Chapter 9
Truth as Composite Correspondence

Gila Sher

Abstract Is a substantive standard of truth for theories of the world by and for humans possible? What kind of standard would that be? How intricate would it be? How unified would it be? How would it work in "problematic" fields of truth like mathematics? The paper offers an answer to these questions in the form of a "composite" correspondence theory of truth. By allowing variations in the way truths in different branches of knowledge correspond to reality the theory succeeds in rendering correspondence universal, and by investigating, rather than taking as given, the structure of the correspondence relation in various fields of knowledge, it makes a substantive account of correspondence possible. In particular, the paper delineates a "composite" type of correspondence applicable to mathematics, traces its roots in views of other philosophers, and shows how it solves well-known problems in the philosophy of mathematics, due to Benacerraf and others.

9.1 The Problem

The problem that motivates me arises from a constellation of factors pulling in different, sometimes opposing directions. Simplifying, they are:

1. The complexity of the world;
2. Humans' ambitious project of theoretical knowledge of the world;
3. The severe limitations of humans' cognitive capacities;
4. The considerable intricacy of humans' cognitive capacities.

Earlier versions of this paper was presented at the *Truth at Work* Conference in Paris, 2011 and at the philosophy colloquium at UC Santa Barbara the same year. I would like to thank the audiences at both events for very constructive comments. This paper continues my earlier work on truth, knowledge, and logic, e.g., Sher (2004, 2010, 2011).

G. Sher
University of California, San Diego, USA
e-mail: gsher@ucsd.edu

T. Achourioti et al. (Eds.), *Unifying the Philosophy of Truth,* Logic, Epistemology, and the Unity of Science 36, DOI 10.1007/978-94-017-9673-6_9

Given these circumstances, the question arises whether a serious notion of truth is applicable to human theories of the world. In particular, I am interested in the questions:

a. Is a substantive standard of truth for human theories of the world possible?
b. What kind of standard would that be?
c. How intricate would it be?
d. How unified would it be?
e. How would it work in "problematic" fields of truth (e.g. mathematics, logic)?

Viewed constructively, the task is to develop a substantive theory of truth (and a standard of truth as part of it) that addresses itself to humans' desire to know and understand the world in its full complexity on the one hand and to their intricate yet limited cognitive resources on the other. Such a theory will be both critical and constructive. It will take a critical stance toward the epistemic project of discovery, understanding, and justification, yet it will contribute to, rather than interfere with, this project. And it will be both normative and descriptive. Normative in setting constraints on the pursuit of knowledge, descriptive in providing an informative account of humans' cognitive relation to the world (or certain aspects thereof). As such, the theory of truth will itself be a theory of the world, bound by the same veridical, justificatory, and pragmatic standards as other theories, and facing the same challenges.

Of course, in trying to tackle the problem of truth we are not starting from scratch. Nor do we purport to provide a complete, let alone final, solution to this problem. But the state of truth today makes our task urgent. Many philosophers have given up the goal of a substantive theory of truth, and even those who have not (like Wright 1992; Lynch 2009) have compromised the unity of truth as well as the connection between truth and reality.[1]

I should note that my focus on theoretical truth (truth of theories and statements within them) is not intended to drive a wedge between theoretical and non-theoretical truth. On the contrary. Inasmuch as I strive for a unified theory of truth, my aim is to encompass both. This, I believe, is possible because the project of theoretical knowledge is a continuation of the non-theoretical project of knowledge. The reason for focusing on the theoretical aspect of truth first is partly methodological. Philosophers have customarily started with everyday truths, yet this left them with many unsolved problems. It pays to see whether by starting with more complex truths we will not make progress in solving some of these problems.[2]

[1] The compromise concerning unity results from their willingness to assign altogether different standards of truth—e.g., correspondence vs. coherence standards—to different disciplines. The compromise concerning the connection between truth and reality arises from their willingness to assign a non-correspondence standard of truth to allegedly "problematic" disciplines. (It's important to note, however, that neither Wright nor Lynch is in principle averse to assigning a correspondence standard to any discipline).

[2] Indeed, given that we have more options in treating the simple cases than the complex cases, combinatorically it makes sense to start with the latter.

9.2 Methodological Flexibility

It's very easy to mismanage our project by rigidly following traditional templates of thought concerning truth. In particular, the "either-or" template, and its "if-then" correlate, which are very common in philosophy, make the space of options available to us unnecessarily limited. A few instances of this template that I find especially counterproductive are:

1. Either you provide a substantive *definition* of truth (a strict "if and only if (iff)" standard of truth), or you have not succeeded in developing a substantive theory (standard) of truth at all.
   There is no need to provide a definition *(an iff standard) of truth; it's fine to provide a more loosely structured account (standard) or even a family of such accounts (standards), so long as they are accurate and informative.* (Sher 2004)
2. If you are serious about a substantive theory of truth you cannot allow either circularity or infinite regress in your theory. More generally: If you are a substantivist about truth you cannot be a holist with respect to truth.
   Holism is perfectly compatible with a substantive theory of truth, and so are non-vicious circularity and infinite regress, which it sanctions. (Sher 2010)
3. If you acknowledge the plurality of truth, you cannot be a correspondence theorist with respect to *all* fields of truth.
   You can. You can say that different fields of truth are based on different (though, ideally, interconnected) correspondence principles. (Sher 2004)
4. Either you hold a copy or an isomorphism view of truth, or you are not a correspondence theorist at all.
   There can be other, possibly more intricate, correspondence conceptions of truth. In this paper I highlight "composite correspondence".
5. If you are a correspondence theorist of truth, you must allow only *one* pattern of correspondence.
   You can allow a variety of correspondence patterns, though you may wish to connect (unify) them in significant ways.
6. If you are a correspondence theorist with respect to logic and mathematics, you must be either a Platonist or an empiricist with respect to them.
   There is no need to postulate a separate, abstract, reality in order to affirm that objects and properties in the world (even physical objects and their physical properties) have formal features (like self-identify and cardinality) and that these (worldly) features, or the laws governing them, underlie logical and mathematical truth. But there is also no need to regard these features (laws) as empirical. It's an open question how mathematical truths are connected to formal features of reality. It's sufficient that they are systematically connected to them in some way that explains their correctness, not necessarily the traditional way that mandates the existence of mathematical individuals, for example.
7. Either you regard knowledge of logical and mathematical truths as apriori or you do not regard it as obtained primarily through use of reason.

It's possible that logical and mathematical knowledge is obtained largely *through reason without being* completely *isolated from (independent of) experience, i.e., without being apriori.*

8. Either you regard logic as grounded in reality, or you regard it as grounded exclusively in the mind.
   You can regard logic as grounded both in reality and in the mind (as you should all other veridical disciplines). (Sher 2011)

With this advance notice of my deviations from common practice, I am almost ready to proceed to my solution of the problem. First, however, I need to clarify an important methodological issue: *holism*. I have touched on holism in (2) above, but holism plays such a central role in my proposed solution to the problem that I need to clarify what exactly I mean by holism, and what my motivation for holism is.

Holism is my chosen methodology for philosophical theorizing. It contrasts most sharply with foundationalism. I don't reject all aspects of foundationalism: I think the search for the (or a) foundation of knowledge, truth, logic, morality, etc., is a worthwhile and legitimate task of philosophy. It's a search I am engaged in this paper (with respect to truth). But I think foundationalism goes about the foundational project in the wrong way. Foundationalism sets a strict ordering requirement on the grounding relation, one that renders this methodology self-defeating. Foundationalism requires that in grounding X we limit ourselves to resources more basic than X, and eventually to resources limited to a certain fixed trove—the so-called *base*. The problem is that it's impossible in principle to ground—or even to seriously justify—the base itself. We arrive at our holistic methodology by relaxing the strict ordering requirement of the foundationalist methodology.

Now, it's common to think of the renouncement of foundationalism as the renouncement of the foundational project itself. I think this is a mistake. The foundational project, as a dual, justificatory and explanatory project, can be carried out in multiple ways. By removing the unreasonable strictures of one methodology, foundationalism, holism provides us with a potentially unlimited number of ways to pursue it. Elsewhere (Sher 2010) I called this conception of holism as a foundational methodology "foundational holism". Among its advantages in pursuing the foundational project are:

- We need not worry about circularity or infinite regress per se. (Not all circularity and infinite regress are vicious.) ((2) above).
- We can use any resources that are helpful to us, in any combination (spanning any number of theories or fields of knowledge), to solve any problem.
- We can take multiple routes to reality—both routes of discovery and routes of justification (explanation)—including composite and/or circuitous routes.
- The justification/explanation process is a step by step process. Partial justification/explanation is worthwhile and superior to no justification/explanation.

This holistic method is not just an appropriate method for philosophical inquiry, but, in light of the four circumstances noted in the opening paragraph of this paper, also an appropriate method for humans to build their "system of knowledge" of the world.

As such, it is nicely captured by the Neurath-boat metaphor, though with one proviso. The boat, as we envision it, is not floating aimlessly in the water and is not isolated from the world. On the contrary, its sailors are engaged in exploration of the world (the sea and its environs), and their survival and success depend on taking the world into account and, indeed, focusing on the world. The Neurathian boat is sometimes associated with coherentism and rejection of the correspondence theory of truth. On our view, the boat pursues a discovery, understanding, and justification project, and its standard of truth is a correspondence standard: a standard of measuring up to (marine) facts. This leads to our proposed solution to the problem of truth.

9.3 Composite Correspondence

If the world is highly complex relative to our cognitive capacities and we nevertheless seek to know it in its full complexity, this requires stretching our cognitive endowments, devising multiple means for reaching its less accessible regions, improvising, experimenting, tinkering, exercising our imagination, etc. In short, it's quite likely that we use a wide variety of routes to reach the world cognitively, and some of these are complex, indirect, jagged. What this means for the study of truth is that it's seriously possible that there are multiple routes of correspondence between true cognitions and reality, and that some of these routes are quite intricate. This possibility concerns language as well. A proper use of, say, a *singular term* need not target an *individual* in the world. It may target something else (say, a property of individuals, a property of properties, etc.) that for one reason or another we reach through the use of a singular term and, possibly, an intermediate, posited, individual that is systematically connected to it.

Given this possibility, it's an open question what form correspondence takes in different areas of knowledge, and it's unreasonable either to assume or to require that it take the form of a copy, or a mirror-image, or even an isomorphism. At the same time it's also the case that in developing theories of the world humans aim at unity and systematicity, and that in seeking to develop a normative-descriptive theory of truth we, too, aim at these things. Put together, this means that the project of constructing a substantive theory of truth is not simple. It requires a balance between unity and diversity, between observing and proposing, between describing and constructing, between being critical and understanding. I will call a theory of truth that requires a substantial correspondence (of one kind or another) between true cognition and reality, allows multiple—including intricate—routes of correspondence from language to reality, yet seeks maximal unity and systematicity, a "composite correspondence" theory.

In attempting to develop a composite-correspondence theory of truth I use the holistic, Neurathian methodology delineated above. While keeping in mind a few milestones of the philosophy of truth (e.g., Tarski's T-Schema, his recursive definition of truth in terms of satisfaction, his semantic definition of logical truth), I will begin, in accordance with my policy of going from the complicated to the simple, with a hitherto problematic field of truth, specifically, mathematics. Before attempting to deal with this field, however, I would like to briefly examine another conception of correspondence, one that is similar to mine in certain important respects yet does not extend to mathematics.

9.4 Horgan's Indirect Correspondence

Lynch (2001) summarizes Horgan's conception of correspondence as follows:

> In Horgan's view. . . truth in any discourse is determined jointly by the world and the semantic standards of the discourse. In short, truth is semantic correctness. Semantic correctness is a realist notion of truth, since it involves a type of correspondence with the world. . . . Nonetheless, the type of correspondence can vary according to what we are talking about. This is because the semantic norms governing truth vary with the context. Thus, there is a spectrum of ways in which statements can correspond to the world. On one end of the spectrum are statements governed by maximally strict semantic standards. Such statements are true just when they directly correspond, via causal/referential relations, to mind-independent objects and properties. On the other end are statements whose truth is determined almost entirely by the semantic standards alone. In between sits the majority of statements we make in life, such as those about corporations and works of art, which **indirectly** correspond to entities and attributes that are in many cases mind-dependent. [*Ibid.*, p. 13. My bolding]

Horgan himself describes his theory as a combination of metaphysical realism and a liberal approach to the correspondence theory of truth. Metaphysical realism, for him, is essentially the view that the world consists of a definite totality of discourse-independent objects and properties. Correspondence is the view that truth depends, at least in part, on the way the world is, and liberal correspondence is the view that this dependence need not involve a direct, 1-1 correlation between singular terms and objects in the world, predicates and properties in the world, quantifiers and sets of objects in the world (those over which they range), etc. The basic idea is that truth depends both on the world, independently of the mind, and on the mind in the sense of the totality of our semantic standards. Those standards vary from one context to another, and as a result there is a whole spectrum of ways in which, and degrees to which, a statement's truth value can depend upon the world. At one end of the spectrum are statements and contexts for which the semantic standards require a *direct* connection between true statements and reality, at the other end statements and contexts for which the semantic standards require no more than empty connection with reality, and the intermediate zone contains contexts and statements for which the semantic standards require indirect connections with reality of various types, levels, and degrees. Statements lying in the first region are subject to the strictest semantic standards: their truth requires a direct correspondence between their referential apparatus (singular terms, predicates, and quantifiers) and individuals, properties, ontologies (ranges of quantifiers) in the world. Statements lying in the second region are subject to the weakest semantic standards: their truth is altogether independent of the world; i.e., they exhibit null correspondence. Statements lying in the third, intermediate, region are subject to intermediate semantic standards:

their semantic correctness significantly depends on how things are with the world, but this dependence is *indirect*—does not involve a direct correlation between their referential apparatus and individuals, properties, or ontologies. Their truth (when they are true) is based on *indirect correspondence*.

Horgan's conception of correspondence, however, is empiricist. His empiricism manifests itself on three levels: (a) The ultimate ontology of the world is empirical: there are no abstract objects or properties. (b) The relevant relations between language and the world, as far as truth is concerned, are empirical: specifically, causal. (c) Disciplines whose statements do not obtain their truth-value from causal connections to empirical reality are viewed as governed by a *non-correspondence* standard of truth. For example, pure mathematical truths as well as moral truths are viewed as based on a non-correspondence standard of truth.

To see an example of a truth based on indirect correspondence, according to Horgan, consider the sentence

1. Beethoven's fifth symphony has four movements. (Horgan 2001, p. 73)

The truth of (1), Horgan points out, "does not require that there be some ENTITY answering to the term 'Beethoven's fifth symphony' and also answering to the predicate 'has four movements"' (*ibid.*). Rather, it's sufficient that there are "other, more indirect, connections between [(1)] and THE WORLD" (*ibid.*), connections with, in particular, Beethoven's and others' musical behavior: Beethoven's "composing his fifth symphony" (*ibid.*), his earlier compositional activities ("in virtue of which his later behavior counts as composing his *fifth* symphony" (*ibid.*)), "a broad range of human practices (including the use of handwritten or printed scores to guide orchestral performances) in virtue of which such behavior by Beethoven counts as 'composing a symphony' in the first place" (*ibid.*), and so on.

One consequence of accepting indirect correspondence is expanding the domain of correspondence. Even if you are a strict nominalist or empiricist who denies the existence of abstract objects like symphonies, you can acknowledge that a sentence like (1), which talks of abstract objects and properties (symphonies, movements of symphonies, the property of being fourth in a series, the property of having four parts), have a definite truth value, that their truth value is to a significant degree a matter of fact—a matter of how things are in the world. Another consequence is that indirect connections with reality are as significant for truth as direct connections.

As for (1) itself, it is linked to reality both directly, through a causal link to a bona fide physical object—Beethoven, and indirectly, through causal chains ending with the abstract terms "fifth symphony" and "four movements". But (1) is not linked to reality through any independent connections of "fifth" and "four" with reality, direct or indirect. Pure ordinals and cardinals are not connected to reality, either directly or indirectly. Instead they are connected to pure conventions, i.e., to the mind. While the abstract terms "symphony" and "movement" are anchored in reality on Horgan's conception, "five" and "four" are not. It's here that the empiricist bias of his correspondence theory comes into expression. His theory allows language to be connected to reality in two ways: through direct causal links to explicitly-mentioned empirically-acceptable objects and properties, and through indirect causal links to mentioned and unmentioned empirically-acceptable objects, properties, events, etc. In both cases correspondence is limited to *empirical* reality and its routes are limited to *causal* routes.

One virtue of Horgan's conception is its ability to expand the range of correspondence acceptable to empiricists. Horgan shows that even if you are a strict empiricist or a nominalist who denies the existence of abstract objects like symphonies, you can acknowledge that a sentence like (1), whose subject matter is this abstract object, is true or false in the correspondence sense. A significant limitation of his approach (from our perspective) is that it leaves out of the correspondence account important parts of our system of knowledge, for example, logic and mathematics. On our own approach, *all* truths—including logical and mathematical truths—are based on correspondence.[3]

9.5 Mathematical Correspondence

The difficulties encountered by existent theories of mathematical truth—for example, difficulties concerning the identity of, and the access to, mathematical objects (Benacerraf 1965, 1973)—have led many philosophers to doubt that mathematical theories are true in the correspondence sense at all. In what follows I will present a tentative account of mathematical truth that avoids many of the extant objections to mathematical correspondence and is capable of doing the explanatory work that we would like a theory of mathematical truth to do.

In presenting this account, I will take a non-traditional approach. Discussions of truth and correspondence usually center on relatively small units of cognition, single sentences or units of a similar size. It seems to me that in investigating whether mathematical truth is based on correspondence we need to look at the discipline of mathematics as a whole. The main questions are whether mathematics aims (or large parts of mathematics, or some important parts of mathematics, aim) at *discovering facts*, what kind of facts these are, and whether such facts are in principle discoverable by humans. Before we can answer these questions, however, we must ask: Is there something in reality for mathematics to be true or false of? Does reality have mathematical features? The last question is rarely asked in contemporary discussions of truth, but I think it ought to be. Whether reality has features whose study requires a mathematical discipline, and whether the study of such features is an integral part of our body of theories of reality, is crucial for figuring out whether there is a need for correspondence truth in mathematics, and whether mathematical theorems are connected to reality. A positive answer to this question will not establish the existence

[3] It should be noted, though, that Horgan approaches truth from a somewhat different direction than we do. In particular, Horgan is interested in solving the problem of vagueness while we are interested in solving the epistemic problem of truth. These problems are not completely disconnected, but the difference is in certain ways insignificant.

of mathematical correspondence, but it will show that it's reasonable to investigate the possibility of correspondence in mathematics just as it is in physics.

My answer to the question "Does reality have mathematical, or mathematical-like, features?" is positive. You don't need to be a Platonist to realize that individuals (including physical individuals) have properties like self-identity, that properties of individuals (including physical properties) have properties like cardinality, that relations of individuals (including physical relations) have properties like reflexivity, symmetry, or transitivity, that cardinality properties (like the 2nd-level properties ZERO, ONE, TWO, THREE . . .) stand in an ordering relation, and so on. A natural name for such properties (relations) is "*formal properties (relations)*". It's hard to deny that objects in the world have some (indeed, many) *formal* properties. This is our starting point. Our next step is to try to understand what is unique about the formal.

One way to characterize the formal is as *invariant under isomorphisms.* An extended explanation is given in, e.g., Sher (1991, 2008). But briefly, we can see what this means by considering an example of a formal property, say, a cardinality property, $\mathbb{C}$. $\mathbb{C}$ is invariant under isomorphisms because given any isomorphic structures <U,P> and <U',P'>, where U,U' are universes[4] and P,P' are 1st-level properties, P has the property $\mathbb{C}$, (in U) iff P' has the property $\mathbb{C}$, (in U'). In contrast, the 2nd-level property "P is a property of some humans" is not invariant under all isomorphisms, hence it's not a formal property.

Now formal properties, like physical properties, are presumably governed by laws, or systematic regularities. (It's quite clear that cardinalities, for example, are governed by laws.) It stands to reason that humans are interested in discovering these laws, so the question arises: Which discipline studies these laws?—It's reasonable to presume that *mathematics* does, because mathematics does study things like cardinalities, reflexivity, symmetry, transitivity, etc, and it would be very strange if mathematicians knew there were "real" cardinalities and thought they were governed by laws yet studied the laws of other, non-real, imaginary, cardinalities. But if mathematics studies the laws governing formal objects in the world, the appropriate standard of truth for mathematical theories would be a *correspondence standard* of some kind, i.e., a standard that requires a systematic connection between true mathematical statements and laws governing the formal behavior (features) of objects in the world.

Here, however, we seem to encounter a problem: Many formal laws that govern objects in the world are higher-level laws, but the mathematical theories that study them are lower-order theories. For example, the laws of cardinality are laws of 2nd-level properties, but standard arithmetic and set-theory study them as laws of individuals (0-level objects). How can mathematical statements about individuals be true if what they say is true due to facts about higher-order properties? When put this way, the question appears difficult to answer. But if we put it in a different

---

[4] A universe is a non-empty set of individuals. The structures <U, P> and <U',P'> are isomorphic iff there is a 1-1 and onto function *f* from U to U' such that P' is the image of P in U' under *f*.

way, say, "How can laws of individual numbers *correspond* to laws of 2nd-level cardinality properties?", it's fairly easy to answer. In the same way that large objects can be represented by small objects (e.g., in a small-scale model), so higher-level objects can be represented by lower-level objects. 1st-order theories are capable, in principle, of correctly describing 2nd-level objects and their laws, *if we allow indirect representation* of these laws and their objects. 1st-order statements about individual numbers can correspond to facts, laws, phenomena involving 2nd-level properties, *if we allow indirect or composite correspondence*.

Why would humans use composite correspondence in studying these laws rather than simple correspondence? Why not opt for higher-order mathematics? This depends on the circumstances. One possible reason for preferring 1st-order theories might be that we, humans, are so wired that we are better at figuring out the formal laws governing structures of objects when we treat these structures as lower-level structures, say, structures of individuals. In that case, it would be advantageous to us to study higher-level cardinalities by 1st-order theories. There could, of course, be other reasons. The important thing is that this option is open to us.

How would humans go about constructing 1st-order theories of higher-level cardinality properties? One way they could go about it is by introducing a *postulated* level of *individual cardinals* which are systematically connected to *2nd-level* cardinality properties (*in the world*). Mathematical truth and reference would then be composite: Numerical singular terms would refer_1 to posited individual numbers and refer_2 to 2nd-level cardinality properties which are systematically represented by their referents_1. Similarly, 1st-order statements expressing laws of cardinal individuals would correspond_1 to posited 1st-level cardinality laws and correspond_2 to higher-level cardinality laws which their correspondents_1 systematically represent. The reference and correspondence relevant to a theory of mathematical truth are reference_{1+2} and $\text{correspondence}_{1+2}$. Put in terms of standards: an appropriate standard of truth for mathematics might be a standard of composite correspondence, like our $\text{correspondence}_{1+2}$. In short, an appropriate standard of truth for mathematics (or for any other discipline) need not be simple or direct, but it must measure mathematical theories and their statements against that facet of reality which is their target, and it must require mathematical theories to systematically represent this facet.

Systematic connection, however, does not mean translatability to an *equivalent* theory. Although we can translate 1st-order number statements to higher-order number statements, 1st-order arithmetic is *not equivalent* to 2nd-order arithmetic: 1st-order arithmetic has a logically complete proof system (the proof system of standard 1st-order logic) while 2nd-order arithmetic does not; 2nd-order arithmetic is categorical while 1st-order arithmetic isn't. The two are not equivalent, yet they are systematically connected. But even this kind of connection is not mandatory. A systematic connection between 1st-level posits and higher-level phenomena does not require the existence of a worked-out higher-order theory. It's acceptable if for some higher-order formal phenomena there is only a 1st-order theory that describes them, so long as this theory is systematically connected to them. Systematic connection can take multiple forms, and it allows indirect and incremental demonstration, especially for holists.

This is our idea of composite correspondence: n-step correspondence ($n \geq 1$), possibly involving auxiliary posits. Composite correspondence does not give up the desideratum of simplicity, either on the normative or on the descriptive level (or, indeed, as a principle that might be at work in the world). But it regards it as secondary to the more important desiderata of setting up a realistic standard of correspondence and correctly accounting for the way human beings do, or may, cognitively reach the world, both on the level of theory and on the level of common-sense thought. Although our theory is still in its initial stages, we can already point out some of its methodological virtues. Three of these are: strong problem-solving capabilities, interesting connections with related philosophical theories, and potentially fruitful extensions.

9.6 Problem-Solving Capabilities

The composite account of mathematical correspondence, together with the underlying holistic approach to truth and knowledge, enable us to solve, or at least make some significant progress toward solving, a few widely discussed problems for mathematical correspondence. A full discussion of these problems would have to be quite lengthy, but briefly we can indicate the problems and solutions (or their direction) as follows:

a. *The Identity Problem* (Benacerraf 1965). This problem concerns the identity of mathematical individuals. For example: Is Zermelo's 2—namely, {{Ø}}—or von Neumann's 2—namely, {Ø, {Ø}}—the real 2? Our proposal easily dissolves this problem: If numbers (as individuals) are posited representations of real cardinality properties, there is no question of which individual is the *real* 2. Systems of posits are measured by their fruitfulness, systematicity, and representational success, not by their reality. Since Zermelo's and von Neumann's systems are equally fruitful, systematic, and representationally successful, it doesn't matter whether we choose one or the other.
b. *The Applicability Problem.* (Wigner 1960) The problem is to explain how abstract mathematical laws apply to empirical objects (events, phenomena, situations). The problem is especially difficult for traditional correspondence theorists of mathematics, who are platonists. It's hard to see how mathematical laws which govern a Platonic reality, completely separated from our mundane physical reality, can apply to physical objects. But this problem does not arise for our proposal. First, our proposal, being holistic, assumes one, interconnected reality, which has both physical and formal features. Second, formal features themselves, on our proposal, are in principle features of objects of various kinds, including physical objects. For example, cardinality properties are (direct) properties of physical properties of (physical) objects. The applicability of mathematical laws in the physical domain is, therefore, unproblematic.

c. *The Large Ontology Problem.*[5] The problem is to justify mathematics' claim to a super-large ontology of individuals (e.g., cardinals and ordinals). This problem is dissolved by our proposal: mathematical individuals are intermediary posits rather than real objects; as such their number is subject to standards of fruitfulness, not of reality or existence. Indeed, our proposal does more than just dissolve this "problem": it has the capacity to explain why mathematics needs a super-large collection of posited individuals. Mathematics, as a theory of the formal, is a theory of laws governing features of objects. But laws in general have a certain degree of necessity and as such require a counterfactual ontology. Formal laws have an especially strong degree of invariance; as such they require an especially large counterfactual ontology.[6]

d. *The Epistemic Access Problem.* (Benacerraf 1973) The epistemic access problem is the problem of how humans can cognitively access those aspects of reality that are relevant for mathematical truth and knowledge. This problem is especially difficult for those who regard mathematical truth as requiring a *real* ontology of mathematical individuals. It's harder to explain how we can *see* the number *3* than how we can see that there are *three* books on the table. Our account, however, does not require mathematical individuals: there is no need for humans to see the number 3 or any other numerical, set-theoretical, or geometrical individual. The problem of epistemic access is also extremely difficult for Platonists. It's difficult to explain how creatures who exist in one reality can access another reality. This problem, too, does not arise for us. Reality, for us, is one; it has both physical and formal features; there is no need to cross realities in order to access the formal. Finally, the problem of epistemic access is very difficult for empiricists. It's hard for empiricists to account for abstract properties, let alone for their laws. Most importantly, it's hard for empiricists to acknowledge the central role intellect (reason) plays in mathematical knowledge. None of these problems arise for us. Our approach to mathematical knowledge is neither empiricist nor apriorist. As holists we sanction multiple resources for reaching reality that are not available to the empiricist, including multiple configurations of intellect and sense perception that can be used to access the formal.

e. *The Mathematics-as-Algebra Problem.*[7] The problem is that some mathematical theories are not naturally viewed as theories of any particular formal feature (or family of features) of reality, but seem to engage with abstract structures that might have a variety of applications or no applications at all. Put otherwise, these "algebraic" theories create free-floating models: models that might represent multiple real structures (or none). This problem, too, is relatively easy for us to handle. First, we see no radical difference between a theory and a model. Theories

[5] This problem is raised by nominalists of various stripes (e.g., Goodman and Quine 1947), mathematical finitists, supporters of $V = L$, and others who feel uncomfortable with the huge ontology of contemporary (classical) set-theory. Here I focus on the issue of *size*.

[6] For further discussion of the philosophical ramifications of invariance, see Sher (1999a, 2008).

[7] The issue of algebraic vs. non-algebraic mathematical theories is discussed in, e.g., Shapiro (1997).

can involve the postulation of entities, which is what models do; and models, like theories, can account for a given set of phenomena in the world either accurately or inaccurately, either systematically or not systematically. Indeed, our dynamic holism sanctions fluctuations in the status of theories as being about the world or being mere algebras. In the course of history a theory like Euclidean geometry can turn into an "algebra", while an "algebra" like Riemannian geometry can turn into a theory of the world. When it comes to truth, the difference is between being *true simpliciter* and being *true of*; on our account there is no radical difference between the two.

9.7 Relation to Fictionalism and Other Philosophical Views

Our conception of mathematical truth as based on composite correspondence has points of contact with a number of other philosophical conceptions of mathematics, from Aristotle's to contemporary fictionalists'. I will not be able to trace all these contacts here (or, indeed, any one of them with great detail), but I will briefly discuss a few.

Aristotle Our treatment of mathematical truth bears some similarities to Aristotle's, especially as construed by Lear (1982). Reality, according to Aristotle, has multiple aspects, and this is reflected in the variety of features possessed by physical objects. In particular, physical objects have mathematical features (like being spherical), and it's these features that are the subject-matter of mathematics. Mathematics, therefore, studies *real* features of *real* objects, and its theories are genuinely true (or false). How does mathematics study these features? By separating them (in thought) both from the physical objects that possess them and from the other features these objects possess, and by studying them on their own. Thus, Aristotle says:

> Obviously physical bodies contain surfaces, volumes, lines, and points, and these are the subject matter of mathematics. . . . [T]he mathematician . . . separates them, for in thought they are separable from motion, and it makes no difference nor does any falsity result if they are separated. [Aristotle, *Physics*, Book II, Chapter 2: 193b23–35. Cited in Lear: 162]

This effective way of studying mathematical features might involve the positing of mathematical entities:

> The best way of studying [such features] would be this: to separate and posit what is not separate [i.e., what is not separate in reality but is separable in thought], as the arithmetician does and the geometer. [*Ibid. Metaphysics*, Book M, Chapter 3: 1078a21–3. Cited in Lear: 165. My square brackets.]

Under this conception, mathematical objects exist, but they exist in a special way: namely, as abstractions from physical objects. This relation between mathematics and physics explains how mathematics applies to physical objects. Lear concludes:

> For Aristotle, mathematics is true, not in virtue of the existence of separated mathematical objects to which its terms refer, but because it accurately describes the structural properties

> and relations which actual physical objects do have. ... [There is no need] to explain mathematical truth ... via the existence of mathematical objects. One can understand how mathematics can be true ... by understanding how it is applicable. (Lear 1982, p. 191)[8]

Our account is similar to Aristotle's in several significant ways: Both accounts start from the observation that objects in the world have formal/mathematical features and that one central task of mathematics is to study these features; both sanction the postulation of mathematical objects that represent such features; both regard mathematical truth as genuine truth; both regard mathematical truth as based on some kind of circuitous correspondence (Aristotle implicitly; we explicitly); both allow cognitive access to formal/mathematical features of reality; and both have resources for explaining the application of mathematics to physics in a relatively straightforward manner.

But there are also significant differences between the two accounts. Our own account is not committed either to Aristotle's special ontological theory of real, non-posited, objects, or to his views on the variety of formal features of objects in the world. Our account explicitly talks about laws governing the behavior of formal features of reality, and it sanctions laws that, I gather, are farther reaching than those Aristotle had in mind (for example, on our conception it's not unreasonable to expect that something like full-scale set theory is required for a comprehensive account of some formal laws). Our account has resources (e.g., Fregean resources), that were not available to Aristotle, for dealing with arithmetic truth, as well as resources (like invariance) for explaining the necessity of mathematical laws. Another difference is methodological: if, and to the extent that, Aristotle's methodology is foundationalist, our own, holistic, methodology provides us with a wider variety of tools for grounding mathematical theories in reality and for explaining mathematical truth and knowledge.[9]

Frege Our conception of cardinalities as 2nd-level properties, our focus on systematic connections between cardinality properties and numbers as individuals (0-level objects), our view of mathematics as objective, our emphasis on a close relation between mathematics and logic (see next section)—all have at least some roots in Frege. But there are also important differences between our approach and Frege's.

Frege gives more weight to language as a key to ontology than we do. For him, humans' use of singular terms to indicate numbers is a sign that numbers are individuals. (See, e.g., Frege 1884). For us, the relation between language and ontology is far less binding and direct. Natural language is a multi-dimensional tool, developed in a messy and often haphazard way, and there is no reason to assume a simple 1-1 correlation between its terms and things in the world. Theoretical language is less messy and haphazard, but it, too, cannot be taken as a guide for ontology. First,

---

[8] This account (and explanation) applies smoothly to geometry, but Aristotle uses a somewhat different (and arguably weaker) account for arithmetic. For us, post-Fregean philosophers, however, it's natural to extend Aristotle's account of geometry to arithmetic, by viewing numbers as representing cardinality properties of physical objects.

[9] For a discussion of this point, though not as it relates to Aristotle, see Sher (2010).

theoretical language is influenced by many things: natural language, earlier states of our knowledge, etc., and as such might not accurately reflect our current views of ontology. Second, theoretical language is bound by humans' limitations, and as such reflects the indirect and at times improvised routes that humans forge in an attempt to reach reality. In addition, theoretical language has multiple goals, including simplicity, efficiency, unity, and systematicity, and these might stand in some tension with the goal of ontological transparency. Finally, humans have the cognitive capacity to posit objects of various kind, including "fictional objects" systematically related to non-fictional objects, and there is no reason to think they don't exercise this capacity in developing theoretical (and other) languages. Indeed, not only do they have the capacity to posit new objects, they may very well have good reasons to do so, as we explained above. If, as we suggested above, (i) reality has formal features, (ii) these features are governed by laws, (iii) humans have difficulty in figuring out these laws directly, (iv) humans can figure out better, or more easily, what these laws are if they approach the task indirectly, using posited objects—then it's quite reasonable for them to develop languages that refer to these posited objects. This is something humans might do either instinctively and unconsciously or deliberately and in a planned manner, and therefore it might be reflected both in their natural language and in their theoretical languages. For that reason, we allow *posited* individual numbers where Frege requires *real* numbers

Another difference from Frege concerns the status of mathematical truths. While Frege characterizes mathematical truths as analytic and apriori, we, as holists, renounce the analytic-synthetic and apriori-aposteriori distinctions, thereby rejecting Frege's characterization.[10] Still another difference concerns logic. Although we share Frege's view that mathematics and logic are closely related, we differ on the precise nature and structure of this relation.[11] Finally, we reject Frege's claim (in, e.g., Frege 1918) that truth is primitive and unexplainable. Our account of truth as composite correspondence is based on a substantivist, non-primitivist approach to truth.[12]

Quine The idea that positing objects is central to human knowledge is a Quinean idea. Not just mathematical objects but also abstract physical objects, everyday common-sense objects, and even subconceptual sense data are posits, according to Quine—theoretical, conceptual, and evidential, respectively. What is the point of positing such diverse types of object?—For Quine, the motivation is pragmatic. In the case of molecular particles, for example, "the particles are posited for the sake of a simple physics" (Quine 1955, p. 250). Mathematical objects are posited to expedite scientific knowledge in general, and they are justified by their *indispensability* to (empirical) science. What can we conclude from the fact that objects of all types are posited?—"[T]hat posits are not *ipso facto* unreal" (*Ibid.*, p. 251). In fact, posits are essential for our very notion of reality: "it is by reference to [everyday bodies, which

[10] This is a significant issue. For discussion see Sher (1999b, 2010).
[11] This will be briefly discussed in the next section. See references there.
[12] This approach is argued for and explained in Sher (1998–9, 2004).

are posits] that the very notions of reality and evidence are acquired" (*Ibid.*, p. 252). In this way, the boundary between the real and the posited disappears.

Our own approach is similar to Quine's in its emphasis on the basic (and positive) role of posits in knowledge, but is different on other counts. First, we are not committed to the view that all objects are posits. Second, we think of posits as playing a broader (and more fundamental) role in knowledge than a purely pragmatic role. This is because posits, in our view, enable us not just to simplify our theories but also to do more fundamental things like *figure out* certain aspects of reality that we might not be able to figure out without them. Finally, while Quine regards mathematics as subsidiary to empirical science, we see it as standing on its own. Mathematics' primary role is to provide knowledge of the laws governing formal features of reality, and its role in physics, though very important, is secondary to this role.

Contemporary Fictionalists Contemporary fictionalists, like Field (1989), claim that theories need not refer to *real* objects in order to be "good". Theories whose ontology is *fictional* can make a significant contribution to knowledge and have the advantage of being immune to difficulties facing their *realistic* counterparts. Field is especially interested in fictional theories of a particular type: *mathematical* theories which are *conservative* with respect to (physical) *science*. A mathematical theory M is conservative with respect to science iff for any nominalistic scientific theory N and a nominalistic sentence S, M+N logically implies S only if N (by itself) logically implies S (*ibid.*, p. 58). The idea is that mathematics plays a purely *pragmatic* or *instrumental* role with respect to science, i.e., it does not contribute anything substantial to science. Science can dispense with mathematics altogether, at least in principle. This, however, does not mean that we can eliminate mathematics through direct translation to real-object language (*ibid.*, p. 7). Nor does it mean that there is a problem with accepting those parts of mathematics that have no application to science, like the higher reaches of set theory. These, Field says, are natural ways of extending the "story" told by the useful parts of mathematics (*ibid.*, p. 10). Field is flexible with respect to justifying his account. Justification, he emphasizes, is not an all-or-nothing thing (*ibid.*, p. 17). Among the methodological principles he appeals to is "inference to the best explanation" (*ibid.*, pp. 14–20).

Our view is similar to Field's in the central role it assigns to posits in knowledge, in its acknowledgment of the creativity of human cognition, in its acceptance of the non-applicable parts of mathematics as legitimate, in its appreciation of such methodological principles as inference to the best explanation, and in its rejection of the view that justification is an all-or-nothing affair. But we differ from Field in other ways, including his nominalistic outlook on mathematics, his lack of interest in mathematics as a branch of knowledge in its own right, and his approach to logic.

Field's basic approach to knowledge is physicalistic, and although he doesn't ban all abstract objects (e.g., he endorses space-time points), he is a strict nominalist when it comes to mathematics. We, in contrast, leave it an open question whether there are mathematical individuals, and, in the absence of compelling arguments to the contrary, we are committed to the reality of formal properties and formal laws. Unlike Field, we do not measure the epistemic value of mathematics just by its contribution to (or instrumental value for) physical knowledge, but view it as an independent source of knowledge. As a result we, unlike Field, view mathematical laws as genuinely true, true in the sense of correspondence with reality, albeit composite correspondence. Finally, our view differs from Field's with respect to logic. Field's instrumentalist fictionalism puts a heavy burden on logic: on the one hand, mathematical fictions help the scientist to derive scientific truths from scientific truths using logic; on the other hand, the dispensability of mathematics is due to the fact that all nominalistic results arrived at using mathematics can be derived from other nominalistic results using only logic. Logic itself Field regards as "real", in contrast to mathematics. But logic, for him, is not real in the sense of being anchored in reality: logic is disengaged from reality. In this respect his view of logic is more traditional than ours. While we share Field's view that logic is real, we regard it as real in the sense of being anchored in reality, as will be made clear below.

Field's fictionalism is subject to several objections, most of which do not apply to us. One potential problem for fictionalism, however, discussed by Yablo (2001),[13] might be thought to apply to us as well. This problem can be formulated as follows: on the one hand the fictionalist is ready to assert that there is an even prime number, hence, that there are numbers; on the other hand, the fictionalist does not believe in the reality of numbers, hence is committed to asserting that there are no numbers. So the fictionalist is both committed to the view that there are numbers and committed to the view that there are no numbers. (Another way to put it is to say that the fictionalist is committed to the self-defeating sentence "The number of numbers is 0" (*ibid.*, p. 80).) Yablo dissolves the problem by showing it results from overlooking a simple distinction. This distinction he successively formulates as "representational aids" vs. "things-represented" (*ibid.*, p. 81), "engaged" vs. "disengaged" modes of speech (*ibid.*, p. 83)[14], "basic" vs. "parasitic" language-games (*ibid.*), "objectual" vs. "assertional" reality (*ibid.*, p. 85), and "figurative" vs. "non-figurative" speech (*ibid.*). The crucial point is that number words "can travel back and forth between the two categories"; indeed "they can do it ... within a single sentential move" (*ibid.*, p. 81).

Something like Yablo's solution is available to us as well. When we say that there are individual numbers we refer directly to the posits and indirectly to reality (cardinality properties); when we say that there are no individual numbers we refer directly to reality, and not at all to posits. (When we say that the number of numerical individuals is 0 we use "numerical individual" as a term ranging over real individuals and "0" as a term denoting a posit, 0, which represents a 2nd-level property, ZERO.) So there is no contradiction. It is possible, of course, that someone else speaks in a way that leads to a contradiction, but our holism provides us with resources for distinguishing these two ways of speaking.[15] We can move to a higher standpoint

---

[13] An earlier version, directed as *modal* fictionalism, is due to Rosen (1990).

[14] Our speech is engaged when we speak from within a given game, disengaged when we speak from outside it.

[15] For further discussion of our holistic, Neurathian conception of knowledge see Sher (1999b, 2010).

on Neurath's boat, or to a higher language in Tarski's hierarchy of languages, a standpoint or a language from which we can see both the world and the ways different people use language (object language) to speak about it. Some of these ways lead to a contradiction; others don't. Our claim is that on reflection, mathematics' contribution to knowledge is captured by the use of words spelled out above.

9.8 Truth in Logic and Beyond

Our conception of truth as based on composite correspondence is expandable to other fields besides mathematics, and as such it potentially contributes to the unity of the theory of truth and to the universality of the correspondence principle. Furthermore, our specific conception of mathematical truth can be used to establish the unity of logic and mathematics on a new ground.

It's not common to think of logic in the following way, but logic is a field that, under most of its conceptions, breaks the unity of truth and undermines the universality of correspondence. If truth in the empirical sciences is commonly viewed as based on correspondence, and truth in mathematics is sometimes viewed as based on correspondence, truth in logic is almost never viewed as based on correspondence. On the common view, therefore, not all truth is based on the same thing: some truth is based on correspondence, and other truth is based on something else. Composite correspondence enables us to restore the unity to truth and make a significant step toward removing counter-examples to correspondence. Furthermore, it enables us to do this in a way that explains the close connection between logic and mathematics (for example, the fact that every logical truth is an image of some mathematical truth)[16]. How can this be done?

Briefly, we could proceed by pointing out that: (a) Logic works—and has to work—in the world. (b) To work in the world—for example, to succeed in transmitting correspondence truth (truth that depends on the world) from premises to conclusion—logical laws cannot conflict with the basic laws governing the behavior of objects and structures of objects in the world. (c) If logic is grounded in certain universal laws governing objects and structures of objects in the world, this will explain how it works in the world (how it succeeds in transmitting truth from sentences grounded in the world to sentences grounded in the world). Furthermore, if the laws in question have an exceptionally strong modal force, this will explain the exceptionally strong modal force of logic. (d) Formal laws are universal. (e) Formal laws have an exceptionally strong modal force. (f) All logical laws have formal correlates. All this suggests that logical truth is based on correspondence with formal

[16] The logical truth "*Pa V* $\sim$*Pa*" is an image of the mathematical truth that $\underline{a}$ is in the union of $\underline{P}$ and its complement (in the given universe), the logical inference "*PaVQa,* $\sim$*Pa; therefore Qa*" is the image of the mathematical inference that if $\underline{a}$ is in the union of $\underline{P}$ and $\underline{Q}$, yet is not in $\underline{P}$, then it is in $\underline{Q}$, and so on.

laws.[17] It would also explain the close connection between logic and mathematics: mathematics *studies* formal laws, while logic *applies* them in/through language).[18]

What about other areas?—Composite correspondence carries some promise for other areas as well. Take ethics, for example. Our approach suggests that in investigating truth in ethics we don't start by trying to fit ethics into the standard template of correspondence, inspired by Tarski's theory, with its emphasis on individuals and their properties. Instead, we try to understand morality in its own terms. We try to figure out whether there is, or should be, something objective in our moral judgments, something that transcends our subjective and communal emotions (preferences, etc.), and if there is, what kind of thing it is. And we try to understand how humans reach this objective ground of morality, what standard (pattern) of correspondence is involved, and how it relates to the standards (patterns) of correspondence in other fields. In this way, if there is truth in ethics it will be based on correspondence—composite correspondence, like all other fields.

References

Benacerraf, P. (1965). What numbers could not be. *Philosophical Review, 74,* 47–73.

Benacerraf, P. (1973). Mathematical truth. *Journal of Philosophy, 70,* 661–680.

Field, H. (1989). *Realism, mathematics & modality*. Oxford: Basil Blackwell.

Frege, G. (1884*). The foundations of arithmetic: A logico-mathematical enquiry into the concept of number* (trans: J. L. Austin). Evanston: Northwestern. (1968).

Frege, G. (1918). Thoughts. In P. T. Geach (Ed.), *Logical investigations* (trans: P. T. Geach & R. H. Stoothoff) (pp. 1–30). Oxford: Basil Blackwell.

Goodman, N., & Quine W. V. (1947). Steps toward a constructive nominalism. *Journal of Symbolic Logic, 12,* 105–122.

Horgan, T. (2001). Contextual semantics & metaphysical realism: Truth as indirect correspondence. In M. Lynch (Ed.), *The nature of truth: Classic & contemporary perspectives* (pp. 67–95). Cambridge: MIT.

Lear, J. (1982). Aristotle philosophy of mathematics. *Philosophical Review, 91,* 161–192.

Lynch, M. (2001). Realism and the correspondence theory: Introduction. In M. Lynch (Ed.), *The nature of truth: Classic & contemporary perspectives* (pp. 7–15). Cambridge: MIT.

Lynch, M. (2009). *Truth as one & many*. Oxford: Oxford University Press.

Quine, W. V. (1955). Posits & reality. *The ways of paradox and other essays* (pp. 246–254). Cambridge: Harvard University Press. (1976). (Revised and Enlarged Ed.).

Rosen, G. (1990). Modal fictionalism. *Mind, 99,* 327–354.

Shapiro, S. (1997). *Philosophy of mathematics: Structure & ontology*. Oxford: Oxford University Press.

Sher, G. (1991). *The bounds of logic: A generalized viewpoint*. Cambridge: MIT.

Sher, G. (1996). Did tarski commit 'tarski's fallacy'? *Journal of Symbolic Logic, 61,* 653–686.

Sher, G. (1998–9). On the possibility of a substantive theory of truth. *Synthese, 117,* 133–172.

Sher, G. (1999a). Is logic a theory of the obvious? *European Review of Philosophy, 4,* 207–238.

[17] This correspondence would be properly composite (i.e., will involve more than one step) if logical laws are construed as directly concerning linguistic entities and indirectly the world.

[18] For a detailed discussion of this approach to logic and mathematics from the perspective of logic, see, e.g., Sher (1996, 1999a, 2008).

Sher, G. (1999b). Is there a place for philosophy in quine's theory? *Journal of Philosophy, 96,* 491–524.

Sher, G. (2004). In search of a substantive theory of truth. *Journal of Philosophy, 101,* 5–36.

Sher, G. (2008). Tarski's thesis. In D. Patterson (Ed.), *New essays on tarski and philosophy* (pp. 300–339). Oxford: Oxford University Press.

Sher, G. (2010). Epistemic friction: Reflections on knowledge, truth, and logic. *Erkenntnis, 72,* 151–176.

Sher, G. (2011). Is logic in the mind or in the world? *Synthese, 181,* 353–365.

Wigner, E. P. (1960). The unreasonable effectiveness of mathematics in the natural sciences. *Communications in Pure & Applied Mathematics, 13,* 1–14.

Wright, C. (1992). *Truth and objectivity*. Cambridge: Harvard University Press.

Yablo, S. (2001). Go figure: A path through fictionalism. *Midwest Studies in Philosophy, 25,* 72–102.