory account of logic? In order to answer this question we have to find out, first, whether the claim is true (whether logic is really obvious), and second, whether there are overriding reasons for demanding an explanation of logic.

The view that logical truths and inferences are obvious is open to various challenges. First, it is difficult to rule out the fallibility of human intuition. In any given case we may fail to see that something obvious is obvious or that something unobvious is unobvious; in some cases there may not be a fact of the matter, or there may not be an agreement, or there may not be a way of deciding whether something is obvious; our sense of obviousness may be reliable in some areas but not in others, or our sense of obviousness may be systematically skewed. Descartes solved these problems by positing a

7. Not all foundationalists regard logic as obvious and not all those who regard logic as obvious are foundationalists. (Carnap and Quine are two notable exceptions.) But I believe the view that logic is obvious is naturally associated with a foundationalist epistemology.

benevolent god whose responsibility it was to guide our judgments of clarity and distinctness. But most contemporary writers cannot accept solutions of this kind.

Second, it has never been demonstrated that obviousness is a property of logic, let alone of all of logic. Indeed, recent meta-logical results show that even the view that logical truths are *mediately obvious*, i.e., arrived at by finitely many applications of obvious rules to obvious truths, is problematic. Regardless of whether and how we can establish the obviousness of the basic rules and truths, no reasonable logical system is decidable and many non-elementary logical systems (e.g., full 2nd-order logic or various 'generalized' 1st-order logics[8]) are incomplete. Thus, in no reasonable system is there an *effective* method for establishing logical truths based on 'first' logical principles, and in many non-elementary logical systems some logical truths cannot be established *at all* by finite applications of obvious rules to obvious truths.

Third, there is no evidence that obviousness is restricted to logic, i.e., that no nonlogical truths are obvious. Quine, in particular, argues that with regard to obviousness logical truths are on a par with a host of other truths: 'utterances of "It is raining" in the rain' (1970, p. 97), statements like 'there have been black dogs' and 'bachelors are unmarried' (1960, p. 66), etc.[9] Obviousness, for Quine, is a property induced by behavior, and human behavior is partly culture dependent. Each culture has its own array of obvious truths, and these include many of its everyday truisms in addition to (some or all of) its logic.

Finally, the view that logic is intrinsically obvious has (as far as I know) never been established by a rigorous argument. This view is suggested by various natural associations of ideas (as noted above), but these are rather loose, impressionistic associations, based on bits and pieces of largely outdated theories, theories whose particular claims as well as underlying principles (e.g., foundationalism) many of us feel compelled to reject.

It is, however, not just the shaky foundation of the view that logic is obvious that motivates an explanation of logic. Rather, the very features that make logic resistant to explanation—its unbounded generality, its ultimate

8. See, e.g., Barwise and Feferman 1985.
9. Quine talks in terms of 'stimulus analyticity', but stimulus analyticity is, for him, connected with obviousness (in the behavioristic sense).

basicness, its supreme normative force and its absolute certainty—also make an explanation of logic vital to our knowledge. An error in some relatively isolated area of knowledge will not undermine our system, but a serious error in logic will. The greater the generality, the basicness and the authority of a given science, the greater the need to establish its truths, justify its norms, determine its exact relationships with other sciences, and provide tools for detecting and correcting its errors. It is thus due to logic's centrality to our system of knowledge (which, in turn, is due to the magnitude of its properties), that a critical explanation of logic is imperative.

But is a critical-explanatory account of logic possible? Can we, in spite of the difficulties expounded in section 2, construct an account of logic that will uncover the source of its normativity, explain the nature of logicality and generate critical tools for evaluating, improving, and, if need be, revising existent logical theories?

In the remainder of this paper I will attempt to construct an outline of such an account. The key to avoiding the difficulties raised in section 2 is renouncing the foundationalist methodology within which these difficulties arise. Logic, on the proposed (moderately holistic) approach, stands in a multitude of normative, conceptual, practical and theoretical relationships to other sciences, and these relationships provide the critical and explanatory tools needed to justify, explain, criticize and revise it. The hierarchical picture of knowledge with logic at its base no longer holds, and logic is placed alongside, rather than above—or below—the sciences. Needless to say, not all aspects of logic are equally well explained by the present account: in particular, the proof-theoretical dimension of logic is largely beyond the present (semantic) analysis.

5. Truth as a Source of Normativity

What is the source of the normativity of logic? One of the simplest, most satisfying and most unifying answers is: 'Truth'. The logical laws are true, and their truth is the source of their authority.[10] Thus, in the same way that the truth of '1+1=2' licenses (or plays a major role in licensing) the inference schema: '*a* is a B; *b* is a B; a ≠ b; Therefore there are at least two B's',

10. In discussing the truth of the logical laws I disregard the fact that these laws are usually formulated schematically and in this form are not properly true (or false).

so the truth of the law of excluded middle licenses (plays a major role in licensing) the inference schema: 'If A then B; If not-A then B; Therefore B'. There is more to be said about how logical inference is grounded in truth, but first let us examine the nature of logical truth.

What are the logical truths true of? Two natural answers are: (a) the logical truths are true of the world, (b) the logical truths are true of language or thought. The view that logical truths are true of the world can be interpreted in two ways: (a1) There is a correspondence relation between the logical constants and objects (properties, relations) in the world, and it is through this correspondence relation that the logical truths are true. (a2) Logical truths correspond to reality not through a term-by-term relation, but in a holistic, unanalyzable way (Wittgenstein 1921). Likewise, the second view of what the logical truths are true of is divided into two views: (b1) the laws of logic are laws of the mind, (b2) the laws of logic are laws of language. These views could be combined within a single theory, but traditionally they have led to two distinct accounts of logic: (1) logical laws have to do with the structure of thought, (2) logical laws reflect linguistic conventions. The view that the normativity of logic is grounded in convention was advocated by Carnap (see, e.g., his 1939). The truth of the law of excluded middle is, according to Carnap, grounded in our decision to use 'or' and 'not' in accordance with the classical truth tables. Had we decided to use them based on different truth tables or based on rules that do not allow a truth-table representation, we would have ended up with different logical truths and different logical inferences. Finally, the view that the logical laws are laws of thought can also be divided into two: (1.1) the logical laws are descriptive, i.e., the logical laws describe the way we actually think (Fodor 1975); (1.2) the logical laws are prescriptive, i.e., the logical laws tell us how we ought to think (Frege 1879).

I believe that none of these views is adequate by itself, but an exhaustive theory of logical truth must incorporate elements from all. Truth in general is constrained both by the way the world is and by the way our mind operates (or, more generally, by our biology, psychology, and environment in a broad sense). These constraints, however, do not curtail our freedom altogether and we do have a certain amount of choice regarding our language: its lexicon, its logical apparatus, and so forth. Furthermore, although language contributes to truth largely through the reference of its terms, structure is an additional irreducible factor. Not in the sense of the early

Wittgenstein, but in the sense in which the ordering of quantifiers, for example, plays an independent role in determining the truth conditions of so-called branching, or partially-ordered, quantifications. (Henkin 1959 and others)

I will not attempt to explain here the contribution of all these diverse factors—language and the world, reference and form, thought and convention, actual and idealized mental operations—to logical truth. Instead, I will concentrate on the correspondence aspect of logical truth, i.e., on the role played by the world on the one hand and the reference of our terms on the other in determining logical truth.

6. Laws of Formal Structure as the Source of Logical Truth

Consider the statement:

(1) If some American child is hungry, then some child is hungry.

In virtue of what is (1) logically true? Before we consider the question 'In virtue of what is (1) *logically* true?' let us simply ask: 'In virtue of what is (1) *true*?'. Intuitively, my answer is this: The truth of (1), like the truth of any other sentence, is a function of two things: (a) what (1) says about the world, and (b) how the world is. (1) says that if some American child is hungry, some child is hungry, and in the world this indeed is the case. However, the truth of (1) has nothing to do with the state of hunger in the world or with the meaning of terms referring to this state. What the truth of (1) essentially depends on is the *formal* content of (1) and the *formal* state of the world. Formally, (1) says that if an intersection of three sets is not empty, then the intersection of two of these sets is also not empty, and in the world this in fact is the case. In the world, a non-empty intersection of any three sets is included in the non-empty intersection of any two of these sets. The truth of (1) does not depend on the reference of its non-formal terms ('American', 'child', 'hungry'), but it does depend on the reference of its (explicit and implicit) formal terms: 'and', denoting the operation of intersection; 'some', denoting the property of non-emptiness; 'if ... then', denoting the relation of inclusion. In short: it is in virtue of the correspondence between the formal content of (1), i.e., the content displayed by

(2) $(\exists x)(Ax \& Bx \& Cx) \supset (\exists x)(Bx \& Cx)$,

and certain formal features of the world (the inclusion of one kind of intersection in another), that (1) is true.

We have seen in virtue of what (1) is *true*. But in virtue of what is it *logically* true? One reason that (1) is logically true is the *formal* nature of its truth: (1) is true in virtue of a formal feature of the world, rather than a physical or a biological or ..., or a hunger feature. But not all sentences true in virtue of a formal feature of the world are logically true. For example, the sentence,

(3) There exist at least two things,

or, in logical representation,

(4) $(\exists x)(\exists y)x \neq y$,

is not. What distinguishes (1) from (3)—or (2) from (4)—is the fact that (1) is true in virtue of a *formal law*, while (3) is true in virtue of a *formal accident*. It is a law of formal structure that if the intersection of three collections of objects is not empty then the intersection of any two of them is not empty, but it is not a law of formal structure that the cardinality of structures is larger than 1. The truth of (1) is not only formal but also *necessary*, whereas the truth of (3) is only formal. Logical truths are grounded in *laws of formal structure*, and it is this that distinguishes them from all other truths.

But what is a law of formal structure? I characterize a law of formal structure as a *universal property of formally possible structures of objects.*[11] It is a property of all such structures that if the intersection of any three of their sets is not empty, then the intersection of any two of these sets is not empty, etc. What is a formally possible structure of objects? The idea of a formally possible structure of objects can be systematized in various ways. One of its most fruitful systematizations is that of contemporary model theory which, in turn, is based on ZFC. (See, e.g., Chang and Keisler 1973). My account of logic is not committed to any particular systematization of formality (or to any particular underlying mathematics), but for the sake of clarity I will use contemporary model theory as my background theory of formal structure. I should emphasize, however, that by availing myself to the

11. For a response to the question: "Why can't universal properties of formally possible structures of objects be accidental?" see Sher 1996b, p. 681.

resources of other sciences I am not violating the principle of explanation. Rather, I am pursuing the task of providing an explanatory account of logic within a non-foundationalist framework, in accordance with the considerations raised in section 4.

7. The Principle of Logicality

In more details, my account is as follows:

A. Subject matter. Logic does have a specific subject matter: its subject is *the role of formal structure in reasoning.* To account for this subject logic provides (a) a theory of the role of formal structure in truth, and (b) a theory of its role in valid inference (consistent theory, etc.).[12]

B. Logical constants. The formal structure of a sentence is determined by the arrangement of its logical and non-logical constituents. The logical constituents of a sentence are its logical constants (which in natural languages are not always exhibited by its 'surface' grammar (see (1) above)). Both logical and non-logical constants are referential, but the logical constants refer to objects of a special kind: namely, *formal* objects.[13]

C. Formal objects. What is a formal object? First, a formal object is not an individual. A formal object is an object with structure, and individuals (by definition) lack structure. A formal object is, then, a property, relation or function (or a structure of objects corresponding to one of these). What kind of property (relation, function) is formal? Intuitively, a property is formal if it 'takes into account' only the formal features of objects in its domain. One interpretation of 'formal feature' is 'feature which does not depend on the identity of individuals in a given universe' (Mostowski 1957). A more precise rendition of this condition (in terms of invariance) will be

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12. As the reader can see, I include certain basic parts of meta-logic in the province of logic proper.
13. The view of logical constants as referential is one of the distinctive characteristics of my account. Most philosophers (e.g., Quine 1970) draw a sharp distinction between 'form' and 'content': logical constants give a sentence its *form*, nonlogical constants give it its *content*. On my account, both kinds of constant contribute to the content of a sentence: the logical constants to its *formal* content, the nonlogical constants to its *nonformal* content. The notion of 'form' is a relative notion: by 'form' we mean the pattern displayed by highlighting certain constituents of a sentence. Thus, a sentence has a logical form, a modal form, a grammatical form, etc.

given below. In the meantime I will just note that any 2nd-level property whose satisfaction by a 1st-level property is determined by its cardinality (i.e., the cardinality of its extension) is formal; but a 2nd-level property which distinguishes between two 1st-level properties of the same cardinality is not formal. Thus, the 2nd-level property of non-emptiness is formal, since it is satisfied by a 1st-level property P iff P satisfies the 'formal' condition of having at least one element in its extension; but the property of being a property of humans is not formal, since it distinguishes between two properties of the same cardinality according to whether their extension does or does not consist of humans.

D. Criterion for logical constants of a lexicon $\mathcal{L}$.

Lexicon. A lexicon $\mathcal{L}$ is a collection of constant symbols. For the sake of simplicity I restrict myself to lexicons with constants of orders ≤2: indvidual constants (order 0), predicates and functions (orders 1 and 2; 2nd-order predicates and functions are viewed, in the Fregean tradition, as quantifiers).

Extension; extensional constant. Each constant C of $\mathcal{L}$ has a *meaning* and an *extension*. The exact meaning of 'meaning' is not important to us here; the extension of C in a universe A is its denotation in A if C is an individual constant; the set of objects satisfying it in A, if C is a 1-place 1st-level predicate, etc. I will symbolize the extension of C in A by C^A. The meaning of C may or may not coincide with a function delineating its denotation/extension in each universe A. If it does, I will say that C is an *extensional* constant, or that the meaning of C is extensional. For example, the 1st-level identity predicate—'x=y'—is an extensional predicate, while the 1st-level predicate 'x loves y' is not an extensional predicate: its meaning is not exhausted by a 'list' of pairs of objects satisfying it in different universes.

Argument-structure. For each 1st- or 2nd-order n-place constant C of $\mathcal{L}$, an *argument-structure* for C is a structure of the form $\langle A, \beta_1, \ldots, \beta_n\rangle$, where A is a universe, and for $1 \leq i \leq n$: if C_i (the i-th argument-place of C) is a place for an individual symbol, $\beta_i \in A$; if C_i is a place for an n-place 1st-order predicate symbol, $\beta_i \subseteq A^n$; if C_i is a place for an n-place 1st-order function symbol, β_i is a function $\subseteq A^{n+1}$.

Criterion for logical constants of $\mathcal{L}$— (LC)

Let C be a constant of $\mathcal{L}$ Then C is a *logical constant* of $\mathcal{L}$ iff (a)–(c) below hold:

(a) C is of order 1 or 2;
(b) C is an extensional constant;
(c) C is invariant under isomorphic argument-structures, i.e., for any two isomorphic argument-structures for C, $\langle A, \beta_1, \ldots, \beta_n\rangle$ and $\langle A', \beta_1', \ldots, \beta_n'\rangle$: $\langle \beta_1, \ldots, \beta_n\rangle \in C^A$ iff $\langle \beta_1', \ldots, \beta_n'\rangle \in C^{A'}$.[14]

Informally: *C is a logical constant iff C is an extensional predicate or function which does not distinguish between isomorphic argument-structures.*[15]

Among the logical constants of familiar languages are the existential and universal quantifiers (thought of as the quantifiers of non-emptiness and universality), the identity relation, the numerical quantifiers ('There are at least / exactly /at most α x such that ...', for every cardinal number α), the quantifiers 'finitely many', 'indenumerably many', both the 1-place 'most' ('Most x are ...') and the 2-place 'most' ('Most x such that ... are—'), the quantifier '... is a well ordering' (a quantifier over relations), etc. Among constants not satisfying (LC) are all the individual constants, the 1st-level predicates 'x is human', 'x is a number', 'x is taller than y', ..., the 2nd-level predicates, 'X is a property of Napoleon', 'X is a property of humans', 'X is an empirical relation', etc. The 1st-order membership relation ('x∈ y', where 'x'

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14. Two structures, $\langle A, \beta_1, \ldots, \beta_n\rangle$ and $\langle A', \beta_1', \ldots, \beta_n'\rangle$ are isomorphic iff there is a 1–1 function *f* from A onto A' such that for $1 \leq i \leq n$, β_i' is the image of β_i under *f*. (ii) In accepting (c) as a criterion of formality I follow a mathematical tradition exemplified by Lindenbaum and Tarski 1934–35, Mautner 1946, Mostowski 1957, Tarski 1966, and Lindström 1966. These authors use the invariance criterion directly as a criterion of logicality, but their characterization fits nicely with my idea of formality.
15. This definition of logical constants of L is a definition 'from the top down'. We can also define logical constants "from the bottom up":

    Definition of logical constants from the bottom up: Let S be a collection of argument-structures of a given type. Let S* be the closure of S under isomorphisms. Then S* determines a (*potential*) *logical constant*, C, whose extension in each universe A is the restriction of S* to structures with A as their universe. We will say that C is *constructible by closure under isomorphisms.*

    Claim. Every logical constant of $\mathcal{L}$(based on (LC)) is constructible by closure under isomorphisms.

    For more details and proof see Sher 1991, Chapter 4 and Appendix.

and 'y' are individual variables) is not logical, but the 2nd-order membership relation ('x∈Y', where 'x' is an individual variable and 'Y' is a set or property variable) is. The quantifier 'the number of planets' is formal if extensional (i.e., if, in the given language, its meaning coincides with its extension); not formal otherwise.

Are the truth-functional connectives logical? I will begin my answer by observing that these connectives function both as sentential and as 'objectual' operators. Thus, take the conjunction, '&', for example. Syntactically, '&' is an operator both on predicates (open formulas) and on sentences (closed formulas). As an operator on predicates '&' represents an operation on sets, namely, the Cartesian Product operation (a particular instance of which is the operation of intersection: $a \in A \cap B$ iff $\langle a,a \rangle \in A \times B$); as an operator on sentences '&' represents an operation on truth values, namely the binary truth function f such that $f(X,Y)=T$ iff $X=Y=T$. Now, the 'objectual' operator '&' satisfies (LC), but the sentential operator '&' is not the kind of constant considered by (LC). We can add sentential operators to our account in two ways: We can introduce a new criterion for logical connectives based on 'sentential argument-structures', or we can use the same criterion by allowing argument-structures to include truth values as elements and by extending our notion of isomorphism. The second option was taken by Lindström 1966, the first by Sher 1991. Either way, the truth-functional connectives come out logical. (Intuitively, the truth-functional connectives are formal in the sense of taking into account only the bare pattern of truth values of their arguments). Neither the modal nor the propositional attitude operators are logical according to our criterion. Both take into account more than just *formal* features of objects in our sense. Thus, as sentential operators the modal operators take into account more than the bare truth-value of their arguments, and as objectual operators (e.g., 'It is necessary for x that ...') they take into account more than the formal features of their arguments (i.e., more than such features as cardinality).[16]

16. By saying that the modal and proportional-attitude operators are not logical I do not mean to suggest that these operators are in any way inferior to the "logical" operators. All I mean to say is that their contribution to truth and inference is based on different principles.

8. Ontology and Explanation

I have explained the nature of logicality in terms of formality, and I have provided a precise definition of formality which delineates the scope of logical terms. My notion of logical term is relative to lexicon, but by abstracting from the limitations of particular lexicons, we can view the definition as a definition of logical terms in general.[17] My account identifies the logical with the formal, and the formal is 'the mathematical' in the structural sense in which mathematics is concerned with 'patterns of objects possessing properties and standing in relations' (see, e.g., Resnik 1981). As a science of individuals, mathematics is not logic, but as a science of patterns, it is. The individual constant 'one' is not logical, but the 2nd-order 'one' is.

My answer to the question: 'What is a logical term?' involves a criterion (LC) delineating the *totality* of logical terms. In delineating the totality of logical terms my goal is not to produce a 'maximal' logic. Rather, by delineating the scope of logical terms my goal is to arrive at an *informative explanation* of the nature of logicality. The practice of explaining the nature of concepts by delineating their scope often produces extravagant ontologies. Thus, the familiar theories (explanations) of 'number', 'set', and even 'logical connective' are extravagant in the sense that the majority of the objects postulated by these theories have no practical or theoretical interest. Nobody will ever contemplate, let alone find a practical use for, the majority of objects in the set-theoretical hierarchy or even the different number series or the Boolean universe of truth functions. Yet to curtail the ontologies of these theories is to curtail their explanatory power.[18] Were we to characterize the logical constants by enumerating two or three or even ten of them, their nature would have remained a mystery to us; but by accepting a rich ontology of logical entities, we gain insight into their nature and the principles governing them. Our economical loss is our explanatory gain.

The gain in explanation can be seen on several levels: (1) The rich ontology allows us to account for a great many valid inferences that other, more

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17. The independence from language is especially clear in the version delineated in footnote 15 above.
18. Even in the case of the Boolean theory of the logical connectives whose infinite ontology is reducible to a modest finite ontology, it is the general criterion of truth-functionality rather than the theorem of reducibility that does the main job in explaining the logicality of these connectives.

frugal logics, cannot account for. Even such a simple inference as 'Exactly 5 A's are B's; Therefore, Finitely many A's are B's', or 'All A's are B's, Most C's are A's; therefore, Most C's are B's', cannot be accounted for by the frugal theory called 'standard 1st-order logic'. Standard 1st-order logic relegates to set theory the proof of this and other valid inferences, but the ontology of set theory is no less extensive than our logical ontology. The economical advantage of standard 1st-order logic is, thus, largely imaginary. (2) While our account is ontologically uneconomical, it achieves considerable economy in philosophical inquiry. Our account has the virtue (which it shares with Logicism) of reducing two of the most baffling questions of philosophy, namely, the question of the nature of logic and the question of the nature of mathematics, to one question: the nature of the formal. And while most philosophers today have given up the hope of an explanatory account of logic, this is evidently not the case for either mathematics or the formal. (3) Finally, by reducing logic to the formal we will be able to throw new light on its most distinctive (and, as we have seen before, most problematic) features: its extraordinary generality, basicness, normative force, and certainty. (See section 11.)

In the next section I will show how the new account of logical constants can naturally be incorporated into the existent definitions of truth and logical validity. In presenting these definitions I will pay special attention to the informative way in which they explain (or can be made to explain) the role of formal structure in truth and inference. Thus, my goal is not just to show that a substantive account of the nature of logic is possible, but that the logical element in truth and inference itself is a substantive element.

9. The Contribution of Logical Structure to Truth

The truth value of a sentence is determined by a number of factors. The truth of, say, 'Something is red and round', is determined by factors having to do with color and shape as well as with formal structure. To provide a full account of the truth conditions of this sentence we have to specify (a) the conditions under which an object satisfies the predicate 'x is red', (b) the conditions under which an object satisfies the predicate 'x is round', (c) the conditions under which an object satisfies the predicate 'x is Y and x is Z' for arbitrary Y and Z, and (d) the conditions under which an object satisfies the predicate 'something is X'. Now, we can specify these conditions either individually or 'en masse', either informatively and in great detail, or superficially and with little attention to detail, either taking the differences between the four conditions into account or not. We can, for example, simply say that in general an object satisfies a given predicate if it falls in its extension, or we can specify the special conditions for falling in the extension of each predicate. In logic we are interested in the contribution of formal (logical) structure to truth, so our definition provides an informative account of the particular satisfaction conditions of each formal (logical) predicate, but for the nonformal (nonlogical) predicates a general, schematic principle as mentioned above (i.e., a principle glossing over their differences) suffices.

The logical definition of truth for a formalized language L[19] is based on a strict division of lexicon and expressions to logical and nonlogical. Expressions containing at least one occurrence of a logical constant are 'logically-structured', other expressions are 'logically-atomic'. In stating the formation rules of terms and formulae, all nonlogical constants of the same syntactic category (individual constants, *n*-place predicates, *n*-place functions) are treated en masse, but each logical constant is treated individually (is assigned an individual formation rule). As a result, each logical constant is viewed as generating a unique *type* of term or formula, while all nonlogical constants (of the same syntactic category) are viewed as generating the same type.[20] To the extent that L contains at least one iterative logical constant, the formation rules identify an infinite number of distinct logical structures of L.

In preparation for the definition of satisfaction for L, each logical constant is assigned a rule specifying its exact satisfaction conditions in an informative and general manner, i.e., not just for the specific universe of discourse of L, but for any universe. These rules are, in effect, rules of *meaning* for the logical constants.[21] In contrast, the meaning of the nonlogical con-

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19. By a formalized language L I mean a logical language representing the logical structure of some interpreted natural or artificial language L'. The extension and meaning (where relevant) of the constants of L, as well as the universe of discourse of L, are determined by L'. (Of course, the same logical language can be used to represent different natural and artificial languages, but here we assume that L is associated with a particular interpreted language L'.)
20. Technically, of course, we could group the logical constants together, but their individual treatment indicates that these constants are *singled out in the syntax for a special (specific, informative) treatment in the semantics.*
21. In 'satisfaction conditions' I include conditions for logical functions if applicable. See Sher 1991, p. 58.

stants of L is of no interest to logic: only their extension in the given universe of discourse is needed. The rules for the logical constants are incorporated in the entries for the logically complex formulae, and these entries do not vary from one formalized language to another, i.e., the logical portion of the definition is *not* relative to language. Due to the recursive nature of the definition, the satisfaction conditions of *each* logical (i.e., formal) structure—simple or complex—are specified in detail. When we turn to *truth* proper, the definition provides an exact and informative account of the role played by logical (formal) structure in the truth or falsity of every logically-structured L-sentence.

I will now formulate a definition of the logical constituent in truth for a 1st-order language L. To emphasize the logical nature of the definition I will mark the entries dealing with logical constituents by asterisk.[22]

SYNTAX

I. Vocabulary

A. *Non-logical constants*

1. Individual constants: 'c_1', ..., 'c_k';
2. 1-place 1st-order predicates: 'P_1', ..., 'P_m';
3. 2-place 1st-order predicates: 'R_1', ..., 'R_n';
4. 1-place 1st-order functions: 'h_1', ..., 'h_p'.

*B. *Logical constants*

*5. Identity: '$\approx$';
*6. Negation: '$\neg$';
*7. Conjunction: '&';
*8. Conditional: '$\supset$';
*9. Existential quantifier: '$\exists$';
*10. Finiteness quantifier: '$\mathcal{F}$';
*11. Majority quantifier ('most', 2-place): '$\mathcal{M}$';
*12. Well-ordering quantifier: '$\mathcal{W}$'.

22. I assume the reader is familiar with the standard notation and terminology of logical syntax and semantics. The reader may wish to compare the definition of truth formulated here with Tarski's 1933 definition of truth, Carnap's 1947 definitions of truth, and the standard definition of "truth in a model".

C. *Auxiliary symbols*

1. Individual variables: 'x_1', 'x_2', 'x_3', ...;
2. Parentheses: '(', ')'.

II. Term

1. 'x_i', $i \geq 1$, is a term.
2. 'c_i', $1 \leq i \leq k$, is a term.
3. If t is a term, then $\ulcorner h_i(t) \urcorner$, $1 \leq i \leq p$, is a term.
4. Only expressions obtained by 1–3 above are terms.

III. Formula

A. *Logically-atomic formulae*[23]

1. If t is a term, $\ulcorner P_i t \urcorner$, $1 \leq i \leq m$, is a formula.
2. If t,t' are terms, $\ulcorner R_i t,t' \urcorner$, $1 \leq i \leq n$, is a formula.

*B. *Logically-structured formulae*

*3. If t,t' are terms, then $\ulcorner t \approx t' \urcorner$ is a formula.
*4. If Φ is a formula, then $\ulcorner \neg\Phi \urcorner$ is a formula.
*5. If Φ,Ψ are formulae, then $\ulcorner (\Phi \& \Psi) \urcorner$ is a formula.
*6. If Φ,Ψ are formulae, then $\ulcorner (\Phi \supset \Psi) \urcorner$ is a formula.
*7. If Φ is a formula, then $\ulcorner (\exists x_i)\Phi \urcorner$, $i \geq 1$ is a formula.
*8. If Φ is a formula, then $\ulcorner (\mathcal{F} x_i)\Phi \urcorner$, $i \geq 1$ is a formula.
*9. If Φ and Ψ are formulae, then $\ulcorner (\mathcal{M} x_i)(\Phi,\Psi) \urcorner$, $i \geq 1$, is a formula.
*10. If is a formula, then $\ulcorner (\mathcal{W} x_i x_j)\Phi \urcorner$, $i \neq j \geq 1$, is a formula.
*11. Only expressions obtained by 1–10 above are formulae.

IV. Sentence

A formula with no free occurrences of variables is a sentence. ('Free occurrence' is defined as usual.)

SEMANTICS

*I. Rules of Satisfaction (Meaning) for Logical Constants

23. If the language contains 1st-order functional logical constants, some of the formulae falling under the present category will be logically-structured. In such languages we will distinguish between terms and formulae which are 'governed by a logical constant' and those which are not.

For each logical constant, C, let f_C^A be a rule describing the satisfaction conditions of C in an arbitrary universe A. f_C^A is a 'rule of meaning' for C.

$*(\approx)$ $\forall a,b\in A$: $f_{\approx}^{A}(a,b)$=T iff a=b.
$*(\neg)$ $\forall v\in\{T,F\}$: $f_{\neg}^{A}(v)$=T iff v=F.
$*(\&)$ $\forall v_1,v_2\in\{T,F\}$: $f_{\&}^{A}(v_1,v_2)$=T iff $v_1=v_2$=T.
$*(\supset)$ $\forall v_1,v_2\in\{T,F\}$: $f_{\supset}^{A}(v_1,v_2)$=T iff v_1=F or v_2=T.
$*(\exists)$ $\forall B\subseteq A$: $f_{\exists}^{A}(B)$=T iff $|B|>0$.
$*(\mathcal{J})$ $\forall B\subseteq A$: $f_{\mathcal{J}}^{A}(B)$=T iff $|B|<\aleph_0$.
$*(\mathcal{M})$ $\forall B,C\subseteq A$: $f_{\mathcal{M}}^{A}(B,C)$=T iff $|B\cap C|>|B-C|$.
$*(\mathcal{W})$ $\forall B\subseteq A^2$: $f_{\mathcal{W}}^{A}(B)$=T iff B is a linear ordering and every non-empty subset of *Field*(B)[24] has a least element.

II. Denotation of Nonlogical Constants in A*

For each nonlogical constant C, let d specify its extension in A* (the intended universe of L). Note that $d(c_i)\in A^+$, $d(P_i)\subseteq A^+$, $d(R_i)\subseteq A^{+2}$, and $d(h_i)$:$A^+\rightarrow A^+$. We will symbolize d(C) by $\overline{C}$.

III. Assignment functions for Variables

Let G be the set of all functions g: V $\rightarrow$ A^+, where V is the set of variables of L. We call each g in G 'an assignment function for L.' We refer to g("x_i") as g_i.

IV. Denotation under $g - d_g$

1. $d_g(x_i)=g_i$.
2. $d_g(c_i)=\overline{c_i}$.
3. $d_g(h_i(t))=\overline{h_i}(d_g(t))$.

V. Truth-value under $g - v_g$ (satisfaction by g)

A. *Logically-atomic formulae*

1. $v_g(P_it)$=T iff $d_g(t)\in\overline{P_i}$.
2. $v_g(R_it,t')$=T iff $\langle d_g(t),d_g(t')\rangle\in\overline{R_i}$.[25]

24. If B is a binary relation on A (a set of pairs of elements of A), then *Field*(B) = $\{a\in A: \exists b[b\in A \;\&\; (\langle a,b\rangle\in B \vee \langle b,a\rangle\in B)]\}$.
25. Note that by $d_g(t)\notin\overline{P_i}$ and $\langle d_g(t),d_g(t')\rangle\notin\overline{R_i}$ we mean: $d_g(t)\in A^*-\overline{P_i}$ and $\langle d_g(t),d_g(t')\rangle\in A^{*2}-\overline{R_i}$.

*B. *Logically-structured formulae*

*3. $v_g(t \approx t')=\mathrm{T}$ iff $f_{\approx}^{A+}[dg(t), d_g(t')]=\mathrm{T}$.
*4. $v_g(\neg\Phi)=\mathrm{T}$ iff $f_{\neg}^{A+}[v_g(\Phi)]=\mathrm{T}$.
*5. $v_g(\Phi\&\Psi)=\mathrm{T}$ iff $f_{\&}^{A+}[v_g(\Phi),v_g(\Psi)]=\mathrm{T}$.
*6. $v_g(\Phi\supset\Psi)=\mathrm{T}$ iff $f_{\supset}^{A+}[v_g(\Phi),v_g(\Psi)]=\mathrm{T}$.[26]
*7. $v_g[(\exists x_i)\Phi]=\mathrm{T}$ iff $f_{\exists}^{A+}[\{a\in A^{+}: v_{g(a/x_i)}(\Phi)=\mathrm{T}\}]=\mathrm{T}$.[27]
(iff $f_{\exists}^{A^*}[\mathrm{Ext}(\Phi)]=\mathrm{T}$, i.e., iff the extension of Φ is not empty).
*8. $v_g[(\mathcal{F}x_i)\Phi]=\mathrm{T}$ iff $f_{\mathcal{F}}^{A+}[\{a\in A^{+}: v_{g(a/x_i)}(\Phi)=\mathrm{T}\}]=\mathrm{T}$
(iff $f_{\mathcal{F}}^{A^*}[\mathrm{Ext}(\Phi)]=\mathrm{T}$, i.e., iff the extension of Φ is finite).
*9. $v_g[(\mathcal{M}x_i)(\Phi,\Psi)]=\mathrm{T}$ iff $f_{\mathcal{M}}^{A+}[\{a\in A^{+}: v_{g(a/x_i)}(\Phi)=\mathrm{T}\},\{a\in A^{+}:$
$v_{g(a/x_i)}(\Psi)=\mathrm{T}\}]=\mathrm{T}$
(iff $f_{\mathcal{M}}^{A+}[\mathrm{Ext}(\Phi),\mathrm{Ext}(\Psi)]=\mathrm{T}$, i.e., iff the majority of things in the extension of Φ are in the extension of Ψ).
*10. $v_g[(\mathcal{W}x_i x_j)\Phi]=\mathrm{T}$ iff $f_{\mathcal{W}}^{A+}[\{\langle a,b\rangle\in A^{+2}: v_{g(a/x_i)(b/x_j)}(\Phi)=\mathrm{T}\}]=\mathrm{T}$[28]
(iff $f_{\mathcal{W}}^{A+}[\mathrm{Ext}(\Phi)]=\mathrm{T}$, i.e., iff the extension of Φ is a well ordering relation).

VI. Truth in L

A sentence σ of L is true iff for some/all $g\in G$: $v_g(\sigma)=\mathrm{T}$.

10. The Contribution of Logical Structure to Valid Inference

Our definition of truth for L identifies the logical, or formal, component in the truth and falsity of sentences of L. The passage from this definition to the notions of *logical truth* and *logical validity* is straightforward.

A. *Logical Truth*. Some sentences of L are so constituted that the nonlogical component in their truth or falsity is null: their truth value is fully determined by their formal content on the one hand and by the laws governing formal structures of objects in the world on the other. These sentences are *logically* true or false. Among the logical truths and falsities of L are:

26. Note how the combination of entries 1 and 2 with 4, 5, and 6, generates definitions of satisfaction for the "objectual" logical operators corresponding to the logical connectives: complementation, intersection, Cartesian product, etc.
27. $g(a/x_i)$ is an assignment which assigns a to x_i and otherwise is the same as g.
28. $g(a/x_i)(b/x_j)=[g(a/x_i)](b/x_j)$.

(5) $(\exists x_1)(P_1x_1 \,\&\, P_2x_1) \supset (\exists x_1)P_1x_1$,
(6) $(\mathcal{M}x_1)(P_1x_1,P_2x_1) \,\&\, (\mathcal{F}x_1)P_2x_1 \supset (\mathcal{F}x_1)P_1x_1$, and
(7) $P_1c_1 \,\&\, \neg P_1c_1$.

Their truth value is determined by such *formal laws* (i.e., *laws of formal structures*) as: '$X \cap Y \subseteq X$', 'The union of two finite sets (or of a finite set and a smaller set) is finite', '$X \cap \overline{X} = \emptyset$', etc., and is independent of the denotations of their nonlogical constants. We can determine whether a given L-sentence satisfies these conditions by checking whether the statement obtained by abstracting from the specific denotations of its nonlogical constants is a *formal law*, i.e., whether *it holds in all formally possible structures of objects.* (For a characterization of formal laws see section 6 above.)

The notion of formally possible structure of objects is, we have assumed, systematized by some mathematical theory. (For the sake of simplicity we take this theory to be ZFC or some other set theory.) Using the resources of this theory, we formulate the notion of *model for* L. A model for L is a structure $\mathfrak{A}$ of the form $\langle A, \beta_1, \ldots, \beta_s \rangle$, where A is a non-empty set, called the 'universe' of $\mathfrak{A}$, and $\beta_1, \ldots, \beta_s$ are elements of A—members of A, subsets of A, etc.—corresponding to the nonlogical constants of L. A model for L thus represents a formally possible structure of objects relative to L. Now, it is easy to convert the definition of truth for L to a definition of 'truth in a model (for L)': the definition of truth specifies the satisfaction conditions of the logical constants by rules applicable to any (formally possible) universe, and the model itself provides denotations for the nonlogical constants. We define: *A sentence σ of L is* logically true *iff it is true in every model for L; A sentence σ of L is* logically false *iff it is false in every model for L.*

B. *Logical Validity.* Validity is a property of arguments or inferences. An argument is valid iff it is truth preserving. Every finite *valid* argument corresponds to a *true* conditional (and vice versa), and the same factors that make the conditional true make the argument valid. Since there are many factors in the truth of sentences, there are many factors in the validity of arguments. Arguments based on laws of nature—e.g., '*a* is a physical object, therefore: the speed of *a* is lower than 187,000 miles per second'—are 'nomically' valid; arguments based on laws of formal structure are *logically valid.* The inferences

(8) $(\exists x_1)(P_1x_1 \,\&\, P_2x_1)$; Therefore $(\exists x_1)P_1x_1$

and

(9) $(\mathcal{M}x_1)(P_1x_1,P_2x_1)$, $(\mathcal{F}x_1)(P_2x_1)$; Therefore $(\mathcal{F}x_1)(P_1x_1)$

are based on laws of the latter kind.

Using the locutions '$\mathfrak{A}$ is a model of σ' and '$\mathfrak{A}$ is a model of Σ' to stand for 'σ is true in $\mathfrak{A}$' and 'all the sentences of Σ are true in $\mathfrak{A}$', respectively, we define:

> The argument '$\sigma_1, \ldots, \sigma_n$; therefore σ' is *logically valid* iff every model of $\{\sigma_1, \ldots, \sigma_n\}$ is a model of σ.

Generalizing, we define:

> The sentence σ is a *logical consequence* of the set of sentences Σ (possibly an infinite set) iff every model of Σ is a model of σ.

We can use the notion of truth in a model to define other logical notions: 'A set of sentences is *logically consistent* iff it has at least one model', etc.[29]

C. *From Semantics to Proof Theory.* I will not be able to give a complete account of how the semantic relation of logical consequence leads to the syntactic relation of derivability, but in principle, the formal laws responsible for semantic consequence may also be responsible for syntactic consequence (provability). Some basic laws of formal structure may be encodable by finite rules geared to the syntax of a given language (in particular, rules relating to its logical constants), and some logical languages may have a 'complete' system of such rules. In those languages every logically valid argument (in the semantic sense) is *provable* and there is an *effective method* for checking the correctness of putative proofs. Two examples of such languages are the family of languages of standard 1st-order logic and the family of languages of Keisler's 1st-order logic with the indenumerability quantifier (1970). A systematic investigation of the relation between formal rules of syntax and formal rules of objects is beyond the scope of the present paper. Such an investigation I hope to conduct elsewhere.

11. The Attributes of Logic

Logic, on my account, is a theory of the role played by formal

29. As the reader can see, there is no need to adjust the standard definitions of logical truth, validity, etc. to our extended notion of logic.

structure in truth and reasoning. Whether the formal originates in the mind or in the world—whether the mind imposes certain formal 'blueprints' on the world or the world 'causes' certain formal patterns of cognition—is beyond the scope of our investigation. But whatever the origins of the formal, the view that logic is a theory of the formal allows us to explain both the traditional attitudes towards logic and modern challenges to these attitudes: both the 'unbounded generality', 'ultimate basicness', 'supreme normative force' and 'absolute certainty' of logic, and its vulnerability to error and openness to revision. We can explain the traditional attributes of logic by reference to certain 'external' features of the formal: its ubiquitousness in the world, its role in our cognitive (and possibly, noncognitive) ability to maneuver in the world, and so on. But we can also explain them by the 'inner' features of the formal, in particular, the *invariance principle* used to demarcate it.

Properties (relations, functions) in general can be characterized by an invariance principle. An invariance principle for a given type of property tells us what differences between objects (or structures of objects) are *not* discerned by properties of that type. The invariance principle for *formal* properties says that *formal properties do not distinguish between isomorphic structures of objects.*[30] Speaking in terms of predicates, the invariance principle says that the satisfaction conditions of formal predicates do not take into account features of objects that are not preserved under isomorphisms. Physical properties, for example, do not satisfy this principle. Given a physical relation and an argument-structure, $\mathfrak{A}$, satisfying it, there exists an isomorphic argument-structure—say, an arithmetic or set-theoretical 'image' of $\mathfrak{A}$—which does not satisfy it.

We can account for the attributes of logic based on the invariance principle characterizing the *formal* as follows:

A. *Generality.* The fact that formal properties are *invariant* under isomorphic structures means that these properties are *closed* under isomorphisms, and, as a result, that the domain of laws governing these properties, i.e., *the domain of formal laws, is closed under isomorphisms.* Now, suppose

30. I am speaking here of 'invariance under isomorphic object-structures'. More generally, I would speak of 'invariance under isomorphic argument-structures', where 'argument-structure' includes truth-structures and the notion of isomorphism is adjusted accordingly.

the laws of logic are not universal, and let $\mathcal{D}$ be a domain of objects in which the laws of logic do not hold. We know that (i) the laws of logic do hold in the domain of our theory of formal structure, T (since they are defined in reference to that theory); (ii) every formally possible structure of objects, up to isomorphism, is represented by a structure of T (by construction of T); and (iii) all structures of objects in the world and, in particular, all structures postulated by the different sciences, are formally possible. It follows from the closure-under-isomorphism of the formal laws, that these laws apply to $\mathcal{D}$. Since the logical laws are formal, logic applies to $\mathcal{D}$.

B. *Basicness.* Consider any scientific discipline, say, physics or biology. Due to the generality of logic, the laws of logic hold in biology. But, since formal properties—and hence the logical constants—do not distinguish the biological features of objects, our theory of formal structure—hence logic—is independent of biology. Our biological theories have to respect the laws of logic (to be 'logically sound'), but our logical theories do not have to respect the laws of biology (to be 'biologically sound'). Since biology is dependent on logic but logic is independent of biology, logic appears to be more basic than biology. The same relation holds between logic and other sciences.

C. *Normative Force.* The above explanation also explains the (relative) normative superiority of logic.

D. *Certainty.* Due to their strong invariance principle, the formal properties, and hence the laws governing them, are indifferent to most variations between structures of objects, including all variations concerning empirical differences between structures. This means in particular that our account of the formal laws is not affected by specific discoveries made in the sciences, i.e., our theory of formal structure is *highly stable.* But if our theory of formal structure is highly stable, so is logic. Logic is rarely threatened by new discoveries, either internal or external. Logic, in short, is highly certain.[31]

The relative certainty of logic, however, does not means that logic is immune to revision. Since logic is dependent on our theory of formal structure, substantive revisions in the latter will lead to substantive revisions in logic. Changes in our theory of formal structure may be motivated by internal problems and/or new discoveries in various fields of mathematics. For example, Wiles' proof of Fermat's Last Theorem allows us to add a new rule,

31. These considerations can also be used to explain why logic—and mathematics—are commonly viewed as "apriori" and "nonempirical".

or family of rules, to our logical arsenal. Let $n>2$, $k,l,m>0$ and let '$(k^n x)$', '$(l^n x)$' and '$(m^n x)$' be the cardinality quantifiers 'There are exactly $k^n/l^n/m^n$ x'. Then the following is a valid rule of inference: '$(k^n x)\Phi$, $(l^n x)\Psi$, $(\forall x)(\Phi \equiv \neg\Psi) \vdash \neg(\mathrm{m}^n x)(\Phi \vee \Psi)$'. Developments in science and philosophy may also affect our theory of formal structure. For example, current model-theory assumes an ontology of discrete, numerable and 'determinate' individuals (individual which are fully determinate with regard to all the predicates of a given language). But the indeterminacy of quantum-mechanical phenomena might lead to a new theory of formal structure, based on a new kind of ontology. We might end up either replacing the 'old' logic by a 'quantum logic' (see, e.g., Birkhoff and von Neumann 1936), or working with two different logics side by side. Finally, general methodological considerations pertaining to our system of knowledge may motivate changes in any specific theory, including our theory of formal structure and with it logic. (We may, for example, find it pragmatically profitable to use trivalent structures rather than bivalent ones.)

But the idea that logic is a theory of the formal can survive all such changes. This idea explains both the relative stability of logic and its revisability: relative to a given state of our knowledge logic is more general, more basic, more authoritative and more certain than other sciences, but these attributes do not eliminate either the need for a critical evaluation of logic or the possibility of revision in logic. Nor do these attributes preclude the possibility of an explanatory account of logic or necessitate the view that logic is (in some mysterious sense) 'obvious'. We do not have to step into some logically impossible zone in order to give an informative characterization of logic. We can characterize logic internally in terms of such notions as 'formal property', 'structure of objects', 'invariance', 'isomorphism', and so forth. We can think of logic and its formal basis as developed in a cumulative process: Starting with some very simple principles of formal structure we construct a 'basic' logic, and using this logic we develop a richer theory of formal structure and with it a richer logic. This processes can go on indefinitely, resulting in richer and richer formal theories and richer and richer logics.[32]

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