ON THE EXPLANATORY POWER OF TRUTH IN LOGIC

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I. Introduction: Truth- or Proof- Theoretic Explanation of Logic?

Philosophers are divided on whether the proof-theoretic or truth-theoretic approach to logic is more fruitful. The proof-theoretic approach has its roots in Gentzen (1934-35) and Prawitz (1965). The truth-theoretic or semantic approach has its roots in Tarski (1936). More recently, the proof-theoretic approach has begun to encroach on semantics itself, with Dummett (1991), Brandom (2000), and others advocating *proof-theoretic* or *inferentialist semantics,* which they contrasted with *truth-theoretic*, and in particular *truth-conditional*, *semantics*. Thematically, proof-theoretic semantics is associated with verificationism, the meaning-as-use approach to language, assertibilism, anti-realism, anti-representationalism, pragmatist approach to truth, and/or epistemic approach to logic. Truth-theoretic semantics is often associated with a truth-conditional theory of meaning, representational approach to mind and language, realism, correspondence truth, and/or metaphysics. Although the debate on the preferable approach to semantics goes beyond logic, it is often focused on logic − logical constants, logical inference, etc.

My aim in this paper is to demonstrate the considerable explanatory power of a truth-based approach to logic. I will show that, and how, by employing a robust notion of truth (of a kind I will specify) we are able to provide:

- Explanatory characterization of logical inference, distinguishing it from other kinds of inference.
- General, explanatory, and systematic criterion for logical constants and operators.
- Explanatory account of logic's role or function in knowledge.
- Explanation of the veridicality of logic and its applicability in science.
- Explanation of the formality and strong modal force of logic.
- Explanation of the Normativity of logic.
- Explanation of the generality, topic neutrality, basicness, and (quasi-) apriority of logic.
- Explanation of the relation between logic and mathematics.
- Explanatory account of error, revision, and expansion in/of logic.

- Truth-theoretic explanation of logical proof (rules of proof).

And so on.

The high explanatory power of the truth-theoretic approach does not rule out an equal or even higher explanatory power of the proof-theoretic approach. But to the extent that the truth-theoretic approach is shown to be highly explanatory, it sets a standard for other approaches to logic, including the proof-theoretic approach.

II. Explanatory Methodology

By "highly explanatory" I mean what common sense together with some experience with philosophy suggest it means. A few characteristics of a highly explanatory philosophical account of a given subject-matter are: It is theoretical and systematic. It is not trivial or deflationist. It is not mysterious and it does not rely on what philosophers sometimes call "magic". It does not appeal to "obviousness" or gut feelings. It does not appeal to "self-justifying" ("self-evident") facts or beliefs. It is not based on our immediate intuitions. It is rational and informative. It is critical. In the present case: It does not take for granted the adequacy of any logical system. It does not treat natural-language users as an authority on logic. It focuses on the function and content of logic rather than on the way its central terms happen to be used in natural language. It approaches common beliefs on logic (e.g., the belief that logic is analytic) critically, questioning rather than taking them for granted. And so on. The expression "substantial explanation", as used in this paper, has a similar meaning.

Many philosophers believe that logic cannot be given an explanation of this kind: "An explanatory account of logic is impossible due to circularity", "Logic is too fundamental", "We have to start somewhere", "Not everything can be explained". The key to understanding the error of these reasons for giving up the goal of an explanatory account of logic is methodological. Whether a substantial explanation of logic is feasible depends on our methodological choices. If we acquiesce to the traditional *foundationalist* methodology, or uncritically adopt certain principles characteristic of it, then a substantial explanation of logic is indeed impossible. But if we choose a different methodology, then, depending on what methodology we choose, such an explanation becomes possible. In Sher (2016) I explained why the traditional foundationalist methodology lacks the resources for a highly explanatory account of logic, and I delineated an alternative methodology that provides the requisite resources − a holistic methodology of a special kind, which I called "foundational holism".

Foundational holism is a "foundation without foundationalism" methodology.[1] It licenses the use of holistic tools to achieve a foundational goal. The goal is explanatory justification of human knowledge, including logical knowledge. Foundational holism conceives of justification and explanation, as well as discovery, as dynamic, on-going projects, involving back-and-forth movement. Fields and items of knowledge are not viewed as forming a hierarchy with some disciplines/items lying at the bottom. There are no strict, apriori requirements[2] on the use of resources produced by some fields/items to justify/explain others. Circularity is treated with a critical yet open-minded manner.[3] In particular, partial circularity is recognized as a legitimate move, and some forms of circularity are viewed as *constructive*.[4] We explain logic using resources generated by mathematics, epistemology, metaphysics, the theory of mind, the theory of language, biology, psychology, neuroscience, etc., including resources generated by logic itself. But we use these resources carefully and intelligently, critically examining and evaluating the assumptions we rely on, updating our background knowledge, then going back and updating our explanations, and so on. The explanatory project is an on going project, but at each stage

[1] This is a variation on the title of Shapiro (1991): "Foundations without Foundationalism: A Case for Second-Order Logic".

[2] "Apriori" here means "determined in advance, with no attention to context or circumstances".

[3] For a similar approach see Sosa (2009).

[4] I will give examples of constructive circularity in Section III below.

substantial progress can, in principle, be made.

Foundational holism is contrasted with two other types of holism: (a) "Total" or "one-unit" holism − the view that the smallest unit of justification/explanation is our body of knowledge as a whole. (b) "Arbitrary" holism − the view that all items or fields of knowledge are equally (or automatically) connected to each other.[5]

It is important to emphasize that foundational holism is *not* a coherentist methodology. Foundational holism is designed for justification/explanation of knowledge understood as knowledge of the *world* or some facet(s) of the world. This understanding guides its conception of what counts as justification and explanation. Foundational holism is a methodology for *veridical* justification/explanation, i.e., justification of correctness vis-a-vis the world / explanation of relation to the world, rather than justification/explanation of coherence with our current body of beliefs. The underlying idea is that if all branches of knowledge target the world, then using a branch/item of knowledge K' to justify/explain K is using knowledge of some facets of the world (studied by K') to justify the correctness of K or explain its relation to the world.

When it comes to a truth-theoretic account of logic, two special advantages of the foundational-holistic methodology are: (a) We are not forced to treat the notion of *truth* as *primitive*. Truth itself is open to a substantial explanation, using the foundational-holistic methodology. (b) The common view that the truth-theoretic account of logic is exclusively metaphysical, neglecting the epistemic dimension of logic,[6] is undermined. A foundational-holistic truth-theoretic account of logic recognizes both the metaphysical and the epistemic dimensions of logic.

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[5] (a) and (b) are rightly criticized by Dummett (1973/81) and Friedman (2001), respectively. (Dummett focuses on the extension of (a) to language and understanding.)

[6] See Schroeder-Heister (2012/18).

III. The Power of Truth to Explain Logic

Not every truth-theoretic account of logic is highly explanatory. How explanatory a truth-theoretic account is partly depends on the use it makes of truth. And how it is able to use truth partly depends on its underlying methodology. To demonstrate the explanatory power of the truth-theoretic approach I will turn to a particular account, one that makes a particular use of truth and employs the foundational-holistic methodology. A detailed description of this account appears in Sher (2016). What I will focus on here is how, by starting with truth (and employing the new methodology), we obtain powerful resources for an explanatory account of logic. This is the new, positive contribution of the present paper.

What notion or conception of truth do we use to obtain this account? To arrive at a substantial explanation of logic using truth we need a rich and substantial notion/conception of truth (in contrast to a deflationist, minimalist, or quietist notion/conception). But what type of truth should that be − coherence? pragmatic? correspondence? What type of truth is relevant to logic? The answer to the last question cannot be determined prior to the development of some understanding of logic. That is, the answer to this question is part of the explanatory account itself.

My discussion will be divided into ten subsections. The relevant notion of truth will emerge in the discussion of logic's function in our cognitive life, or one of its central functions, with which I shall begin.

1. *The Epistemic Function of Logic*

One central aspect of humans' cognitive life is the pursuit of knowledge, including theoretical knowledge. Logic is needed to bridge two conflicting aspects of this pursuit. On the one hand our pursuit of knowledge is ambitious: we seek wide-ranging, high-quality theories of the world, theories subject to highly-demanding norms of adequacy and success. On the other hand, our cognitive resources are limited in various ways, hence high-quality knowledge is not easy for us to acquire. Two important ways in which these limitations interfere with our epistemic goals are (a) they make us, and our theories, prone to error, (b) they considerably restrict the amount of knowledge we can acquire as well our ability to establish each new item of knowledge independently, in a way that involves all parts of its content. The first problem explains the role of truth in knowledge, the second explains the function of logic in knowledge (in terms of truth).

The first problem, being prone to error, introduces an urgent concern with correctness. One of the most important norms that high-quality knowledge requires is, therefore, a norm of correctness. Given that we start with a notion of "truth" in our conceptual arsenal, we can recognize this norm as a norm of truth.[7] Which kind of truth? The norm of truth is concerned with the accuracy with which our theories describe the world (or some facet of the world), with whether what they say about the world actually holds. Therefore, the notion of truth involved in this norm is closer to what we understand by "correspondence truth" than by "coherence truth" or "pragmatic truth".[8]

But does the fact that our theories of the world are prone to error require that our norm of correspondence-truth be based on the *traditional* conception of correspondence? Traditional correspondence is committed to the naive, simplistic, and highly restrictive idea that for X to correspond to Y it has to be a "copy" or "mirror-image" of Y, or that it has to be directly

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[7] Notice the difference between this conception of the task of truth and the deflationist conception (Horwich 1990/98).

[8] Note that what we say about the significance of truth for the human pursuit of knowledge does not involve commitment to the traditional definition of knowledge as true justified belief. As a result, it is not subject to the well-known problems associated with this definition (e.g., Gettier problems). When it comes to philosophical notions that are as broad and complex as "knowledge" or "truth", the foundational-holistic methodology favors "theories" or "accounts" over "definitions". The former, having a loose and open-ended structure, are better equipped than the latter for handling the complexities involved in such notions.

isomorphic to Y. As far as the problem of error is concerned, there is no need to adopt this traditional conception. An appropriate norm of truth must require a substantial, systematic connection between each item/unit of knowledge and the relevant facet(s) of the world, but it does not have to require that this connection assume any specific pattern (such as copy or direct isomorphism). Nor does it have to require the existence of special objects − truth-makers, facts, or states of affairs − that true sentences literally correspond to (a requirement attributed to correspondence by many philosophers, traditional as well as contemporary). Once again, as far as the problem of error is concerned, there is no need to commit our correspondence norm of truth to the existence of such objects.

Accordingly, the notion involved in the norm of truth, as a norm of human knowledge, is a robust yet *non-traditional* correspondence notion. It is free from the superfluous encumbrances that led many philosophers to reject the correspondence conception of truth. This notion of truth is committed to the equivalence schema, understood as saying that a sentence is true iff (if and only if) what it says about the world is the case. But it allows that what this comes down to depends on, and varies according, to the complexities of different facets of the world (studied by different theories, targeted by different statements) as well as the peculiarities of our language.[9]

The second problem due to our epistemic limitations concerns the amount of high-quality knowledge we can acquire, given our physical and cognitive limitations. This problem makes the development of reliable shortcuts in the acquisition of knowledge crucial. Logic's epistemic function arises in this context. Given this problem, we will tremendously benefit from the development of devices that enable us generate new knowledge from given knowledge using shortcuts. One important device of this kind is that of *inference.* Logic's function is to develop an

[9] For a fuller description of this conception of correspondence-truth see, e.g., Sher (2004, 2015). For a similar view (in certain relevant respects) see Horgan (2001).

especially powerful method of inference.[10] And having a notion of truth, we can make fruitful use of it for an explanatory characterization of logical inference.

2. *Characterization of Logical Inference/Consequence*

Approaching logical inference from a truth-theoretic perspective, we can characterize it in two steps. First, we will characterize *inference* in general or, to use an expression commonly employed in truth-theoretic semantics, *consequence* in general; next we will characterize *logical inference.* To shorten the presentation, I will sometimes use "truth" instead of "correspondence truth". But "truth" will be always used as an abbreviation of "correspondence truth".

(**I**) An inference/consequence, **I**, is a pair, <X,Y>, where X is a sentence (set of sentences) and Y is a sentence, such that if X is *true* (all the sentences in X are true), Y is *true*. We may formulate the latter condition by saying that an inference, <X,Y>, *transmits truth* from X to Y.

This characterization clarifies what we care about, or what is at stake, in inference as a tool for the expansion of knowledge. What we care about is *truth*. We are not interested in whether X transmits beauty, or shortness, or widespread belief in, or explanatory power, or any of a great many other characteristics of X to Y. We are interested in whether it transmits *truth* to Y.

But this characterization does not distinguish between logical inference and other kinds of inference (biological, physical, metaphysical, etc.[11]). Indeed, it does not distinguish between logical inference and mere material inference: the pair <"New York City is in the United States", "London is in Britain"> satisfies (**I**). (**I**) is too weak for *logical* inference (and, indeed, for many other kinds of inference). Focusing on the task or function of logic in knowledge, logical

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[10] Another epistemic function of logic, connected with the first problem ((a) above), is the prevention of errors of a particularly destructive kind (called "logical contradictions"). The two tasks are related, but for the sake of brevity I will limit myself to the expansion of knowledge here.

[11] An example of a physical inference-schema is: <"*a* exerts force *f* on *b*", "*b* exerts force *f* on *a*">

inference needs to provide an especially strong guarantee of transmission of truth from X to Y. Furthermore, it should generally be based on certain specific elements of the content of X and Y rather than on their entire content (so we can know Y without having to deal with all elements of its content). And logical inference should also be universal, i.e., its schemata should apply to all fields of knowledge, or at least to a great many fields of knowledge.

One way to construe the first two requirements is in terms of *modal force* and *formality*. To give an especially strong guarantee of transmission of truth from X to Y, on this construal, would be to guarantee it with an *especially strong modal force*. And to transmit truth based on some, but not all, elements of the content of X and Y, would be to transmit it based on their *formal content*. At the outset "formal" is merely a stand-in for a specific type of content. The identification of this type of content requires further investigation.

Assuming (for the time being) that universality is expressed by the generality of the characterization (its unlimited applicability to fields of knowledge), this construal leads to the following characterization of logical inference:

(**LI**) An inference <X,Y> is *logical* iff X transmits truth to Y with an *especially strong modal force* and based on *formal* elements of X and Y.

A note on logical inference. The characterization of logical inference given by (**LI**) can be understood in two ways: (a) as distinguishing logical inference from other types of inference, and (ii) as distinguishing correct logical inference from incorrect logical inference. On the first reading, (**LI**) tells us what an inference has to do in order to be logical. On the second reading it specifies the conditions that have to be satisfied for a logical inference to be correct. Below I will use both forms of speech with respect to (**LI**).

A note on modal force. The explanatory account we are discussing interprets "X transmits truth to Y with an especially strong modal force" as "X transmits truth to Y in an especially wide range of counterfactual situations (including all actual situations)", where a counterfactual situation is a way the world could be or could have been. The question: "What is the exact scope of this array of counterfactual situations?” is addressed later on in the investigation.

A note on formality: By saying that a logical inference takes into account certain formal elements of its premises and conclusion, the account does not mean that it takes into account the “form” of the sentences involved as contrasted with their “content”, where *form* and *content* are taken as given in advance (perhaps determined by language). As emphasized by, e.g., Etchemendy (1990), the notion of “form of a sentence” is indeterminate. Any terms of the sentences X and Y can be viewed as determining their “form”. For example: If X is “John is a man” and Y is “John is a person”, then we can view the form of X as “... IS A MAN” and the form of Y as “... IS A PERSON”. This gives rise to the inference <“John IS A MAN”, “John IS A PERSON”> or, more naturally, “John IS A MAN; therefore, John IS A PERSON”. But we have no reason to regard this inference as a *logical* inference or its “form” as relevant to logic. What *form* or *formality* is in the case of logic is something to be investigated. The account we are discussing settles this question later on in its investigation.

3. *The Veridicality of Logic*

The veridicality of logic is deep. To see how deep, consider the following scenario: Someone develops a new logical theory, **L**, and urges you to adopt it as your logic. What considerations should guide you in evaluating the adequacy of **L**? Two important considerations are centered on truth:

(a) **L***'s claims must be true.* In particular, **L** should not claim that an inference <X,Y> is logically valid unless this claim is true, i.e., unless <X,Y> is in fact logically valid, that is, unless <X,Y> in fact transmits truth from X to Y with an especially strong modal force.

(b) **L** *must satisfy the requirement that logical inferences transmit correspondence-truth from premises to conclusion*. If X is, or could be, *correspondence-true*, **L** should not sanction any inference from X to Y unless it transmits *correspondence-truth* to Y. If logic is to fulfil its role in knowledge, its inferences must enable us to infer correspondence-true sentences from correspondence-true sentences.

The second requirement has significant ramifications for our understanding of logic. One of its ramifications is that, contrary to the accepted view, logic is constrained by the world. Let me explain. If X is true in the correspondence sense, then X is constrained by the world. If Y is to be true in the correspondence sense, then Y must be constrained by the world. This means that any inference sanctioned by an adequate logical theory is constrained by the world. If the world satisfies, or could be such as to satisfy, the conditions set on it by X but cannot satisfy the conditions set on it by both X and Y, then <X,Y> is *not* a logically-valid inference, and any logical theory that says it is is inadequate.

Logic, therefore, is constrained not just by our language (concepts, pragmatic considerations, etc.). Logic is also *significantly constrained by the world*. Logic is not a mere game, such as chess or baseball. Nor is it merely conventional. Whatever rules of inference logic sanctions, these rules are constrained by the world. Logic cannot sanction a move from X to Y if the world is, or could be, such that X is correspondence-true but X and Y together cannot be correspondence-true.

Once we realize this point, we have made considerable progress toward a highly explanatory account of generality, modal force, and formality of logic.

4. *Generality, Modal Force, and Formality*

By connecting logic to the world, the truth-theoretic approach (as construed here) brings about a revolution of a sort, or a transformation, in the philosophy of logic, one that, as we shall see below, considerably augments our explanatory resources.[12] Traditional truth-

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[12] The few philosophers who did connect logic to the world, such as the early Wittgenstein, did not see their way to a theoretical, systematic, and explanatory account of this

oriented approaches do not partake in this transformation. For example, the *analytic* approach, which says that logical claims are analytically true, does not. This approach leaves us with, and is possibly one of the sources of, the widespread belief that logic is only constrained by our language, concepts, and/or cognitive makeup. Many adherents of this belief regard logic as trivial.

The view that logic is trivial calls for a critical comment. If the function of logic, or one of its main functions, is to enable us to genuinely expand our existent body of knowledge, then logic cannot be trivial. If logic is trivial, then it essentially leaves our existent knowledge as it is. But can logic significantly expand our knowledge? It can and does. Consider the use of logic in mathematics. In mathematics we commonly distinguish between axioms and theorems. Axioms are what we start with, our existent knowledge; theorems (or more precisely, proper theorems, i.e., theorems that are not axioms) convey new knowledge, knowledge we acquire through logical proofs, i.e., by applying logic to the axioms.[13] Not all theorems provide genuinely new knowledge. But many do. Consider, for example, Cantor's theorem.[14] By any reasonable measure of what counts as *new knowledge*, this theorem provided us with new knowledge over and above the knowledge *immediately* conveyed by the axioms of set theory (i.e., without any use of logic). It tells us that there are many different infinities, increasing in size. One is hard pressed to say that this is just a trivial extension of the standard set-theoretic axioms. Mathematics in general is full of eye-opening theorems whose difference from the axioms is exclusively due to logic. (It is perhaps mainly in teaching elementary logic, where we focus on simple rules of proof and simple

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connection, let alone to a systematic explanation of logic based on this connection.

[13] Here I am speaking in broad strokes. What end up being our axioms is not always what we started with. Various pragmatic norms determine how we divide a given theory into axioms and theorems in later stages of constructing the theory. But it is easy to reformulate the broad picture so these minor deviations are set right.

[14] Cantor's theorem says that for any set *s*, the cardinality of the power set of *s* is larger than the cardinality of *s*.

applications of these rules, that logical proofs seem to impart no genuinely new knowledge.)

The present truth-theoretic approach provides us with new resources for explaining how logic expands our knowledge – how, by applying logic to existent knowledge of the world, we beget new knowledge. It enables us to appeal not just to language, concepts, and mind to explain how logic expands our knowledge of the world, but to the world itself, or some of its facets.[15] The basic structure of such an explanation is this: We know that logic is constrained by the world in the sense that logic cannot say that X logically implies Y if the world is, or could be, such that X is true but X and Y cannot both be true.[16] Now, if the world is such that (a) some of its features are systematically related to each other with an especially strong modal force, (b) these features are universally applicable, and (c) there is a distinct category of expressions that canonically denote these features, then we can single out (all or some of) these expressions as determining the logical form of sentences and build a system of inferences based on the modally-strong connections between these features. I.e., if the world is such that some of its features necessitate others universally and with an especially strong modal force, then these features can serve as a basis for logical inferences (satisfying (**LI**)). Such inferences will be grounded[17] in the *laws* governing (and connecting) these features. And if these features are naturally viewed as *formal*, this can give substantive content to the view that logical inferences are "formal". If, finally, neither the features in question nor their laws are trivial, logical inferences will (generally) be

[15] Some philosophers hold the view that everything that has to do with the world is contingent. This view is challenged by the present account. Some laws governing the world have considerable modal force, as we shall shortly see. These laws hold not just in the actual world but also in counterfactual situations.

[16] Indeed, logic cannot say that X logically implies Y even if the world *could be* such that X were true and Y were false.

[17] In this paper I use "grounding" as a synonym of "foundation" (which, in turn, I interpret as veridical justification in a broad sense that includes theoretical explanation, confirmation, and so on).

non-trivial. And the knowledge gained by such inferences will be non-trivial as well. A system of inference of this kind will substantially expand our knowledge by transmitting truth from sentences that are part of our existing body of knowledge to sentences that are not (yet) part of it. Such a system will fulfill logic's function in knowledge. And we will be able to explain why and how it fulfills it.

An explanation of logic along these lines will be substantial: it will ground logic in objective laws, it will tell us why logical inferences can transmit *correspondence*-truth from sentences to sentences (the laws they are grounded are true in the correspondence sense)[18], and it will show how certain traits of logical inferences render them both highly general and highly necessary (these laws are highly general and highly necessary). But can we in fact explain logic in this way?

The answer to this question is positive. Let me begin by showing how this can be done in a particular case. Consider two inferences that are prima-facie paradigms of logically-valid and logically-invalid inferences. Say:

(1) Something is either green or red; Nothing is green. Therefore: Something is red.

(2) Something is either green or red; Something is green. Therefore: Something is red.

Why is (1) a valid logical inference and (2) an invalid logical inference? The premises of (1) say that the union of two properties is non-empty while one of these properties is empty. Its conclusion says is that the other property is non-empty. The premises of (2) say that the union of two properties is non-empty while one of these property is also non-empty. Its conclusion says that the other property is non-empty. But while the world is governed by a modally-strong law that underwrites the first claim, it is not governed by a modally-strong law that underwrites the second claim. This is the reason the (correspondence-) truth of the premises of (1) necessitates

[18] There are two ways to view laws: (i) as (highly general and highly necessary) regularities governing the world and (ii) as law-like statements that are true of the world. The two are closely related, and I will move from one to the other without further comment.

the (correspondence-) truth of its conclusion, while the (correspondence-) truth of the premises of (2) does not necessitate the (correspondence-) truth of its conclusion. And this, in turn, is the reason that (1) is logically valid while (2) is not.

Note that in explaining the logical-validity of (1) and the logical-invalidity of (2) we did not appeal to the entire content of (1) and (2). We appealed only to what is naturally regarded as the *formal content* of (1) and (2), or the *formal skeleton* of their content. The only elements of the content of (1) and (2) that belong to this skeleton are three formal parameters: union, non-emptiness (of properties), and complementation. These formal parameters are correlated with what we paradigmatically view as the logical constants of (1) and (2), as reflected by their standard logical regimentation:

(1') $(\exists x)(P_1x \vee P_2x)$, $\sim(\exists x)P_1x$; therefore $(\exists x)P_2x$.

(2') $(\exists x)(P_1x \vee P_2x)$, $(\exists x)P_1x$; therefore $(\exists x)P_2x$.

The logical constants of these inferences are "$\exists$", "$\vee$", and "$\sim$", and they denote the 2nd-level property of non-emptiness and, in the context of (1') and (2'), the union operation, and the complementation operation (the complement of the property of being non-empty is the property of being empty), respectively.[19] These logical constants are formal not just in the sense of determining the "form" of the inferences in question, but also in a deeper sense, indeed, in a sense that explains why the laws governing them are highly general and have an especially strong modal force.

5. *Logical Constants*

Most approaches to logic do not offer a general, systematic, and philosophically

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[19] Notes: (i) For individuals and properties (in a broad sense including relations and operators) I use the terminology of "levels". Individuals are objects of level 0, properties of individuals are of level 1, properties of properties of level 1 are of level 2, and so on.

(ii) For the purpose of the present discussion there is no need to settle the ontological status of properties. The discussion is compatible with various views of properties.

explanatory characterization of logical constants. Adherents of the proof-theoretic approach tend to characterize logical constants in terms of *introduction* and *elimination* rules (see, e.g., Dummett 1991). But one is hard pressed to establish a philosophically significant connection between the idea of introduction/elimination rules and the idea of logicality.

The traditional truth-theoretic approach does provide a general, philosophically significant, characterization of some logical constants. It characterizes the logical constants of sentential logic − the logical connectives − as truth-functional. But it provides no comparable characterization of the logical constants of predicate logic. These constants are identified merely by enumeration − an utterly unexplanatory characterization.

One of the virtues of the truth-theoretic approach to logic is its ability to provide a general, systematic, and highly explanatory characterization of logical constants. The characterization we shall discuss here starts with the logical constants of predicate logic and is then extended to the logical constants of sentential logic in a way that coincides with their truth-functionalist characterization. Historically, this characterization is traced to Mostowski (1957), Linsdtröm (1966), and Tarski (1966). Philosophically, there are several ways to present it.

Let us start with our (prima facie) paradigmatic example of a logically-valid inference, (1) above. On our truth-theoretic analysis, the logical constants of this inference denote three intuitively formal properties: the cardinality property *is non-empty*, the *union* operation, and *complementation*.

Now, these properties share a very special trait: they all have an *especially high degree of invariance*. More specifically, they are *invariant under all (uniform) 1-1 replacements of individuals within and across domains*. Using a mathematical term of art, they are invariant under all *isomorphisms*. What does this mean? And how is it relevant to logicality?

Invariance in general, as applied to properties, is a measure of what a given property takes into account and what it does not take into account. Some properties take into account fewer things (in the sense of inclusion) than others.[20] For example, the property *x is a person* takes into account fewer things than the properties *x is a man* and *x is a woman*. It does not take gender into account while they do. This can be expressed in terms of invariance by saying that *x is a person* is invariant under all 1-1 replacements of men (individuals who are men) by women (individuals who are women). Such replacements take objects that have the property *x is a person* to objects that also have this property and objects that do not have this property to objects that do not have it. But the properties *x is a man* and *x is a woman* are affected by such replacements: individuals that have the property *x is a man* are replaced by individuals that do not have this property, and the same holds for individuals that have the property *x is a woman.* More generally, all 1-1 replacements of individuals that do not affect *x is a man/woman* do not affect *x is a person*, but not vice versa. In this sense, the degree of invariance of *x is a person* is higher than that of *x is a man/woman*.

Overall, however, *x is a person* does not have a very high degree of invariance. If we replace persons by, say, numbers, *x is a person* will be affected. The property *x is a person* is not preserved under *all* 1-1 replacements of individuals. Indeed, no biological, physical, psychological, social, or political property is invariant under all 1-1 replacements of individuals. And the same holds for many other properties. But not all. Some properties *are* invariant under all 1-1 replacements of individuals whatsoever. These properties have an especially high degree of invariance. In fact, all the properties denoted by the logical constants of (1) are of this kind. Indeed, all the properties denoted by the standard logical constants of predicate logic are of this kind. (To shorten and simplify the discussion I will focus on 1st-order predicate logic, but later on I will briefly explain why this extends to higher-order logic.)

Consider the 1st-level relation of identity: If we (uniformly) replace any individuals (in our ontology) by any individuals in a 1-1 manner, the relation of identity "will not notice". Any

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[20] Here and below I understand "fewer", "smaller", "larger", etc. in the sense of inclusion. I.e., a collection c_1 of things is larger than a collection c_2 of things iff c_2 is properly included in c_1.

such replacement will take us from a pair of individuals that stand in the identity relation to a pair of individuals that stand in this relation and from a pair of individuals that do not stand in this relation to a pair of individuals that do not stand in this relation. The same holds for the 2nd-level property of non-emptiness (denoted by the 1st-order existential quantifier).[21] Take any 1st-level property P and (uniformly) replace any individuals (in your ontology) in by any individuals in a 1-1 manner. The non-emptiness property will not notice. P has this property iff its image under the given replacement of individuals is non-empty. The property of non-emptiness is invariant under all 1-1 replacements of individuals. So is union: Take any individual *a* and any 1st-level properties P_1 and P_2. Now, replace individuals by individuals in a 1-1 manner. The union operation will not be affected: *a* is in the union of P_1 and P_2 iff its image under this replacement is in the union of the image of P_1 and the image of P_2 under this replacement.

In the case of complementation (the operation denoted by negation in a predicative setting − "~Px"), there is a small wrinkle: here it is essential to specify a *domain*. What belongs to the complement of a given property in one domain may not belong to it in another, smaller domain (in the sense of inclusion). The same holds for universality − the property denoted by the universal quantifier. A property can be universal in one domain but not in another. In particular, a property universal in a given domain may not be universal in a larger domain. So here it is important that we talk about 1-1 replacements of all the individuals of a given domain, where the replaced individuals also form a domain − either a new domain or the same domain. But the principle remains the same. The universal-quantifier property is invariant under any 1-1 replacement of the individuals of any given domain. A 1-place 1st-level property P is universal in a given domain D iff for any 1-1 replacement of all the individuals in D, the image of P under this replacement, P', is universal in D' (= the image of D under this replacement). And this holds

[21] Terminology: "1st-order" applies here to logical systems and their languages, whereas "1st-level" applies to properties. The 1st-order logical constant of identity denotes the 1st-level property of identity, but the 1st-order quantifiers (quantifiers of 1st-order logic) denote 2nd-level properties. This observation goes back to Frege (1884).

for all the logical constants definable from the primitive logical constants of standard 1st-order logic as well as higher-order logic, such as “has cardinality n”, where n is finite (or, using a quantifier expression, “there are exactly n things such that ...”).

Now, if, for the sake of unity, we always express invariance under 1-1 replacements of individuals in terms of domains, we immediately get invariance under “isomorphisms”. The universal- and existential-quantifier properties, for example, are invariant under all isomorphisms of structures of the type <D,B>, where D is a domain (i.e., a non-empty set of individuals), and B is a subset of D, representing a property P as it applies (or is restricted) to D. I.e., if <D,B> and <D’,B’> are isomorphic, then B is universal/non-empty in D iff B’ is universal/non-empty in D’. Another name for this kind of invariance is “invariance under bijections”, bijection being a 1-1 and onto function.

Our next step is to show that, and explain why, all the logical inferences of standard predicate logic are highly formal, highly general, and have an especially strong modal force. We have seen that all the logical constants of this logic have a *special trait*, a trait that distinguishes them from all biological, physical, psychological, sociological, moral, and political constants. The logical constants of standard predicate logic denote properties that have an *especially strong degree of invariance*. And this special trait, we shall presently see, is inherently connected to formality, generality, and strong modal force.

Formality. There is an *intuitive* sense in which the properties denoted by the logical constants of standard predicate logic − complementation, union, identity, non-emptiness, universality, having cardinality n, etc. − are formal. But we are interested in a *theoretical* explanation of their formality. Theoretically, their formality is explained by their strong degree of invariance. The philosophical significance of the fact that these properties are invariant under all 1-1 replacements of individuals is that they do not pay attention to anything but *formal structure*. Formal structure abstracts from the identity of individuals as well as from those features of properties that are sensitive to the identity of individuals. What is left when you take away the identifying characteristics of individuals – those that distinguish any one individual from any other – is *formal structure*. Invariance under isomorphisms is a theoretical characterization of formality in a sense akin to strong structurality.[22] Since the properties denoted by the standard logical constants satisfy this invariance condition, they are formal in this theoretical sense.

Next, let us note that not just properties but also laws have a degree of invariance. The laws of gravity are invariant under replacements of rocks by humans and vice versa, but not under replacements of rocks by numbers. They hold of humans as much as they hold of rocks. But they do not hold of numbers. In contrast, the laws of identity do not distinguish between rocks and numbers and the laws of finite cardinalities do not distinguish between physical properties (properties restricted to physical individuals) and mathematical properties. The laws of identity and finite cardinalities are invariant under all 1-1 replacements of individuals: they have the same degree of invariance as that of the formal properties they are laws of. More generally, the laws governing formal properties have the same degree of invariance as formal properties. Given our theoretical characterization of formality, the laws of formal properties are formal.

Now, logical inferences are grounded in formal laws. In this sense, *logical inferences are formal*.

Generality and Modal Force. Our next observation concerns the connection between the degree of invariance of laws and their scope: The greater the degree of invariance of a given law, the greater its scope. This applies both to the actual and to the counterfactual scope of laws. The laws of physics hold of all actual physical objects as well as of all physically possible objects. But they do not hold of physically impossible objects. Formal laws, which have a greater degree of invariance than physical laws, hold in a larger array of counterfactual situations than physical laws. Formal laws hold not just in counterfactual situations that obey the laws of physics but also in counterfactual situations that do not obey the physical laws. For example, they hold in

[22] There are some affinities between this theoretical conception of formality and Shapiro's (1997) theoretical conception of structurality.

counterfactual situations in which material objects violate the laws of gravitation. They also hold in counterfactual situations that violate metaphysical laws, or laws that are on the border of physics and metaphysics. For example, they hold in counterfactual situations in which an object has two different temperatures at the same time (see Chang 2007) as well as in counterfactual situations in which an object is both all-red and green at the same time. They do not "pay attention" to such non-formal features of objects as heat or color.

The strong degree of invariance of formal laws means that they hold in (a) all actual situations, and (b) an especially large array of counterfactual situations. As a result of (a), the formal laws that ground 1st-order logical inferences, hence the logical inferences they ground, are *highly general*. As a result of (b) they have an *especially strong modal force*.

Altogether, the present account shows that, and explains why, the logical inferences of standard logic are *formal*, *general*, and have an *especially strong modal force*.

But that is not all. The invariance test is not simply a test that happens to be satisfied by the properties denoted by the standard logical constants. It is a systematic test, and the properties that satisfy it form a philosophically significant collection − a philosophical "natural kind", so to speak. These properties include all strongly structural properties: not just the properties denoted by the standard logical constants but many others as well: reflexivity, symmetry, and transitivity, all cardinality properties (infinite as well as finite), and indeed all higher-level correlates of 1st-level mathematical properties that fail the invariance test as they stand.[23] Philosophically, all properties that satisfy the invariance test belong to the natural kind of *formal properties*, and their laws are formal laws. This explains why higher-order logic satisfies the formality, generality, and necessity requirements as well. All the properties definable in terms of its logical constants are formal. (This result can be traced back to Lindenbaum & Tarski 1934-35).

[23] Thus, the 1st-level union operation fails this test, but 2nd-level union passes it. The same holds for the 1st- and 3rd-level property "is even", the 1st- and higher-level membership relation, and so on. But their higher-level versions pass the test. (See, e.g., Tarski 1966)

Treating invariance under 1-1 replacements of individuals as a criterion of formality and formality as a special, distinctive trait of the properties denoted, or denotable, by logical constants, leads to a systematic criterion for (predicative) logical constants:

(**LC**) A constant is, in principle, logical iff it denotes a formal property.

"In principle", here, means two things: (a) It means that there is a systematic connection between logicality and formality (in the invariance sense), a relation that has to do with theoretical philosophical principles rather than with mere impressions, intuitions, or habits. Some such connection was already recognized by Kant (1781/87), Frege (1879), and Tarski (1936), who all emphasized that logic does not distinguish between different objects. (b) It allows that there might be additional principles and considerations involved in determining which constants are treated as logical. These include (i) theoretical considerations pertaining to the inclusion of any constant as a logical constant in a particular type of a logical system, and (ii) pragmatic considerations that have to do with the intended uses of a given logical system. In Sher (1991) I specified a series of theoretical conditions that a constant denoting a formal property has to satisfy in order to serve as a logical constant in a "Tarskian" syntactic-semantic system of the kind found in standard textbooks of so-called *mathematic logic*. Pragmatic considerations might say that there is no point in including a quantifier that denotes the 2nd-level formal property *indenumerably many* in a logical system designed for use in an area where the issue of infinite cardinalities plays no role.

In addition to being systematic, theoretical, and anchored in a general, explanatory account of logic, (**LC**) is also a *critical* criterion. It is not a stamp that simply affirms our accustomed view of logic. It follows from this criterion that there are more admissible logical constants than we are accustomed to, and this forces us to rethink our habitual views on the scope of logic.

Let us take stock. Having approached logic from a truth-theoretic perspective and adopted a holistic foundational methodology, we have shown how the resources generated by this perspective and methodology make a highly explanatory account of logic possible. The account we have focused on begins with an analysis of the function of logic in a central facet of humans' cognitive life, namely the search for knowledge of the world. The account characterizes this function using a notion of truth that is centered on agreement with the world, namely, a *correspondence* notion (which it frees from superfluous traditional encumbrances). The account grounds logic in a particular type of law governing the world, *formal* law, a type of law that, that, due to its formality, has sufficient generality and modal force to ground logical inferences. These special traits of formal laws have the power to explain other characteristics of logic as well. Let us now turn to some of these.

6. ***Topic Neutrality, Basicness, and (Quasi-) Apriority***

Topic Neutrality. The generality of logic, as explained above, includes topic neutrality. The generality of logic includes applicability to all areas of knowledge and their topics. This is just topic neutrality.

Basicness. Logic, on the present account, is not basic in the foundationalist sense (according to which basic items of knowledge can partake in the justification/explanation of other items of knowledge but no items of knowledge can partake in their justification/ explanation). But the formality of logic explains a different sense in which logic is basic. Since the scope of formal laws is larger (in the inclusion sense) than the scope of other laws, the scope of logic, which is grounded in these laws, is larger than that of other disciplines. Accordingly, logic applies to all other disciplines, but they do not apply to it. In particular, logic applies to all empirical sciences, but their laws do not apply to it. In this sense, logic is more basic than the empirical sciences.[24]

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[24] I will not be able to compare the basicness of logic and metaphysics here. Metaphysics is a heterogeneous discipline and its principles vary in scope. Logic is more basic than the narrower parts of metaphysics (as we have seen above with respect to the possibility of all-red and green objects), but the situation is more complex with regard to metaphysics' most general

Quasi-Apriority. Logic is traditionally viewed as purely apriori. Foundational holism, like other types of holism, challenges the traditional bifurcation of knowledge into purely apriori and empirical. But challenging this bifurcation is not tantamount to saying that logic is empirical. Foundational holism emphasizes the multiplicity of ways in which sensory perception and intellect join forces in obtaining, justifying, and explaining knowledge. At the same time, it recognizes that differences in subject-matter affect the balance of these sources of knowledge. In logic's case, the balance tilts away from sense perception. But not completely. Given that physics is bound by formal laws, some errors in physics have their roots in formal errors. In those cases, discovering an error in physics using some combination of sensory and intellectual resources may point to an error that is largely formal. Given humans' cognitive makeup, this method of discovering formal, hence logical errors, is significant as a matter of principle and not just as a matter of accident. This is one reason a foundational-holist views logic as quasi-apriori rather than purely apriori.

But why does logical knowledge tilt toward apriority? The explanation is similar to those given above for logic's generality, modal force, and basicness. Logical inferences are grounded in formal laws, and due to their especially high degree of invariance, formal laws hold in a very broad array of situations, including situations that are inaccessible to sensory perception. As a result, the role of sensory perception in logical knowledge is significantly limited. The role of sensory perception in the discovery, justification, and explanation of *correct* logical claims (as opposed to the refutation and explanation of *incorrect* logical claims) is especially minimal.

7. *The Normativity of Logic and its Applicability to Science*

Logic is naturally viewed as normative in the sense of telling us which inferences are

parts. For comparative comments on the concepts of metaphysical necessity/possibility and formal necessity/possibility see Sher (1996).

good and justified and which inferences are either bad or require extra-logical justification.[25] This is right, but it leaves important issues unaddressed. For example, it says nothing about the *source* of the normativity of logic and it does not explain why the normativity of logic is greater than that of other disciplines.

Before turning to these points, however, we have to acknowledge that the claim that logic is normative has been challenged, primarily by Harman (1986). Harman's challenge, however, does not apply to the normativity of logic as it is understood here. Harman understands normativity as *agent-oriented* and *unexceptional* (allowing no exceptions). He rightly points out that it may not be a good idea for a human agent to draw all the logical consequences of her beliefs; doing so might clutter her mind. He also rightly points out that in choosing a course of action, an agent has to take many norms and considerations into account, not just logical norms/considerations, and that under certain circumstances it might be reasonable to give greater weight to extra-logical norms/considerations. But here, the claim that logic is normative is understood as involving primarily *theory*-normativity and only secondarily agent-normativity (in Harman's sense). And in both cases, the normativity of logic is taken to be *exceptional*. I.e., both theories and agents are viewed as subject to numerous norms − logical and others − which they have to balance. On this understanding, Harman's objections are moot.

Having clarified this point, let us turn to the source of logic's normativity. By approaching logic from a truth-theoretic perspective, we can adopt the Fregean view that all disciplines are normative, and their source of normativity is their truth. Physics is normative with respect to the way we ought to think about the physical world and logic is normative with respect to the way we ought to think about the validity of inferences. But we can go further. We can explain that in the case of logic it is the truth of *formal laws* (formal-law claims) that is the

[25] Logic is also naturally viewed as normative in the sense of telling us that certain claims and theories, namely logically contradictory claims and logically inconsistent theories, are bad. The two senses are directly connected, but to save space, I will not get into the second one here.

specific source of its normativity.

As for the question: “Why is the normativity of logic greater than that of, say, physics?”, this question is, at this stage of our discussion, easy to answer. The reason logic’s normativity is greater than physics’ is the same as the reason that its generality, modal force, and basicness are greater than those of physics. The scope of the physical laws is properly included in the scope of formal laws, hence logic is normative for physics but not the other way around.

8. *Relation between Logic and Mathematics*

The relation between logic and mathematics is, and has been for a long time, a central topic of philosophical inquiry. The most influential doctrine focused on this relation is logicism, developed by Frege (1879), adopted and adapted by Russell (1908), and renewed, under the name “neo-logicism”, by Wright (1983), Hale & Wright (2001), and others. Many of logicism’s problems have been widely discussed in the literature (see, e.g., Tennant 2013/17), but one of its most serious problems is largely ignored. This problem is philosophical and methodological. Philosophically, logicism sets out to provide a theoretical foundation for mathematics in logic, where “foundation” is understood in the traditional foundational*ist* sense. However, logicism does not, and indeed cannot, either provide or help itself to a theoretical foundation for logic. Why? To provide a theoretical foundation for logic, we have to use some *theoretical* resources. But since (i) logicism is a *foundationalist* doctrine, (ii) logic is considered by logicists to be a fundamental discipline (i.e., a discipline that lies at the bottom of the foundationalist hierarchy), and (iii) foundation*alism* requires that the only theoretical resources for a foundation of a discipline X be taken from theories lower down in the foundational*ist* hierarchy than X, there are simply no theoretical resources for a foundation for logic to be used by logicists. As a result, logicism, viewed as a foundationalist philosophical doctrine, rests on shaky grounds.

The *correspondence*-truth-theoretic approach to logic, combined with the *holistic foundational methodology*, enables us to provide an alternative to logicism. As a nonfoundationalist (yet *foundational*) approach, it makes theoretical resources available for a foundation of logic. And as a correspondence-truth theoretical approach, it sanctions the use of truth-related theoretical resources in constructing a foundation for logic. This opens the door to a joint theoretical foundation for logic and mathematics. Logic and mathematics are *jointly* grounded in the *formal structure* of the world. Mathematics studies the laws governing the formal parameters of this structure (formal properties of various kinds); logic constructs a powerful method of inference based on these laws. There is, thus, both a close connection and a division of labor between logic and mathematics. Mathematics studies the laws of union, complementation, intersection, inclusion, finite cardinality, and so on. Logic builds these formal parameters into our language as logical constants and develops a method of inference grounded in the formal laws governing these parameters. Often, mathematical theories study these formal parameters (properties) through representative objects. For example, 1st-order arithmetic studies finite cardinalities (which are 2nd-level properties) by studying numerical individuals: the numbers 0, 1, 2, ... 1st-order set theory studies 2nd-level intersection, union, complementation, ordering properties of relations, etc., by treating their arguments as individuals: sets.

It is important to emphasize, however, that while historically, logic used the resources of specific mathematical theories and mathematics used the resources of particular logical theories, in principle, their foundation does not depend on these choices. What is the best, or even an adequate, mathematical theory of formal structure and what is the best (or a good) logical framework for mathematics are open questions, and a *philosophical* foundation for logic and/or mathematics does not need to settle these questions.

The relation between logic and mathematics on this account is an example of what I earlier called “constructive circularity”. Logic and mathematics are developed in tandem. Schematically, we can view their development as progressing in stages along something like the following lines. Starting with a simple logic-mathematics (e.g., something on the order of a Boolean and syllogistic logic-mathematics) we build a simple mathematical system and, using its resources, we develop more and more sophisticated mathematics, (e.g., up to naive set theory). Using this mathematics we formulate a stronger logic (e.g., standard 1st-order mathematical logic), using this logic we build a more sophisticated mathematics (e.g., axiomatic set theory), using this theory we build a still more sophisticated logic (say, 1st-order logic with generalized quantifiers, such as “most” or “denumerably many”), and so on. In short, we provide a joint foundation for logic and mathematics in the formal layer of the world, where the formal itself is sharpened and explained using resources taken from various disciplines, including logic and mathematics.

How do we detect errors in logic and mathematics, given this circularity? As we have explained above, the foundational-holistic methodology sanctions partially circular patterns of justification, explanation, and discovery, including discovery of error. Take Russell’s discovery of a paradox in Frege’s logic as an example. Russell had to use some logical resources in order to discover the paradox. Indeed, due to the complexity of the paradox, he had to use quite sophisticated logico-mathematical resources. But Russell used these resources carefully and intelligently, relying on some assumptions and suspending others, going back and forth, adding mathematical resources, common-sensical principles of rationality, and so on. Partially relying on logic, including elements that were part of Frege’s logic, did not prevent Russell from discovering the paradox inherent in that logic.

9. *Error, Revision, and Expansion of Logic*

One myth concerning logic is that it is beyond error. Ironically, there are two opposing ways to arrive at this myth. One is based on the view that our logical theory is so clearly true that its falsehood is inconceivable. The other is based on the view that our logical theory is neither true nor false; it is merely conventional. History suggests that the myth is false. First, there is a well documented case of error in one of the most important logical theories in the course of history −Frege’s logic. Second, many philosophers and logicians argue that one or another logical theory is wrong: either absolutely, or in a particular context. For example, adherents of intuitionistic logic tend to regard classical logic as erroneous, period. And adherents of fuzzy logic tend to regard classical logic as erroneous in certain contexts. Finally, almost everyone agrees that certain logical theories err on the side of omission. Aristotelian logic, for example, falls under this category.

The truth-oriented, foundational-holistic account of logic discussed here recognizes the possibility of error in logic. And it focuses our attention on a type of error that is often neglected: error concerning the formal laws that ground logical inferences. If we make a mistake concerning formal laws, we will affirm inferences that are logically invalid, as we have seen in subsection 4 above concerning (2). Indeed, many of the "logical fallacies" studied in elementary classes of logical reasoning fall under this category. Consider a predicative version of the fallacy called "affirming the consequence":

(3) Every A is a B, Something is a B; therefore: Something is an A.

For (3) to be logically valid, being included-in-a-non-empty-property would have to formally necessitate being non-empty, i.e., the connection between the two conditions would have to be a formal law. Since this is not the case, (3) is logically fallacious. A similar explanation applies to Frege's error: the world is such that no formal law grounds Basic "Law" V. This "Law" is erroneous. This explains why a change in Frege's logic (or in one of its background assumptions) was needed.

10. *Explanation of Logical Proof in terms of Truth*

It is questionable whether the proof-theoretic approach has resources for explaining the truth-related features of logic in a *principled* way.[26] But as we shall presently see, the truth-

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[26] An example of a non-principled explanation is one that takes a (relatively small) finite collection of rules of proof, checks them *one by one*, and shows that *intuitively* each has certain truth-related features.

theoretic approach to logic (as it is construed in this paper) has resources for explaining its proof-related features in a principled manner. We have seen how this truth-theoretic approach explains many of the central characteristics of logic, from transmission of truth to modal force. These explanations come together in its explanation of the Tarskian, model-theoretic definition of logical consequence,

(**MT**) An inference (consequence) <X,Y> is *logical* iff X transmits truth to Y in all models.

On this explanation, models represent formally-possible situations with respect to a given language, and the only parameters of the language given fixed denotations in models are its logical constants, whose denotations are formal. Very briefly, the totality of models represents the totality of formally-possible situations, both actual and counterfactual, with respect to the given language. And regularities across models are formal regularities or laws. Accordingly, transmission-of-truth-in-all-models is due to the formal parameters of sentences involved and is grounded in formal laws governing the formal properties denoted by these constants. This ensures that inferences satisfying (**MT**) satisfy

(**LI**) An inference <X,Y> is *logical* iff X transmits truth to Y with an *especially strong modal force* and based on *formal* elements of X and Y.

Now, there is a straightforward way of using the elements of this explanation to explain the proof-theoretic characterization of logical inference which, on one formulation, takes the form:

(**PT**) An inference <X,Y> is *logical* if there is a proof of Y from X,

where a proof is a finite sequence of sentences/formulas such that each sentence/formula is either a member of X (or X itself, if X is a sentence), or is obtained by a rule of proof from earlier elements in the sequence.[27]

Briefly, the explanation appeals to the existence of a partial correlation between formal

[27] It is easy to adjust the explanation below to other forms **PT** commonly takes.

laws and proofs, including rules of proof. Every rule of proof encodes a formal law, and every proof leads from premises to conclusion by a series of rules of proof that together encode a formal law. This explains why adequate systems of rules of proofs yield inferences that satisfy (**MT**). And this, in turn explains why these inferences satisfy (**LI**). Put together, the explanation says that provable inferences are logically valid because they are grounded in formal laws, laws that, being formal, are highly general and have an especially strong modal force. Thus consider our original example of a logically-valid inference,

(1) Something is either green or red; Nothing is green. Therefore: Something is red,

logically regimented as

(1') $(\exists x)(P_1x \vee P_2x)$, $\sim(\exists x)P_1x$; therefore $(\exists x)P_2x$.

From a proof-theoretic perspective, the logical validity of (1) is due to the provability of (1'). One way to construct a proof of (1') is based on the rules of Quantifier Negation, Existential Elimination, Universal Elimination, Disjunctive Syllogism (applied in a predicative setting), and Existential Introduction. It is easy to correlate each of these rules with a relatively simple formal law and the finite sequence of these rules, in the order they appear in the proof, with a more complex formal law. This complex law (described in subsection 4 above) grounds both the model-theoretic validity and the proof-theoretic validity of (1'). Since passing the model-theoretic test is a good indicator of logical validity, so is passing the proof-theoretic test.

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