Although the invariance criterion of logicality first emerged as a criterion of a largely mathematical interest (Mostowski 1957, Lindström 1966, Tarski 1966), it has developed into a criterion of considerable philosophical significance. As a philosophical criterion, invariance has been studied and developed from several perspectives. Two of these are the natural-language perspective and the theoretical-foundational perspective, centered on logic’s role in knowledge. My own work (Sher 1991 to 2016) has focused on the second perspective. I have argued that the invariance criterion of logicality makes important contributions to the development of a theoretical foundation for logic focused on its contribution to knowledge – a dual, normative-descriptive foundation centered on (i) the veridicality of logic and (ii) its strong modal force.

Those who focus on the natural-language perspective concentrate on the descriptive adequacy of this criterion for the study of natural language. Here we have on the one hand philosophers and linguists who study the criterion’s contributions to linguistic semantics (see Peters & Westerståhl 2006 and references there). On the other hand, there are critics of the criterion who base their criticisms on its purported linguistic and intuitive inadequacy (see, e.g., Hanson 1997, Gómez-Torrente 2002, McFarlane 2005/2015, and Woods 2016). Thus, Woods opens his nuanced criticism by saying:

I argue that in order to apply the most common type of criteria for logicality, invariance criteria, to natural language, we need to [require] both invariance of content ... and invariance of character ... . If we do not require this, then old objections ... suitably modified, demonstrate that content invariant expressions can display intuitive marks of non-logicality. [2016: 778, my emphases]

* I would like to thank the participants in the conference “Model Theory: Philosophy, Mathematics, and Language” (Munich Center for Mathematical Philosophy, LMU, 2017), and in particular Gil Sagi and Jack Woods for very helpful comments.
Often, critics focus on natural-language inferences whose logical validity is allegedly sanctioned by the invariance criterion but challenged by speakers’s intuitions (either raw or theory-laden intuitions). Some of these criticisms are directed at the prevalent version of the invariance criterion, while others are directed at the very idea of an invariance criterion of logicality. Still others are directed at the more general idea of a precise, systematic criterion of, or necessary-and-sufficient condition for, logicality, regardless of whether it involves invariance. Among the latter, some opt for a purely pragmatist approach to logicality.

Naturally, there is room for misunderstandings between philosophers who evaluate the invariance criterion of logicality on different grounds and from different perspectives. In particular, there is room for misunderstandings between (i) those who evaluate this criterion on theoretical grounds and those who evaluate it on intuitive grounds, and (ii) those who evaluate it from the point of view of its contribution to a philosophical foundation of logic focused on logic’s veridicality and role in knowledge and those who evaluate it from the point of view of its descriptive adequacy with regard to natural language. In this paper I will try to remove a few misunderstandings concerning the theoretical-foundational perspective on the invariance criterion of logicality. To avoid repetition, I will focus on certain aspects of invariance that I have not expanded on in the past as well as on certain points concerning the theoretical approach to invariance and logicality that have led to misunderstandings. I hope that the clarification of these points will help alleviate the tensions between the theoretical-foundational approach to logicality and the natural-linguistic approach.

1. The General Idea of Invariance

Invariance in general is a relation of the form “X is invariant under all variations Y” (where “variations” can be understood as “changes”, “transformations”, “replacements”, and similar expressions, and “Y” can be read as “in Y”, “of Y”, “of type Y”, “of type Y in Z”, etc.). Invariance, in this general sense, is a very fruitful notion. Three examples (on different levels) of
claims involving an invariance relation, taken from logic, mathematics, and physics, are:

(1) A sentence is logically true iff (if and only if) its truth is invariant (preserved) under all replacements of one model by another.

(2) The different geometries can be characterized in terms of the transformations of space under which their concepts are invariant.

(3) The laws of physics are invariant under all changes of inertial frames of reference.

The first example is a reformulation of the standard semantic (model-theoretic) definition of logical truth. Spelled out in more detail, it says that a sentence is logically true iff it is true (in the/a model representing the actual world or even just true in some model) and its truth is preserved under all variations in models (replacements of any model by another). The second example is based on Klein’s 1872 *Erlangen* program of classifying geometries and explaining the relations between them in terms of the transformations of space under which their characteristic notions are invariant. Thus, the notions of “rigid-body” geometry are invariant under all transformations of space that preserve *distance* between points, while the notions of Euclidean geometry are invariant under all transformations of space that preserve *ratios of distance* between points. Since the latter condition involves invariance under more transformations than the former, Euclidean geometry is more general than rigid-body geometry. One of the most general geometries is topology, whose notions are invariant under all transformations that preserve closeness (open sets). And in principle, geometry $G_1$ is *more general* than geometry $G_2$ iff the notions of $G_1$ are *invariant* under *more transformations* of space than the notions of $G_2$. The third example is taken from special relativity, whose laws are invariant under all variations in inertial frames of reference. Indeed, the notion of *symmetry*, which plays a central role in physics, is an invariance notion. A physical system satisfies symmetry $X$ iff it remains unchanged under transformations associated with $X$. For example, the speed of light has the same value under all transformations of the Poincaré symmetry group, hence is invariant under all replacements of appropriate frames of reference.
What does invariance mean? What is its significance? What does it amount to? We may say that when X is invariant under all variations Y, X “does not notice”, “does not pay attention to”, “is blind to” changes in Y, “is immune” to changes of type Y, or “is not affected” by changes in Y and “cannot be undermined” by discoveries concerning features that vary from one Y to another. Thus, if we regard models as portraying all possible ways the world could have been (in some relevant sense of “possible”), then we may say that logical truths “do not pay attention” to whether the world is as portrayed by one model or by any other. In a similar way, the property of being a Euclidean triangle “is blind” to transformations of space that change distances between points so long that they preserve ratios of distances. (The image of any Euclidean triangle under such transformations is also a Euclidean triangle.) The laws of physics “are immune to changes” in inertial frames, or “are not affected” (“cannot be undermined”) by discoveries concerning the distinctive features of given inertial frameworks, those that vary from one inertial framework to another. And so on.

Accordingly, one of the ways in which invariance is highly significant is that the stronger the invariance conditions a given notion satisfies (or the characteristic notions of a given field satisfy), the stronger or more stable the notion (field of knowledge) is, in relevant respects. “Stronger”, in the cases we consider here, can be characterized as follows: Invariance condition $I_1$ is stronger than invariance condition $I_2$ iff the class of transformations associated with $I_1$ properly includes the class of transformations associated with $I_2$. But if the stronger the invariance conditions satisfied by X, the stronger (in relevant respects) X is, then it is to be expected that if X satisfies especially strong invariance conditions, X is especially strong (in relevant respects). It would thus not be surprising if we could explain the fact that, and the way in which, logical truths and consequences are stronger than other truths and consequences based on their strong invariance. And as we shall see below, it is indeed possible to explain the exceptionally modal force of logical truths and consequences based on the fact that they, and/or some of their constituents, satisfy certain especially strong invariance conditions.
2. Logicality and Veridicality Challenges

I. The Logicality Challenge

The logicality challenge is the challenge of establishing the theoretical viability of a system of genuine logical consequences and explaining how it might be structured. Philosophers may have less and more demanding conceptions of genuine logical consequence. Here I am interested in a relatively demanding conception of logical consequence, a conception associated with logic’s role in knowledge. This role, as I understand it, is to devise a powerful, universal method or system for extending knowledge in any field by moving us from truths – robust, correspondence-like truths – that we may already know to truths (of the same kind) that we may not yet know. In this spirit, I require that a genuine logical consequence satisfy the following strong conditions:

(T) A logical consequence transmits truth from premises to conclusion (where truth is a demanding notion: truth as correspondence in a broad yet robust sense, rather than mere coherence, pragmatic justification, disquotation, etc.).

(M) The transmission of truth is guaranteed with an especially strong modal force.

II. The Veridicality Challenge

The veridicality challenge is the challenge of truth and justification. To be adequate, a logical theory has to say true things about logical truths and consequences. It should not say that a sentence S follows logically from a set of sentences \( \Gamma \) unless S in fact follows logically from \( \Gamma \), i.e., unless the sentences of \( \Gamma \) in fact transmit correspondence-truth – truth in the world – to S and

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1  (i) I understand “disquotational truth” in this paper as exemplifying the view that truth in general takes into account only facts (such as disquotation) concerning language. I understand “robust” as involving demanding requirements concerning the world (generally, the extra-linguistic world).

(ii) In this broad sense, correspondence is free from its traditional association with the naive and simplistic idea of copy, mirror-image, or direct isomorphism. For further explanation of this broad (yet robust) conception of correspondence see, e.g., Sher (2016).
do so with an especially strong modal force. It is not sufficient that our intuitions tell us, or give us the impression, that this is the case; this has to be the case, and we need to theoretically justify the claim that it is the case.

Now, ideally, there would be no need to treat the veridicality challenge as a separate challenge. It would go without saying that an adequate system of logical consequence satisfying the logicality challenge produces consequences that truly or in fact transmit truth from premises to conclusion with an especially strong modal force. But in contemporary philosophy, as we have noted above, philosophers sometimes focus on intuitive rather than theoretical justification.² So it is important to indicate that this is not sufficient. An adequate account of logicality must show that the requisite conditions on logical consequence are in fact satisfied, and this “showing” must be theoretical rather than merely intuitive in the everyday sense of the word. And similar requirements holds for the rejection of given accounts of logicality.

The question we ask in this paper, then, is whether the invariance principle of logicality enables us to establish, theoretically, the viability of a system of consequences that affirms all and only patterns of consequence that in fact transmit truth from premises to conclusions with an especially strong type of necessity.

3. Preliminaries

I. Methodology. The challenges of logicality and veridicality are foundational challenges, challenges that have to do with fundamental philosophical questions concerning logic. But the attempt to deal systematically with such foundational questions raises methodological problems that have to be treated with care. Traditionally, philosophers assumed that the only methodology

² For example, Hanson rejects “modal and formal accounts” of logicality on the ground that they “fail to satisfy our intuitions about logical consequence” (1997: 386, my emphasis). He denies the logicality of a term alleged to be logical by the invariance account on the ground that “it seems bizarre”, i.e., counter-intuitive, “to treat” it as logical (ibid.: 392, my emphasis). And so on.
for dealing with foundational questions is the foundationalist methodology. But the foundationalist methodology makes a theoretical foundation of logic impossible. I have discussed some of the problems it raises and proposed an alternative methodology elsewhere (Sher 2013, 2016), so here I will be very brief. One problem with the foundationalist methodology is its requirement that in giving a foundation for a field of knowledge K we limit our epistemic resources to those produced by more fundamental fields than K (fields lying lower than K in the foundationalist hierarchy). But no basic field of knowledge (one that lies at the bottom of this hierarchy, or at the bottom of one of its paths) can be given a theoretical foundation under these conditions. Since logic is classified by foundationalists as a basic field, this problem applies to logic. In the literature, many philosophers focus on a particular aspect of this problem. Due to the basicness of logic, we cannot provide a theoretical foundation for it without circularity or infinite regress. Since all forms of circularity and infinite regress violate the foundationalist strictures, we cannot provide a theoretical foundation for logic at all.

To investigate logicality theoretically, therefore, we need a different methodology. The methodology I will use here is a holistic methodology of a special kind, called “foundational holism” (see Sher 2016). This methodology is holistic rather than foundationalist, but it differs from various other types of holism in being geared toward foundational studies. Thus, this holistic methodology is world oriented rather than coherentist, it emphasizes inner structures rather than totalities or wholes, and so on. Its holistic nature is reflected in its attentiveness to large and open-ended networks of connections between diverse elements. It recognizes that there are many ways to reach the world cognitively, both on the level of discovery and on the level of justification. In particular, both discovery and justification may exhibit multiple patterns, some hierarchical, others not. Accordingly, not all forms of circularity are forbidden: some occurrences of circularity are innocuous, and some are even constructive. The paradigmatic metaphor of

3 It does not say that we can only study language or knowledge as a whole, but urges us to focus on their inner structure.
foundational holism is Neurath’s Boat. In trying to meet the logicality challenge we go back and forth between various kinds of considerations on various levels, using whatever resources are available to us at the moment, creating new resources, rechecking our original results, arriving at new results, producing new justifications, and so on.\(^4\)

II. Philosophical Theory and Mathematical Background-Theory. In studying logicality theoretically from a philosophical point of view we are faced with a special problem. On the one hand, we aim at a philosophical rather than a mathematical account, and in particular, we wish to avoid commitment to any particular mathematical background-theory. On the other hand, using the resources of some mathematical background-theory may have considerable benefits, enabling us to express our philosophical ideas using precise terms-of-art, bring clear examples and counter-examples, answer questions that are difficult to answer without mathematical resources, and so on. The usefulness of a mathematical background-theory is especially significant in the philosophies of logic and mathematics, due to the formality of the disciplines they study. But using a specific mathematical theory as a background theory might introduce complications. Whereas our philosophical ideas are devoid of problematic mathematical commitments, using the resources of a specific mathematical theory to express them can easily create the false impression that they do carry such commitments. How shall we prevent misunderstandings of this kind? In Sher (2016) I divided my discussion of logicality into two parts. I started by formulating and explaining my ideas in a way that emphasized their philosophical character. Among other things, I did not use any mathematical terms-of-art that might give the impression of mathematical commitments. Once this account was completed, I presented a precisified version of the account, helping myself to the resources of a specific mathematical theory, ZFC. Throughout the discussion I stressed that in principle one could use a different mathematical background-theory,

\(^4\) For an extended discussion, see Sher (op. cit.).
with different mathematical commitments, so ZFC’s commitments are *not inherent* in the account.

Due to limitations of space I will not be able to be as thorough in separating the two accounts here. But to avoid misunderstandings, it is important to be aware of this point. In particular, it is important to realize that the explanation of invariance and logicality given in the present paper is *philosophical* rather than set-theoretical. It is not committed to ZFC; nor does it carry its commitments.

### 4. Two Invariance Principles of Logicality.

In the philosophical literature on logicality, talk of *invariance* is usually directed at one use of invariance – demarcation of *logical constants* – and accordingly, at one type of invariance. But in fact, there is another use, and another type, of invariance in logical semantics as well. This invariance principle appears as my first example of general invariance above. It concerns the use of *models* for demarcating *logical truths and consequences*. I will call it “the first invariance principle of logicality”, or “the model-theoretic invariance principle” (I-M).

#### I. The First Invariance Principle of Logicality (I-M)

The first invariance principle of logicality underlies the standard semantic definition of logical consequence, whose roots go back to Tarski (1936). Consider a collection \( \Gamma \) of sentences of a given language \( L \) and a sentence \( S \) of \( L \). The standard semantic definition of logical consequence can be formulated as:

\[(LC) \quad S \text{ is a logical consequence of } \Gamma \text{ (in } L) \text{ iff in every model (for } L\text{) in which all the sentences of } \Gamma \text{ are true } S \text{ is also true},\]

without commitment to a specific mathematical construal of models. To capture the requirement that the truth in question is of a robust kind, i.e., a broadly correspondence-truth, we can reformulate LC as:
(LC’) S is a logical consequence of \( \Gamma \) (in L) iff in every model (for L) in which all the sentences of \( \Gamma \) are correspondence-true S is also correspondence-true.

Now, although people rarely think of LC as a definition of logical consequence in terms of invariance, the idea of invariance (the same idea as in (1) above) is implicit in it. We can make this idea explicit by reformulating LC as Invariance-under-Models, I-M:

(I-M) S is a logical consequence of \( \Gamma \) iff the transmission of (correspondence-) truth from \( \Gamma \) to S is invariant under all variations in (replacements of) models,

or more precisely,

(I-M’) S is a logical consequence of \( \Gamma \) iff \( \Gamma \) transmits (correspondence-) truth to S in some model \( M \), and this transmission of truth is invariant (is preserved) under all replacements of \( M \) by any other model \( M' \).

Three questions concerning LC, or its reformulation, I-M, concern language, models, and logical constants:

(a) Language. What kind of language is assumed by LC/I-M? Since we are interested in a theoretical account of logicality, we need to think of this language, which we may identify with L above, as a theoretical language. How is L related to natural language? As a theoretical language, L represents certain aspects of language in general, namely, certain aspects of both natural languages and artificial, or semi-artificial, languages (e.g., the languages used in contemporary mathematics textbooks). L, like theoretical representations of objects in general, abstracts from some features of the languages it represents (namely, those deemed irrelevant for understanding logicality) and focuses on others (those deemed relevant). Subject to this caveat, it is reasonable to start with a broad, open-ended understanding of L.  

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5 Is L, and the languages it represents, extensional or intensional? Following Barcan Marcus (1960), I view “extensionality” and “intensionality” as open to a variety of interpretations, so that there is no unique or absolute sense of extensionality or intensionality. Furthermore, in accordance with my open-ended approach to language here, I prefer to leave the particular status of L and the languages it represents with respect to extensionality/intensionality open. Informally, and in relativistic terms, I think that it is reasonable to view natural language as more intensional than either L or the artificial (semi-artificial) language of textbook mathematics.
(b) Models. How shall we understand models, philosophically? Let us go back to the logicality challenge. The challenge is to establish the viability of a theoretical system for identifying logical consequences, where a logical consequence is one that transmits correspondence-truth from sentences to sentences with an especially strong modal force.

Now, to capture this conception of logical consequence using an apparatus of models, models have to satisfy certain conditions. One natural way to construe these conditions is: (i) models should represent all and only ways the actual world could have been, given a relevant understanding of possibility (ways the world could have been),\(^6\) (ii) there should be a model representing the way the world actually is in relevant respects,\(^7\) and (iii) the totality of models should be especially large, i.e., the conception of possibility involved is especially broad, broader than that of physical and even metaphysical possibility.\(^8\) By focusing on the world – the way it is and the ways it could have been – (i) and (ii) ensure that logical consequence transmits the right

\(^6\) (i) For the relevant understanding of possibility, see (iii) below.

(ii) In principle, there is no unique way in which models must represent the relevant ways the world could have been in order to satisfy the logicality and veridicality challenges of Section 2 above. Once again, I prefer to leave this question as open as possible in this paper. I will, however, present certain constraints on the construction of adequate models in what follows, especially concerning the treatment of logical and non-logical constants and the type of possibility adequate models need to represent. It is these constraints that distinguish my own “representational” conception of models from the one discussed, and rightly rejected, by Etchemendy (1990). For a detailed explanation, see Sher (1996).

\(^7\) Some philosophers (e.g., Field 2009) argue that models based on standard set theory as a background theory are incapable of adequately representing the actual world. Whether he is right or wrong, the fact that our conception of models is not tied up to this particular background theory (or, indeed, to any other) exempts it from his argument.

\(^8\) I will identify the relevant type of possibility in Section 5 below, including fn. 23.
kind of truth, namely correspondence-truth (truth-in-the-world), and that the transmission of truth occurs in all relevant situations, actual and counterfactual. By adding (iii) we ensure that logical consequences have an especially strong modal force, i.e., the modal force of logic is greater than that of physics and even metaphysics. This account, however, is incomplete: it does not specify the type of possibility involved in (i) and (iii). Specifying (theoretically) the requisite type of possibility is something the foundational theorist still has to do.

(c) Logical Constants. The universality of logical consequence – its applicability to all fields of knowledge – requires that it take into account only features of sentences that do not vary under changes in (replacements of) fields of knowledge. I.e., logical consequence must be invariant under such changes, or, as commonly said, it must be topic neutral. One feature of sentences that satisfies this requirement is their logical form or content. Logical consequence takes into account only the logical form/content of the sentences involved, not their non-logical content. Now, logical form is a matter of the division of constants of L into two kinds, logical and non-logical. Logical content is a matter of the content of logical constants. The logical form/content of a sentence S is thus largely determined by its logical constants and their ordering in S. For example, the logical form/content of “Not (Everything is red)” is “Not (Everything is ...)”, and it is the same as the logical form/content of “Not (Everything is round)”. But it is different from the logical form/content of “Everything is (Not red)”, which is “Everything is (Not ...)”. What this means is, primarily, that logical consequence holds fixed the content or denotation of the logical constants of L, treating the content or denotation of the non-logical constants as variable (in effect, treating these constants as schematic letters or variables).

The division of constants into logical and non-logical induces a division between logical and non-logical components of the truth conditions of sentences. The logical components include the denotation (content, satisfaction conditions) of the logical constants of sentences (if they have any); the non-logical components include the denotation (content, satisfaction conditions) of their non-logical constants (if any).
When it comes to models, logical constants have a fixed denotation (content, satisfaction conditions) in all models, while the denotation (content, satisfaction conditions) of the non-logical constants varies (vary) from model to model.⁹ As a result, whether a set of sentences transmits truth to a given sentence in all models is a matter of its logical form/content, and in particular of the logical component in the truth conditions of the sentences involved, which is heavily dependent on the division of constants into logical and non-logical. This division is, therefore, crucial for the adequacy of a given system of logical consequence and, indeed, of the model-theoretic definition of logical consequence, LC/I-M, itself. If we were to classify, say, “Tarski” and “is a logician” as logical constants, LC/I-M would fail.

This was already noted by Tarski (1936), who also considered the possibility of classifying traditional logical constants as non-logical. If, for example, we were to classify the material conditional as non-logical, changing its denotation from model to model, Modus Ponens would come out logically invalid. On the other hand, Tarski recognized that we can classify some traditionally non-logical constants as logical without “running into [logical] consequences which stand in sharp contrast to ordinary usage” (ibid.: 419). Tarski himself did not offer a principled demarcation of the logical constants in his 1936 paper, and indeed “consider[ed] it quite possible that investigations will bring no positive results in this direction” (ibid.: 420). From the present perspective, the challenge is to find a demarcation of logical constants that does not lead to a violation of, and perhaps even maximizes the satisfaction of, the main requirement on logical consequence, namely, that it transmit (correspondence-) truth from premises to conclusion with

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⁹ (i) See Sher (1991). This amounts to another important constraint on models.
(ii) As explained in Sher (ibid.), the fixity of logical constants does not mean that they have the same extension in all models (the extension of the universal quantifier in a model with 8 individuals is a set of 8 individuals, while its extension in a model with 9 individuals is a set of 9 individuals). What it means that their extension is determined for all models in advance, by a fixed principle. (In the case of the universal quantifier, this principle says that its extension in any model is the whole domain of that model).
(iii) For an interesting discussion of the fixity of logical constants in the context of current model theory (the current mathematical theory of models), see Sagi (2018).
an especially strong modal force.

The considerations raised in (b) and (c) leave the theoretical philosopher of logic with two important (and, in my view, doable) tasks:

(A) Construct a theoretical criterion for logical constants (a theoretical division of terms into logical and non-logical) in theoretical languages L, and explain why it is appropriate.

(B) Specify a type of possibility suitable for LC/I-M, that is, a type of possibility that determines the totality of models considered by LC/I-M, and explain why this type of possibility is appropriate, given our conception of logical consequence.

In other words, what the theoretical philosopher seeks to do at this point is find, or develop, a theoretical criterion for logical constants and identify a type of possibility that, together, render LC/I-M an adequate criterion of logical consequence. This brings us to the second invariance principle of logicality and the discussion of formal possibility.

II. The Second Invariance Principle of Logicality (I-I)

The second invariance principle of logicality is a criterion for logical constants. This criterion is often referred to as the “invariance under isomorphisms” criterion (I-I).\(^\text{10}\)

The Invariance-under-Isomorphisms criterion for logical constants (I-I) that I will discuss here is the criterion developed in Sher (1991) based on earlier mathematical criteria due to Mostowski (1957) and Lindström (1966).\(^\text{11}\) I-I has two parts, an objectual part and a linguistic part. The latter concerns the constants of the language L, the former – their objectual denotations, and more generally, objects (in particular, extra-linguistic objects).

\(^{10}\) It is also often referred to as the “invariance under bijections” criterion and the “Tarski-Sher thesis”. A related criterion is the invariance-under-automorphisms/permutations criterion (Mostowski 1957, Tarski 1966), but depending how one understands it, this criterion is significantly different from, and inferior to, the invariance-under-isomorphisms/bijections criterion. See McGee (1996) and Sher (1991, 2016).

\(^{11}\) The 1991 criterion was developed in the mid-80’s, before Tarski’s 1966 lecture was published. But it can also be construed as a development of the criterion proposed by Tarski.
A. Objectual Part of I-I. The objectual part of I-I applies to objects of a certain kind. One can think of these objects in various ways. Given my goal, I prefer to think of the relevant objects as properties, where properties include proper properties, relations, and functions of any level and any arity. I-I divides properties into two types: those that do and those they do not satisfy it. Adherents of I-I regard the former as admissible denotations of logical constants, the latter as inadmissible. The formulations of I-I by Mostowski, Lindström, and Tarski are limited to the objectual part of I-I.

B. Linguistic Part of I-I. The linguistic part of I-I does two things:

(a) It tells us that a logical constant must denote a property that satisfies the objectual part of I-I.

(b) It sets additional conditions on logical constants, intended to ensure that logical constants satisfying (a) are adequately integrated into a syntactic-semantic system of logical consequence incorporating LC/I-M.

In this paper I will focus on the objectual part of I-I. (For the linguistic entries of I-I see Sher 1991.) In accordance with my second preliminary comment in Section 3, I will offer two versions of I-I: one that is not and one that is couched in a mathematical background-theory. I will call the non-mathematical version of the criterion “Invariance under 1-1 replacements of individuals” or “invariance under replacements of individuals”, and I will use the abbreviation “I-R” for this version. I-R is intended to be understood in a way that does not involve specific mathematical (including set-theoretical) commitments. Depending on the context, “I-I” will name either the mathematical version of the criterion – I-I proper – or the broader conception of the criterion –

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12 I think of objects in general as divided into individuals (objects of level 0) and properties (objects of level >0). The use of properties in the present discussion does not assume any specific theory of properties, and various theories of properties are compatible with this account. For the purpose of the present discussion we can for the most part disregard current controversies concerning properties.

13 “Linguistic”, here, has nothing to do with “natural language”. The linguistic part of I-I is linguistic in the sense of being about linguistic expressions, and more specifically constants.
I-R.

The non-specifically-mathematical version of I-I, *invariance-under-replacements-of-individuals*, or I-R, can be presented as follows:

(I-R) An n-place property, \( \mathcal{P} \), of level m, is invariant under all 1-1 replacements of individuals iff for any domain of individuals, \( D \), and any argument, \( \beta \), of \( \mathcal{P} \) (in \( D \)), \( \beta \) has the property \( \mathcal{P} \) (in \( D \)) iff the image of \( \beta \) under any 1-1 replacement \( R \) of the individuals in \( D \) has the property \( \mathcal{P} \) (in \( D' \), the image of \( D \) under \( R \)).

Consider the 2-place 1st-level property \( x\text{-loves-}y \). It is quite clear that this property does not satisfy I-R. But the 2-place 1st level property \( x=y \) does satisfy I-R. Consider the 1-place 2nd-level property \( P\text{-IS-A-PROPERTY-OF-HUMANS} \), where \( P \) is a 1-place 1st-level property. This property does not satisfy I-R, but the 1-place 2nd-level property \( P\text{-IS-NON-EMPTY} \) – the existential-quantifier property – does.

The mathematical version of I-R, I-I, is thought of as a precisification of I-R, and as noted above, various variants of I-I can be introduced using various background mathematical theories.

(I-I) An n-place property, \( \mathcal{P} \), of level m, is invariant under all isomorphisms iff for any domains \( D, D' \) and any arguments \( \beta, \beta' \) of \( \mathcal{P} \) in \( D, D' \) respectively: if \( <D,\beta> \) is isomorphic to \( <D',\beta'> \), then \( \beta \) has the property \( \mathcal{P} \) (in \( D \)) iff \( \beta' \) has \( \mathcal{P} \) (in \( D' \)).

One version of I-I will use ZFC as its background theory, another may use Russell’s theory of

\[14\]
(i) \( D \) is any collection of individuals, actual or counterfactual. Since the I-R is not formulated in any specific mathematical background theory, \( D \) does not have to be identified as a set, a proper class, or an entity of any other specific mathematical type. For the sake of simplicity we assume that \( D \) is non-empty.

(ii) Given a \( \mathcal{P} \) and a \( D \): If \( \mathcal{P} \) is a 1-place 1st-level property, its arguments in \( D \) are individuals in \( D \). If \( \mathcal{P} \) is a 2-place 1st-level property, its arguments in \( D \) are pairs of individuals in \( D \). And so on. If \( \mathcal{P} \) is a 1-place 2nd-level property of 1-place properties, its arguments in \( D \) are 1-place 1st-level properties whose arguments in \( D \) are individuals in \( D \). And so on.

\[15\] I use italics for 1st-level properties and small capital letters for 2nd-level properties.

\[16\] \( <D,\beta> \) is isomorphic to \( <D',\beta'> \) iff there is a bijection \( f \) from \( D \) to \( D' \) such that \( \beta' \) is the image of \( \beta \) under \( f \).

\[17\] In this version, \( D, D' \) will be proper sets.
types as background theory,\textsuperscript{18} and still others may have other mathematical background theories. Although in the historical order of discovery I-I was prior to I-R, in the order of philosophical explanation and justification I-R is prior to I-I. This calls for a methodological clarification: My goal in this paper is to explain how the foundational theorist approaches the question of logicality and how invariance enters into her eventual account. To that end, the explanation I provide has the character of a \textit{rational reconstruction} (in a quasi-Carnapian sense). It does not seek to trace the history of the invariance criterion; instead it explains how it is rational to construct it.

I-I as presented so far is, strictly speaking, a criterion for \textit{properties} and \textit{predicates} (including quantifiers). What about \textit{sentential operators and connectives}? As I have shown in Sher (1991, 2016), I-I can be generalized to an invariance criterion of logicality for sentential connectives. Under this generalization, it coincides with their usual criterion, truth functionality. For the purpose of the present discussion, however, it is sufficient to focus on I-I as a criterion for properties/predicates.

We are now ready to complete the tasks (A) and (B) of subsection I above, namely, to explain why the \textit{Invariance-under-Isomorphisms} criterion is an appropriate criterion for logical constants and to specify the type of possibility that must be represented by logical models. This will enable us to explain how the two invariance conditions, I-M (invariance under models, or LC) and I-I, satisfy the two conditions on an adequate notion (system, method) of logical consequence from Section 2 above – T (transmission of correspondence-truth) and M (especially strong modal force).

5. \textbf{Invariance-under-Isomorphisms, Formality, and Modal Force}

One of the distinctive characteristics of the invariance-under-isomorphism criterion – a

\textsuperscript{18} In fact, Russelian type-theory was one of the two background theories used by Tarski for his 1966 version of I-I.
characteristic that distinguishes it from other criteria for, and accounts of, logical constants\textsuperscript{19} – is that it captures a certain especially fruitful philosophical idea. This idea is formality. Not formality in the traditional syntactic sense, or the schematic semantic sense, or the substitutional semantic sense, but formality in an objectual semantic sense. Objects – specifically properties – satisfying I-I are formal in this sense; objects that do not satisfy I-I are not formal in this sense. Any constant can be formal in the syntactic, schematic, or substitutional sense, i.e., be treated as a fixed, distinguished element, partaking in the “form” of sentences (see, e.g., Etchemendy 1990). But only constants that denote properties satisfying I-I are formal in a sense that is relevant to the two conditions on logical consequence noted above, T (transmission of correspondence-truth) and M (especially strong modal force).

How is I-I connected to formality? First, through the idea that mathematics is formal. We can easily see that the objects – namely, properties – satisfying I-I are, or can viewed as, mathematical objects, and that all mathematical objects – mathematical individuals, properties, and structures – can be systematically correlated with objects – properties – that satisfy I-I. For example: The individual object one (the number 1) is correlated with the 2nd-level cardinality property, \textit{one}, which satisfies I-I. The 1st-level \textit{identity} relation satisfies I-I as it is. The 1st-level 1-place property \textit{x-is-even} satisfies I-I when construed as a 3rd-level property of 2nd-level cardinality properties. The 2nd-level properties of \textit{non-emptiness} and \textit{universality} (the existential- and universal-quantifier properties) satisfy I-I as they are. The 2nd-level mathematical properties \textit{r-is-reflexive/symmetric/transitive}, \textit{r-is-well-ordered}, etc. do too. So do the 2nd-level cardinality properties \textit{finite} and \textit{infinite}. And even mathematical structures, such as the structure of the natural numbers, are systematically correlated with quantifier-properties satisfying I-I. In this way, all mathematical objects either are, or can systematically be construed as, or can be correlated with, properties that satisfy I-I. In contrast, all

\textsuperscript{19} From Feferman’s (1999, 2010) invariance-under-homomorphisms criterion to pragmatist, non-invariance accounts (see below).
paradigmatic non-mathematical objects and properties (such as *Archimedes*, *is-red* and *is-a-property-of-humans*) do not satisfy I-I.\(^{20}\)

Second, if we look at what I-I says, we see that it captures the idea of formality as *strong structurality*. Take any property \(\mathcal{P}\) of any level, any domain \(D\), and any argument \(\beta\) of \(\mathcal{P}\) in \(D\). Now take the pair \(<D,\beta>\) and take any pair \(<D',\beta'>\) that has the exactly the same structure as \(<D,\beta>\), or that can be obtained from \(<D,\beta>\) by a 1-1 replacement of the individuals of \(D\). \(\beta\) satisfies \(\mathcal{P}\) in \(D\) iff \(\beta'\) satisfies \(\mathcal{P}\) in \(D'\). I.e., a property satisfies I-I iff it pays attention *only* to *highly structural features* of its arguments, iff it is blind to all features of its arguments but (some of) their highly structural features. What I mean by strong structurality is as follows:

Most properties abstract from some features of their arguments, and as such they satisfy some invariance condition and have some degree of invariance. In this sense, they are at least weakly structural. But I-I is an especially strong invariance condition. Paradigmatically biological, physical, and other properties do not satisfy it. Only highly-structural properties do. Such highly structural properties are formal.

Now, I-I says that logical constants are limited to constants denoting formal properties. Since it is the logical constants, rather than the non-logical constants, that play the major role in determining logical consequences, logical consequences are due to, or based on, certain relations between formal properties or formal structures. For example, the logical consequence

\[
(\exists x)(Ax \lor Bx), \neg(\exists x)Ax; \text{ therefore: } (\exists x)Bx
\]

is based on a relation between two formal structures: a structure of a non-empty union of two properties, \(P_1\) and \(P_2\), the first of which, \(P_1\), is empty, and a structure in which the second property, \(P_2\), is non-empty. This relation is itself formal, so (4) is based on a formal relation

\(^{20}\) To apply I-I to Archimedes, we identify Archimedes with a property, such as *is-Archimedes* or *IS-THE-PROPERTY-OF-BEING-ARCHIMEDES*. Clearly, these properties do not satisfy I-I.
between two formal structures, or on a formal relation between formal features of the situations that make (or would make) the premises and conclusion of (4) true. It is due to this relation that (4) transmits (correspondence-) truth from its premises to its conclusion, and it does so with formal force.

This enables us to specify the type of possibility that should be represented by logical models – the models referred to in our criterion of logical consequence, LC/I-M. If transmission (or preservation) of truth in all models is to determine logical, i.e., formal consequences, then models should represent formal possibilities (with respect to a language L) rather than possibilities of some other kind (e.g., physical or even metaphysical possibilities). And the totality of models should represent the totality of formal possibilities. If these requirements are satisfied, then regularities across all models will be formal regularities or laws. And such regularities or laws will be formally necessary. In particular, transmissions of (correspondence-) truth that are preserved in all models will be formally necessary. Assuming logical models represent the totality of formal possibilities, logical consequences will be formally necessary.

But is formal necessity an especially strong type of necessity, as required by the second condition on logical consequence, M? Yes. It does not apply to biological, physical, or even metaphysical laws (principles, regularities), but only to highly-structural laws, those governing formal properties.

The special strength of formal necessity is connected to the high degree of invariance of formal properties. The I-I criterion associated with formality is an especially strong invariance criterion, and formal properties are therefore distinguished from non-formal properties in having an especially high degree of invariance. Having an especially high degree of invariance (satisfying especially strong invariance conditions) is, as we have noted in Section 1 of this paper, correlated with being especially strong in certain respects. One of these respects is modal

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21 Metaphysical possibilities will be discussed below.
force. The greater the degree of invariance of a given property, the greater the space of possibilities under which it does not vary; hence the greater the space of possibilities in which the regularities or laws governing it hold. But the greater the space of possibilities (including counterfactual possibilities) in which a regularity or a law holds, the greater its modal force. Since formal properties have a significantly greater degree of invariance than other properties, the space of formal possibilities, the possibilities represented by models, is significantly larger than the space of other types of possibilities. Therefore the modal force of formal laws or regularities is greater than the modal force of other laws or regularities. In other words, since logical consequences are grounded in formal laws or regularities, their modal force is greater than that of non-logical consequences, as required by M. I.e., I-I guarantees that the answer to the above question is positive.

This, in a nutshell, is the way the two invariance criteria of logicality, invariance-under-isomorphisms, I-I, and invariance under models, I-M, enable us to establish theoretically the viability of an adequate system of logical consequence. I-I says that logical constants are formal. This suggests that we construe models as representing formally-possible situations. Since the space of formally-possible situations is especially large, the modal force of consequences that are invariant under all variations in models is especially strong. The formality of logical constants, together with the fact that logical consequences, consequences based on these constants, hold in all formally-possible situations, guarantee the especially strong modal force of logical consequences and explain why and how they have this force. Elsewhere (Sher 2016 and works mentioned there) I showed that the formality of logical consequences (in the sense of I-I) also explains their other properties: their considerable generality, topic neutrality, basicness, certainty, and normativity, as well as their quasi-apriority.  

22 Concerning generality, Bonnay (2008) interprets Tarski as saying that I-I is associated with utmost generality rather than with formality. But for reasons presented both in Bonnay (ibid.) and in Sher (2008), I-I does not really capture the idea of utmost generality. It captures the idea of formality which, in turn, is associated with considerable, yet not utmost, generality. For
But this is not all. The development of a theoretical foundation for logic, on our approach, is done using the foundational holistic methodology, and this leads to further clarifications of this foundation. The foundation is developed in a step by step, or stage by stage, manner, going back and forth, in a Neurath-boat style. And these steps enable us to do things that we did not – indeed, could not – do earlier on. For example, when we introduced I-I, we did not specify the totality of actual and counterfactual individuals that we took into account in the 1-1 replacement of individuals underlying I-I. We emphasized that it includes counterfactual individuals, and we implicitly assumed that these are not limited to physically- or even metaphysically-possible individuals. But we did not specify what kind of counterfactual individuals it includes. Now we can do so. The space of counterfactual individuals has to include all formally-possible individuals. How is this totality of individuals larger than the totality of, say, metaphysically-possible individuals? It is larger in the sense that it contains individuals that are metaphysically impossible.

One example of such an individual is an individual that is both all red and yellow. Metaphysically, such an individual is impossible, but formally it is possible. The incompatibility between being all red and being yellow is not formal; the combination of being all red and yellow is not ruled out on formal grounds. It is not ruled out by mathematical or formal laws. Formal laws abstract from most features of individuals, including color and color relationships. Therefore, an individual that is both all red and yellow is formally possible. As such, it is not

Discussion see op. cit.

23 Explanation: Metaphysics is a highly heterogeneous discipline, dealing, on the one hand, with very basic ontological issues, such as what makes something an object, and on the other hand, with less basic issues, such as causality, free will, observable vs. unobservable objects, physical vs. mathematical objects, abstract vs. concrete objects, and so on, including issues like the one mentioned in the example presented in the next paragraph. These less basic (but still quite basic) issues occupy much larger space in contemporary metaphysics than the more basic ones, and my references to metaphysics in this paper concern metaphysical possibilities and impossibilities of the less basic kinds. (As for the relation between the most basic part of metaphysics and logic, I will not address this issue here.)
ruled out either in applying I-I or in constructing models. Accordingly, there are models that represent individuals that have both the property of being all red and the property of being yellow, individuals that are both dead and alive, individuals that do not satisfy either the regularities of biology or the laws of physics or the principles of metaphysics. Many of the alleged counter-examples to I-I neglect the difference between formal possibility and other kinds of possibility, which is crucial for understanding the philosophical significance of both the invariance-under-models criterion (I-M or LC) and the invariance-under-isomorphisms criterion (I-I). These alleged counter-examples often assume an intuitive or a metaphysical notion of possibility, which is weaker than the notion relevant for I-M and I-I. Therefore, they are not genuine counter-examples. These examples are also usually presented as natural-language examples.

This brings us to the relation between the foundational-theoretical perspective on logicality and the natural-linguistic perspective.

6. The Natural-Linguistic and Foundational-Theoretical Perspectives.

So far we have discussed the two invariance criteria of logicality – invariance-under-models (I-M) and invariance-under-isomorphisms (I-I) – as theoretical criteria, criteria whose goal is to contribute to the construction of a logical system satisfying the two theoretical conditions on logical consequence introduced in Section 2 above: transmission of (correspondence-) truth (T) and especially strong modal force (M). We have seen that, from this perspective, the combination of the two invariance criteria, I-M and I-I, fares quite well. Theoretically, it ensures the satisfaction of T and M, thereby establishing the viability, in principle, of an adequate system of logical consequence. How does it fare from a natural-linguistic perspective? In particular, is I-I descriptively adequate with respect to natural

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24 Of the less basic type noted in fn. 23.
To answer this question we need, first, to understand what the question means. What, exactly, does descriptive adequacy amount to in this case? How do we establish it in principle? It is hard to find a detailed answer to these questions in the critical literature on I-I.

Two co-authors who raise this question are Peters and Westerståhl (2006). Peters and Westerståhl discuss the possibility of evaluating the invariance-under-isomorphisms criterion (I-I) from a linguistic perspective. They consider approaching this task in a way that is analogous to the one taken in this paper, namely, by examining its adequacy for producing genuine logical consequences. They then ask whether the method commonly used in empirical linguistics, namely, the method of consulting speakers’ linguistic intuitions, is appropriate for this task. This method is widely viewed as appropriate for determining grammaticality; the question is whether it is also appropriate for determining validity and logicality (for example, the validity and logicality of such inferences as “John is a bachelor; therefore, John is unmarried” and “Someone is a handsome bachelor; therefore, someone is a bachelor”). Peters and Westerståhl are skeptical about a positive answer to this question. While linguistic intuitions have been shown to be reliable with respect to grammaticality, they appear to be unreliable with respect to validity and logicality. As a result, Peters and Westerståhl give up the attempt to solve the problem of logicality from a natural-linguistic perspective. They take I-I to be a necessary condition on logical constants in natural language, but they do not try to determine whether it is a sufficient condition, i.e., whether it is a criterion of logicality.

What they do investigate, instead, is whether the invariance-under-isomorphisms criterion, I-I, enables us to better understand linguistic phenomena that are difficult to understand without it. Their answer to this question is positive. They show that and explain how non-standard logical quantifiers sanctioned by I-I enable us to explain phenomena concerning determiners and complex quantifier-structures in natural language. For example, the non-standard monadic logical operator most, sanctioned by I-I, explains the behavior of the
determiner “most” in sentences such as “Two critics reviewed most films”; the polyadic operator which we may call most-and-most, sanctioned by I-I, explains branching-quantifier structures in natural language such as “Most of the boys in my class and most of the girls in your class have all dated each other”; and so on.

In my view, Peters & Westerståhl’s approach is reasonable. On the one hand, studying the ways the invariance-under-isomorphisms criterion, I-I, provides new resources for understanding linguistic structures, both in natural language and in artificial languages, makes good sense. But relying on speakers’ intuitions to determine validity and logicality does not. Validity and logicality are significantly different from grammaticality, and employing the same method for both requires careful justification.

But natural-linguistic intuitions are widely used in attempts to undermine I-I. Examination of such attempts suggests that they often disregard crucial theoretical considerations pertaining to I-I and sometimes appeal to philosophical theories whose relevance to logicality is questionable. Let me explain these points by reference to two alleged counter-examples to I-I due to Gómez-Torrente (2002, 2003): the “unicorn” and “male widow” examples.

Gómez-Torrente claims that the properties is-a-unicorn and is-a-male-widow are empty “in all possible universes” (Ibid.: 2002, p. 18). As such, he says, they satisfy the invariance-under-isomorphisms criterion, I-I. Accordingly, the linguistic expressions “is a unicorn” and “is a male widow” come out logical. This, in turn, implies that “There are no unicorns” and “There are no male widows” are logically true. But these sentences “are intuitively not logically true” (Ibid.: 2003, p. 204). Hence, according to Gómez-Torrente, I-I is not an adequate criterion of logicality.

I explained why this criticism is incorrect in Sher (2003). But there I focused on the fact that the linguistic expressions “x is a unicorn” and “x is a male widow” do not satisfy the extended, linguistic, version of I-I, spelled out in Sher (1991). Here I would like to focus on the properties is-a-unicorn and is-a-male-widow. I would like to point out certain assumptions
underlying Gómez-Torrente’s use of these properties to criticize I-I and explain how these assumptions lead us to think that these properties satisfy I-I when in fact they do not. Let me begin with \textit{male widow}.

Gómez-Torrente claims that \textit{male widow} is empty in “all possible universes”. What is the basis for this claim? My understanding is that this claim is based on our ordinary intuitions. But this approach to the issue \textit{neglects} the fact that the notion of possibility involved in both the invariance-under-isomorphisms criterion (I-I) and the invariance-under-models criterion (I-M) is a specific and especially broad notion of possibility, namely, the notion of \textit{formal possibility}, whereas the notion of possibility employed in the claim that \textit{male widow} is empty in all possible universes is a non-specific notion of possibility, one that is usually understood in a way that makes it weaker than formal possibility. This explains why this example cannot be used to undermine I-I. The incompatibility between being male and being a widow is \textit{not a formal incompatibility}. Therefore, it does not rule out the \textit{formal possibility} of situations in which the property \textit{male-widow}, like the property \textit{both-all-red-and-yellow}, is \textit{not} empty. \textit{Male-widow}, then, does \textit{not} satisfy I-I, and “There are no male widows” is \textit{not} true in all models, hence does \textit{not} come out logically true on the invariance account of logicality incorporating I-I and I-M.

What about the property \textit{unicorn}? Why would anyone think that this property is empty in all possible universes? The claim that \textit{unicorn} is empty in all possible universes is, if I understand Gómez-Torrente correctly, based not on natural-linguistic intuitions but on a particular philosophical theory that is naturally viewed as belonging to the philosophy of language or to metaphysics, due to Kripke (1972/80). But first, this theory is irrelevant to I-I, and second, this theory does not really say that \textit{is-a-unicorn} is necessarily empty; it says that this would-be-property, being a mythological species property, is, like all other mythological species properties, \textit{not a genuine} property. I will not go into Kripke’s reasons for
this claim here. But if one accepts his claim, one cannot bring *is-a-unicorn* as a counter-example to I-I, since I-I does not deal with would-be-properties that are in fact non-properties.

There are other linguistic/intuitive grounds on which some philosophers have tried to deny I-I. For critical discussions of these grounds see, e.g., Paseau (2013), Sagi (2015), and Sher (1991, 2003, 2016).


A number of philosophers -- e.g., Hanson (1997, 2002) and Gómez-Torrente (2002, 2003) -- prefer a pragmatist approach to logicality over a theoretical approach. Two main weaknesses of the pragmatist approach to logicality are: (i) its neglect of *veridicality*, and (ii) its neglect of *theoretical explanation*. These, I believe, are pragmatism’s main weaknesses in all theoretical branches of knowledge. If, and so long as, we view the search for knowledge as a search for *truth*, if we require *veridical justification* and *evidence* for theoretical claims, if we aim at *genuinely explanatory* theories, then we cannot be content with a pragmatist approach to knowledge. In the philosophy of logic, or those parts of the philosophy of logic that are discussed in this paper, the question of truth arises in multiple places and on multiple levels: What should logical consequence transmit from premises to conclusion given its role in knowledge? Is a given claim of logical consequence true? (Does it in fact transmit truth from premises to conclusion with an especially strong modal force?) What is (are) the source(s) of truth of logical-consequence claims? Everything else being equal, is it true that a system of logic based on I-I and I-M satisfies the requirements of transmission of truth and modal force on logical consequence? Does the formality of logic, articulated in terms of I-I, provide a theoretical explanation of the special features of logic -- necessity, generality, topic-neutrality, etc.? And so on. All these are theoretical questions of truth and explanation that, in principle, have veridical theoretical answers.

This is not to say, however, that pragmatic considerations play no role in theoretical
knowledge, including philosophical knowledge. Where are pragmatic considerations likely to enter into the invariantist account of logicality? They are likely to play a partial role in choosing the overall best background theories for the account. (Such a choice is needed when, e.g., we have no decisive veridical basis for choosing between two candidates for a background theory.) They are likely to play a partial role in deciding which logical system licensed by the invariantist account of logicality to choose in a particular context or given a particular goal. (For example, it is pointless to choose a system that includes high-infinite-cardinality quantifiers if our interest is limited to everyday inferences or even to inferences in physics.) They may be used in deciding on certain details of our system of logical consequence. (For example, the decision whether to limit models to structures with non-empty universes.) And so on. But pragmatic considerations should be used alongside, and be balanced by, considerations of veridicality and theoretical explanation, not in lieu of these. Or so I believe.

8. Conclusion

Invariance plays a central role in many fields of knowledge. In logic, it has plays a central role in the account of logicality, both on the level of logical constants and on the level of logical consequence, although in the second case its role has been largely overlooked. Logicality, however, can, and has been, approached from different directions, and this could, and has, led to unnecessary disagreements concerning the role of invariance in logic. In this paper I have tried to put some of these disagreements in perspective. In particular, I have explained the foundational-theoretical perspective on logicality as distinguished from the natural-linguistic intuitive perspective.

The foundational-theoretical perspective starts with a conception of logic’s role in the advancement of human knowledge, and proceeds to the requirement that logical consequences transmit truth from sentences to sentences with an especially strong modal force. It shows how the two invariance criteria of logicality, invariance-under-models and invariance-under-
isomorphisms, give rise to a logical system that grounds logical consequences in a particular facet of the world, namely formal laws, which have the requisite modal force. A central aspect of this account is the connection between invariance, formality, and modal force. Logical constants represent formal properties, properties that have an especially high degree of invariance and as such do not distinguish between most situations (including situations that are physically and even metaphysically possible and impossible). Logical consequences are based on laws governing the relations between such properties, laws that hold in all formally-possible situations, which are represented by the totality of models. As such, their modal force is greater than the modal force of laws and principles of other disciplines, whose scope falls short of all formally-possible situations.

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