Chapter 6

The General Science

1. We have seen that the development of the encyclopedia presupposed knowledge of the general science, that is, of a universal method applicable to all the sciences; and that little by little the project of an encyclopedia gave way to the more limited project of the *Elements of General Science*, in which Leibniz would have revealed the principles of his method. This general science constituted, in short, his complete logic.[1] This is what we now have to examine.[2]

Leibniz conceived of logic in the broadest sense as the "art of thinking":[3] it is not only the art of judgment and demonstration, as was Aristotle's analytic; it is also, and above all, the art of invention, like the Cartesian method.[4] Originally, Leibniz himself would even have placed in it the art of memory, or mnemonics, since in order to think well it is necessary to have "the mind present" and to know how to recall in a timely manner knowledge already acquired in order to deduce more from it by means of new combinations.[5]

In general, however, logic is composed of two essential parts for Leibniz: the first, which he also calls the *method of certainty*[6] or the *elements of eternal truth*,[7] will serve to demonstrate already discovered truths and to verify doubtful or contested propositions. The second will serve to discover new truths by a sure and almost infallible method, and in a progressive and systematic order, whereas until now discoveries have been made by groping haphazardly and almost at random. The first will have to establish scientific truths of every order in the manner of mathematical theorems, with the same rigor and in the same logical sequence; the second would show how to resolve problems of every sort by reducing their solution to known problems, as in geometry. The first, for example, would study whether such-and-such a given machine in fact produces such-and-such an expected or presumed effect; the second, by contrast, would permit the invention of a

---

[1] "*Logic* is the general science." 1683 (LH XXXV, 1, 26 a).

[2] Descartes was originally to have given the title *Plan for a Universal Science* to his *Essays* of 1637

[2] Descartes was originally to have given the title *Plan for a Universal Science* to his *Essays* of 1637 (including *Discourse on the Method*): "In this *plan* I reveal one part of my method." Descartes to Mersenne, March 1636; Adam-Tannery, I, 339).

[3] See *Phil*., VII, 183 (end of *Discourse Concerning the Method of Certainty*).

[4] See letter to G. Wagner, 1696 (*Phil*., VII, 516).

[5] See the fragment *On Wisdom* (*Phil*., VII, 84), and *New Method for Learning and Teaching Jurisprudence* (1667), §22 (Note VII). Cf. Leibniz to Koch, 1708 (*Phil*., VII, 476) and *Plan for a New Encyclopedia*, June 1679, where mnemonics appears between logic and topics (LH IV, 5, 7 Bl. 4 recto). In various places, Leibniz gave rules for the art of memory. He chiefly relied on divisions and classifications (Leibniz to Wagner; *Phil*., VII, 516-7). However, he did not disdain even the most artificial memory techniques (see LH IV, 7B, 3 Bl. 7). In his unpublished manuscripts, there is a file relating to mnemonics (LH IV, 6, 19).

[6] *Phil.,* VII, 183.

[7] *Phil.,* VII, 49, 57, 64, 125, 296; Erdmann, 85a.

machine to produce such-and-such a projected and desired effect.[8] The one, therefore, proceeds from principles to consequences, from causes to effects; the other moves backwards from given consequences to desired principles, from known effects to unknown causes.[9] Thus the former follows a progressive and synthetic course, the latter a regressive and analytic course, with the result that we can assimilate them to synthesis and analysis, in the sense in which geometers understand these terms.

2. This indeed is the way Leibniz conceived of the parts of logic, at least to begin with.[10] However, he soon recognized that the art of invention is synthetic as well as analytic: for if it is analysis that serves to resolve a *given* problem by moving backward from the proposed effect to the unknown cause, it is by synthesis (that is, the combination of known ideas and truths) that we discover new propositions and invent new problems.[11] Thus, in most of the fragments relating to the encyclopedia, the art of invention is divided into two parts: the combinatory, which is synthetic, and analysis proper.[12] Later, Leibniz observed that for him the art of judgment also employs analysis and synthesis in turn: analysis, when it is a question of verifying a problematic proposition by reducing it to known truths; synthesis, when a desired and foreseen consequence is progressively deduced from given principles. The result is that he finally recognized that both in the art of invention and in the art of judgment we employ analysis and synthesis at the same time.[13] It is undoubtedly for this reason that the distinction between the two parts of logic progressively loses its importance and little by little disappears in the fragments relating to the characteristic. Thus we often see Leibniz identifying the general science in its

[8] Erdmann, 86a. Cf. the fragment LH IV, 6, 12f Bl. 28: "The synthetic method is proper for those who want to construct sciences; for others, it can yield the tables and inventories that are established thereafter. The analytic method is for the use of those who want to solve some problem, although the science to which the problem pertains has not yet been perfected or, indeed, perhaps not formulated; and so it can also be of use to those who have not learned the science or who, though eager, cannot be fully at liberty" (Bodemann, 90).

[9] We know that for Leibniz, as for the Cartesians, the terms "principle" and "cause" are synonyms, as are "consequence" and "effect"; he always understands "cause" and "effect" in a logical sense.

[10] In *Judgment on the Writings of Comenius* (Note XIII), on the other hand, he conceived of the art of invention as synthetic and combinatory and the art of demonstration as analytic and resolutory. Cf. *New Method of Learning and Teaching Jurisprudence* (1667), in which the *analytic or art of judgment* is opposed to *topics*, conceived as the art of invention (Note VII); we shall later see the reason for this.

[11] In the *Plan for the Investigation of Nature* (1676), we read: "The form or order" (of the encyclopedia) "consists in the conjuction of the two greatest arts of invention, analysis and combinatory." (Foucher de Careil, VII, 123). Thus analysis and synthesis are both methods of invention. Cf. LH XXXV, 1, 26 c: "There are two methods, the synthetic, *via* the combinatory art, and the analytic. Both can demonstrate the origin of invention; thus this is not the privilege of analysis." Elsewhere, Leibniz shows that even algebra, considered as an analytic method of invention, depends on combinatorial synthesis (LH XXXV, 1, 26 d). Cf. the fragment entitled: *Synthesis. Analysis. Combinatory. Algebra* (LH XXXV, 1, 27 a), and a fragment entitled *Combinatory*: "Algebra and combinatory differ in my view as analysis and synthesis…. Many human discoveries are made by synthesis rather than by analysis…. The combinatorial method is from causes to effects, or from the means to the end, or from the thing to its use; the analytic method is from effect to cause, from end to means. Both can be scientific, since they plainly direct us to the proposition we seek" (LH XXXV, 3A, 26 c). For the relations of algebra and geometry, see Chap. VII, §3.

[12] *Phil.*, VII, 49-50, 57; *Math.,* VII, 17; Erdmann, 86 a.

[13] Leibniz to Koch, 1708: "And in fact, in both judgement and invention, it is proper to use both analysis and synthesis." (There is here a mention of *On the Art of Combinations*.) "Combination pertains to synthesis" (*Phil.*, VII, 477).

entirety with the art of invention.[14] These two parts, which employ the same methods, are distinguished only by the use made of them, or rather by the intention directing their employment, which depends on the purely subjective fact that the truth to be demonstrated is either known or unknown.

3. It follows from this that the true division of logic is actually the distinction between synthesis and analysis, in the sense in which these are understand in mathematics.[15] It is therefore a generalization of the mathematical method that constitutes the general science, just as it already constituted the method of Descartes. Thus the logic of Leibniz presents itself from the start as an extension and perfection of the Cartesian logic. Without a doubt, Leibniz is critical of the latter from very early on,[16] but we know why he objects to it: it is not that it is inexact or false; it is simply insufficient and ineffective. It consists of rules so vague and open-ended that one needs another method in order to follow them exactly and surely.[17] It is this other method that Leibniz claims to furnish; it is therefore destined to complete and confirm the Cartesian method, rather than replace it.[18]

Leibniz thus begins by accepting the Cartesian rules, while adding his own to them, in a fragment entitled *On Wisdom*, which must date from his early youth since in it he acknowledges the triple division of the art of thinking (or *wisdom*) into the *art of reasoning well*, the *art of invention* and the *art of remembering what one knows at a given moment*.[19] "The *art of reasoning well* consists in the following maxims": the first is Descartes's first rule, which advances the criterion of being evident;[20] the second states that "when there appears to be no way to achieve this assurance, it is necessary to be satisfied with probability." Already we see emerging the divergence between the rigid mind of Descartes[21] and the more supple mind of Leibniz, who was taught by his studies of jurisprudence and theology that it is necessary to be content with probabilities in nearly all questions of a practical and empirical order.[22] The third rule, entirely Cartesian, concerns deduction, and prescribes that a succession without interruption be observed in it.

---

[14] *Phil.*, VII, 168, 169, 172, 173. Cf. *Phil.*, IV, 292.

[15] "But the analytic method is when some problem that is assumed to be under investigation is resolved into simpler notions until its solution is reached. And the synthetic method is when we proceed from simpler notions to more complex ones, until we come to what has been assumed" (LH XXXV 3A, 26 c).

[16] See Chap. IV, §6.

[17] This is the characteristic, which furnishes infallible means to the two parts of the general science: to the one, "sensible marks for judging truth"; to the other, the "sure thread of the art of invention" (*Phil.*, VII, 59; cf. p. 47: "*On the Art of Invention*, or the sensible thread for directing inquiry").

[18] In the definitions that appear at the beginning of *On the Art of Combinations*, Leibniz names Descartes as the inventor of analysis, that is, algebra (*Phil.*, IV, 35), which shows that from 1666 he knew of and appreciated the Cartesian method.

[19] *Phil.*, VII, 82-5. We see how Erdmann (p. xxv) was mistaken in ordering this fragment chronologically (albeit with some doubt) after the *Theodicy* (1710).

[20] A criterion that Leibniz had already rejected in his *New Method* (1667). See Note VII.

[21] Who prescribed in his *Rules* only to study objects of which we can have a certain and indubitable knowledge (Rule II).

[22] On the subjects of probabilities, Leibniz notes the following rules: (1) "It is necessary to distinguish degrees in probabilities"; (2) a consequence can never be more probable than the principle from which it follows; (3) if a consequence is derived from several probable principles, it is less probable than each of them.

As for the *art of invention*, it consists in the following maxims: "First, in order to know a thing, it is necessary to consider all its requisites, that is, all that suffices to distinguish it from every other thing. And this is what we call *definition, nature, reciprocal property*."[23] The second rule stipulates that we apply the first rule to each condition or requisite which enters into the discovered definition, and seek "the requisites of each requisite."[24] The third rule states simply that "when one pushes the analysis to the end,… we arrive at a *perfect knowledge* of the thing in question." These three maxims constitute for Leibniz the rules of "the true *analysis* or the distribution of the problem into several parts, which has not yet been explained." This is the novel part of Leibniz's method, which he opposes to, and substitutes for, the Cartesian analysis.[25]

The other maxims of the art of invention are borrowed from Descartes: among these are the fourth, which recommends that "we have this perfect knowledge present to the mind all at once," and for this purpose prescribes repeating the analysis several times until we see it all completely "in a single stroke of the mind";[26] the sixth, which counsels us to begin investigations with the easiest things, that is, the most general and most simple;[27] the seventh, according to which it is necessary to proceed in order from easy things to difficult ones, and to try to uncover some progression in the order of our thoughts;[28] and the eighth, which stipulates that we omit nothing in all our classifications and enumerations (for which purpose dichotomies serve well).[29] Finally, the ninth and tenth rules summarize what is for Leibniz the highest aim of the general science: by numerous and varied analyses, we will come to construct "the catalogue of simple thoughts," and as soon as we possess the latter, "we will be in a position to start again *a priori* and to explain the origin of things from their source," by means of "a perfect order and an absolutely complete combination or synthesis."[30]

In sum, analysis consists in decomposing all concepts into their simple elements by means of definition; and synthesis consists in reconstituting all concepts starting from these elements using the art of combinations. We have a *perfect knowledge*[31] of a thing when we have completely analyzed its concept; from then on we are in a position to discover deductively and *a priori* all its properties. This is what the fifth rule requires:

"The mark of perfect knowledge is when nothing appears of the thing in question for which a reason could not be given, and there is no occurrence such that we could not *predict* the event in advance."

---

[23] For an explanation of these words, see below, §9.

[24] "A *requisite* is that which can enter into a definition" (LH IV 7B, 2 Bl. 58).

[25] "For although they have said that it is necessary to divide the problem into several parts, they have not given the art for doing so, and they have not noticed that there are some divisions that confuse more than they clarify" (*Phil*., VII, 83). Cf. Leibniz to Galloys (*Phil*., VII, 21); *Phil*., IV, 330, 331, 347; Erdmann, 86b. As we know, what is here in question is the second rule of the *Discourse on the Method*.

[26] This is the process by which deduction progressively becomes intuition. Cf. Descartes, *Rules for the Direction of the Mind*, Rules VII, XI.

[27] This rule corresponds to the third Cartesian rule; cf. *Rules*, Rules VI, IX.

[28] This rule is included in Descartes's third rule; cf. *Rules*, Rules V, X.

[29] This is Descartes's fourth rule (called the rule of enumeration). In the *Elements of General Science*, an allusion is also made to Descartes's second and fourth rules, and to the process of perfection that the second demands (Erdmann, 86b).

[30] *Phil*., VII, 84.

[31] Leibniz will later say: "adequate idea."

We see that *On the Art of Combinations* contains the seed and the principle of this entire logic, and provides the key to this dual method of analysis and synthesis.

4. Still, this method is not completely expounded in the fragment *On Wisdom*; a fundamental distinction is missing that is introduced only after the fact in the form of a passing remark: "It is very difficult to carry the analysis of things through to the end, but it is not so difficult to complete the analysis of whatever truths we need. This is because the analysis of a truth is completed when one has found the demonstration of it, and it is not necessary to complete the analysis of the subject or predicate in order to find the demonstration of the proposition. Most frequently, the beginning of an analysis of the thing is sufficient for an analysis or perfect knowledge of the truth that one knows about the thing."[32]

In order to understand this remark, we must recall that in *On the Art of Combinations*, Leibniz considers concepts as products of simple elements, and that in every true proposition the predicate must enter as a factor in the subject. This being so, in order to ascertain the truth of a proposition it is not necessary to analyze either the subject or the predicate completely: it is enough to establish that the subject contains the predicate as a factor, which is generally recognized as soon as we begin to decompose it. This is why the analysis of truths is shorter than the analysis of ideas and why it does not presuppose that the latter be completely realized.

This remark also allows Leibniz to escape the difficulty raised by Pascal,[33] namely, that we can demonstrate nothing absolutely if we must proceed backwards indefinitely from principle to principle, without ever discovering a first principle.[34] In fact, the demonstration of a proposition can be perfect and absolute, so long as the partial resolution of the subject shows that it includes the predicate, whereas the perfect definition of this same subject requires that the resolution be complete.[35] Thus the analysis of concepts is more difficult than the analysis of truths;[36] but were it impossible (that is, infinite), the latter would remain for that no less possible or fruitful.

Thus, analysis is applied at the same time to concepts and to propositions; the analysis of ideas consists in definition, the analysis of truths consists in demonstration. This is why Leibniz proposes replacing all of Descartes's rules by these two: "Admit no word without definition, nor any proposition without demonstration."[37] Now demonstration itself, we have just seen, is carried out by decomposition of the terms of the proposition to be demonstrated, so that the analysis of truths is reduced to the analysis of concepts, that is, in short, to definition. But this analysis can be finite or infinite: if it is finite, it will lead to simple elements, to primitive concepts which form part of the

[32] No. 5a of the maxims of the art of invention, which was inserted later (*Phil*., VII, 83-4).

[33] In the treatise *On the Geometrical Spirit*, Section I, of which Leibniz had undoubtedly had knowledge in Paris through Arnauld.

[34] "On the difference between perfect and imperfect concepts, where Pascal's difficulty concerning continued resolution is met, and it is shown that a perfect demonstration of truth does not require perfect concepts of things." Unpublished Plan for an Encyclopedia, *ca*. 1680 (LH IV 7A, Bl. 26).

[35] See LH I, 6, 12f Bl. 23; LH XXXV, 1, 2 (*Demonstration of the Axioms of Euclid*, 22 February 1679).

[36] See *Introduction to a Secret Encyclopedia* (LH IV 8 Bl. 2 verso; quoted n. 84).

[37] *New Method*… (see Note VII). This formula recalls the rule stated by Pascal in the fragment *On the Geometrical Spirit*.

alphabet of human thoughts; if it is infinite, it will at least disclose without limit new simple elements, whose enumeration could never be complete; there will always be a complex remainder to analyze. Likewise, and consequently, the analysis of truths can be finite or infinite: if it is finite, it will lead to some simple principles from which the proposition in question is deduced; if it is infinite, it will proceed backwards from proposition to proposition without ever reaching a truly simple and primitive principle.

5. What is the nature of these principles which serve as the starting point for deduction? We have just seen that every demonstration rests on the definition of terms. At the outset, therefore, Leibniz admits only definitions as first principles.[38] This is the thesis he maintains in his correspondence with Conring, and it is interesting to follow its development. Leibniz begins by affirming that demonstration is only a succession of definitions.[39] And indeed, every demonstration is carried out by decomposing each term into its elements, that is, by substituting its definition for it. The art of demonstration consists in two things: the art of definition, which is analysis, and the art of combining definitions, which is synthesis.[40] Without a doubt, we can demonstrate a proposition by reducing it to a simpler proposition, and so on; but this reduction itself is only carried out thanks to a partial analysis of terms, that is, a definition. The result is that in the final analysis the only primary propositions on which the entire demonstration depends are definitions.

Yet here Conring objects that there exist indemonstrable propositions, namely axioms. Leibniz denies this: he agrees that we could, that we must even, for ease and for the progress of science, admit axioms or postulates without demonstration; but he maintains that all axioms admitted in this way must be demonstrable. And indeed, from where else would their certainty come? It cannot come from experience, for induction could not justify any universal and necessary proposition.[41] It is necessary, therefore, that it rest on the principle of identity or contradiction (the only *a priori* principle that Leibniz recognizes). And he boldly concludes that it must be possible to demonstrate all truths with the exception of *identical* propositions (reducible to the principle of identity) and *empirical* propositions (known by experience).[42]

---

[38] This thesis is in agreement with the doctrine of Hobbes: "Only definitions are universal primary propositions." *De Corpore*, Part I: *Computation or Logic*, Chap. VI: *On Method*, §13.

[39] Leibniz to Conring (1671?): "For demonstration is only a chain of definitions." (*Phil*., I, 174). Cf. *Judgment on the Script of Comenius* (1671?), in which Leibniz says that "demonstration is nothing but the combination of definitions, as I have shown in the art of combinations." (See Note XIII.)

[40] "I have always thought that a demonstration is only a chain of definitions, or, in place of definitions, of propositions already demonstrated earlier from definitions or accepted as certain. But analysis is nothing but the resolution of the defined into a definition, or of a proposition into its demonstration" Leibniz to Conring, 3 January 1678 (*Phil*., I, 185). Cf. Leibniz to Conring, 19 March 1678: "But the definition of any complex idea is a resolution into its parts, just as a demonstration is only a resolution of a truth into other already known truths" (*Phil*., I, 194).

[41] *Preface to Nizolius* (1670), *Phil*., IV, 161; cf. *Phil*., VI, 490, 495, 504; VII, 553. See below, §37.

[42] Leibniz to Conring, 3 January 1678: "Axioms are not, as you say, *indemonstrable*, but still I do not think that it is generally necessary for them to be demonstrated. I indeed consider it certain that they are demonstrable. For how else would their truth be decided by us? Not, I think, by induction, for in this way all knowledge would be rendered empirical…. Therefore, all certain propositions can be demonstrated, except for those that are identical or empirical" (*Phil*., I, 188). Leibniz to Conring, 19 March 1678: "From this it follows that all identities are indemonstrable, but all axioms… are… demonstrable; or, therefore,

In order to respond to the objections of Conring, Leibniz develops the same theory in his next letter. Every demonstration, he says, rests on definitions, axioms or postulates, demonstrated theorems, and truths of experience. But the theorems have been demonstrated by the same method, and cannot count as primitive truths; as for the axioms or postulates, they must all be reduced to identical propositions. Therefore, in the final tally, all truths are resolved into definitions, identities, and empirical propositions.[43] And as rational and purely intelligible truths cannot depend on experience (for the reason indicated above), they are finally reduced to definitions and the principle of identity.[44]

6. We thus see how the theory of demonstration is developed and elaborated: deduction rests not only on definitions whose substitution in some mechanical way suffices to reveal the hidden truth (the identity), but also on identical propositions and truths of fact. Leibniz does not continue any the less for that to maintain his initial thesis: "a demonstration is a chain of definitions."[45] And in a certain sense he is right: for, as he insists in a final reply, mathematical truths depend on definitions, axioms and postulates; but axioms and postulates in turn derive from definitions, in the sense that they become obvious as soon as we understand their terms, which occurs by substituting the definitions for the defined.[46]

However, Leibniz here seems to play on words: for axioms are not resolved purely and simply into definitions, and the proof of this is that they are not arbitrary but necessary. But what is the foundation of their necessity? Leibniz knows only one: the principle of contradiction. The necessary is that whose denial implies a contradiction, "which is the true and singular character of impossibility."[47] Thus necessary propositions alone are identical propositions; the only impossible or absurd propositions are propositions contradictory in themselves. In sum, axioms can indeed be demonstrated *by means of* definitions; but the foundation of their truth is not in the definitions: it is in the principle of identity.[48]

---

since they are ultimately understood from concepts (that is, by substituting a definition in place of the defined), it is obvious that they are necessary or that the contrary implies a contradiction" (*Phil*., I, 194).

[43] "*It is obvious that in the end all truths are resolved into definitions, identical propositions and experiences*." Leibniz to Conring, 19 March 1678 (*Phil*., I, 194). Earlier, Leibniz had acknowledged only definitions of words and experience as primary propositions; see *Preface to Nizolius*, 1670: "using definitions and experience alone, all conclusions can be demonstrated" (*Phil.,* IV, 137). Cf. *Judgment on the Script of Comenius* (1671?), Note XIII.

[44] Leibniz to Conring, 19 March 1678 (*Phil*., I, 194).

[45] Leibniz to Conring, 19 March 1678 (*Phil*., I, 194, 205).

[46] Leibniz to Conring, 19 March 1678 (*Phil*., I, 194).

[47] Leibniz to Conring, 1678: "It is evident that in the science called pure mathematics, everything depends on definitions; that is, they are obvious as soon as the terms are understood…. Analysis is nothing but the substitution of simples in place of composites, or principles in place of derived truths; that is, the resolution of theorems into definitions and axioms, and, if necessary, of the axioms themselves into definitions…. And so, if anyone considers the matter carefully, it cannot be doubted that demonstration, and thus synthesis and analysis, if not expressly then certainly implicitly, is nothing but a chain of definitions" (*Phil*., I, 205).

[48] Leibniz later opposed his principle to that of empiricists (Locke) in these terms: "I hold to the truth that the principle of principles is in some way the good use of ideas and experiences; but in studying the matter thoroughly one will find that with regard to ideas it is nothing other than to connect definitions by means of axiomatic identities" (*New Essays*, IV.xii.6).

We will note how far Leibniz is from the nominalism of Hobbes, while at the same time continuing to maintain the same thesis verbally. For Hobbes, every definitions is *nominal* and, consequently, arbitrary; it consists in adopting by convention a word to represent and replace a group of words; demonstration consists in always substituting the definition for the defined, that is, in replacing words by paraphrases; and, as it is only a chain of definitions, the demonstrated proposition has only an arbitrary verbal significance, like the definitions themselves.

For Leibniz, on the other hand, a definition expresses the *real* decomposition of a complex concept into simple concepts; from this it follows that the substitution of the definition for the defined no longer takes place by virtue of an arbitrary convention, but by virtue of the principle of identity; it is therefore this principle which constitutes the heart of every demonstration, and which creates the truth of demonstrated propositions.[49] This truth is no longer nominal and subjective as in Hobbes, for whom it was entirely relative to the definitions of words, that is, to our linguistic conventions; it is real and objective, for it rests not only on definitions (which, moreover, as we have just seen, are not arbitrary), but also on axiomatic identities which give it the character of necessity.[50]

7. Leibniz is thus led to a theory of concepts and definition that is as different from the nominalist doctrine as is his theory of truth and demonstration. Both are basic to his system and entirely explain its formation. We see this theory of definition realized and developed in his correspondence with Tschirnhaus, particularly in a very important letter dating from the end of May 1678. On it Leibniz wrote the following note, which summarizes the letter's content and reveals its interest:

"In this letter I already explained to Tschirnhaus my general method for investigating quadratures, as well as the mark of real definitions, which is possibility."[51]

Between nominal and real definitions Leibniz establishes a distinction that is not at all in conformity with usage or etymology, but which has a fundamental importance in his theory of knowledge. A definition is *nominal* when it indicates certain distinctive characteristics of the thing defined, in such a way as to allow us to distinguish it from everything else; but a definition is *real* only if it reveals the possibility or the existence of the thing. It is the last type only that Leibniz regards as *perfect* and *adequate*.[52] And he

[49] In his first letter to Foucher (1679?), Leibniz speaks of necessary truths (such as those of arithmetic, geometry, metaphysics, physics, and morals) "whose convenient expression depends on arbitrarily chosen definitions, and whose truth depends on axioms that I am accustomed to calling identities" (*Phil.*, I, 369).

[50] We now understand why Hobbes admitted only definitions as primary propositions, and why modern nominalism regards principles or postulates as only disguised definitions or simple verbal conventions. Cf. Chap. IV, §11.

[51] *Math.*, IV, 451; *Brief.*, I, 372. Here is what Leibniz said about Tschirnhaus in 1687 in connection with the publication of his *Medicina mentis et corporis*: "He was initially nursed on Descartes. But when he often conversed with me in Paris, I showed him certain better foundations; in particular, the difference between nominal and real definitions, which consists in the fact that from a real definition we can recognize whether or not a thing is possible." (There follows a critique of the ontological argument.) "For nothing can be safely concluded from definitions, unless it is determined that they are real or are of a possible thing." Leibniz to Placcius, 10 May 1687 (Dutens, VI.1, 44). Cf. Leibniz to Foucher, 1687 (*Phil.*, I, 392), and Leibniz to Huygens, 3/13 October 1690 (*Math.*, II, 51-2).

[52] "I consider a sure sign of a perfect and adequate definition to be when, *with the definition once grasped, it can no longer be doubted whether the thing understood through this definition is possible or not.*" Leibniz to Tschirnhaus, end of May 1678 (*Math.*, IV, 462; *Brief.*, I, 381). Compare this letter to *On*

gives this reason for it: we can deduce nothing with certainty from any definition if we do not know that the object defined is possible, that is, non-contradictory; for if it were impossible (contradictory), we could deduce from its definition consequences contradictory among themselves.[53] From this it immediately follows that definitions are not arbitrary, as Hobbes claimed. Undoubtedly, a definition is not a truth, but the explication of a term or rather an idea; undoubtedly again, it can be neither demonstrated nor refuted, and we are free to attribute different senses to a given word or notion; but this freedom has a limit, for a definition must not imply an intrinsic contradiction; it must not cause incompatible elements to enter into the comprehension of the concept. In this sense, a definition always implies an axiom or postulate susceptible to demonstration, for before being able to make use of it, it is necessary to prove that its object is possible, that is, non-contradictory.[54] A real definition is not arbitrary like the simple imposition of a name, for it corresponds to a true "essence," to a possible "nature," which does not depend on our intention.[55]

Thus conceived, real definition includes as a special case causal or generative definition; for it is clear that the best way to show the possibility of a thing is to indicate its cause or construction, when this is possible.[56]

8. Leibniz himself states that he has borrowed this criterion of a true idea from geometers;[57] and indeed, the geometrical method demands that we demonstrate the possibility (the ideal existence) of each of the figures we define, either by indicating its

---

Universal Synthesis and Analysis, *or on the Art of Invention and Judgment*. This extremely interesting essay appears to date from the same period as the *Meditations* of 1684, which it completes; at the same time, it is related to the plans for an encyclopedia developed around 1680, from which Gerhardt has wrongly separated it, for, as its title alone indicates, it is the rough sketch of a chapter of the encyclopedia. (See the plan of *Plus Ultra*, *Phil*., VII, 49.)

[53] "By *definitions of the best kind*, I mean those from which it can be determined whether the defined thing is possible, since otherwise nothing can be safely concluded from definitions; for from impossibles, two contradictory propositions can be inferred at the same time." Leibniz to Tschirnhaus, end of 1679 (*Math*., IV, 481-2; *Brief*., I, 405). Cf. Leibniz to Arnauld, 14 July 1686 (*Phil*., II, 63).

[54] LH XXXV 1, 1 b. Cf. Leibniz to Tschirnhaus, 1679: "From this it is also clear that demonstrations are not arbitrary, as Hobbes believed" (*Math*., IV, 482); and *On the Instrument or Great Art of Thinking* (LH IV 7C, Bl. 157).

[55] *On Universal Synthesis and Analysis*: "From this we can also answer a difficulty raised by Hobbes. Hobbes saw that all truths can be demonstrated from definitions, but he believed that all definitions are arbitrary and nominal, since the imposition of names on things is arbitrary. He therefore wanted truths to consist in names, and to be arbitrary" (*Phil*., VII, 294-5). *Meditations*, 1684: "This argument answers Hobbes who maintained that truths are arbitrary, since they depend on nominal definitions, not recognizing that the reality of a definition is not arbirtrary and that not just any concepts can be joined together" (*Phil*., IV, 485). Cf. Leibniz to Malebranche, 1679 (*Phil*., I, 337). See n. 55, and Appendix II.

[56] "The consequences of this criterion are so great that the efficient cause is included in the definitions of those things which have an efficient cause" (*Math*., IV, 482). Cf. *Meditations*, 1684 (*Phil*., IV, 425); *On Universal Synthesis and Analysis*: "It is useful, therefore, to have definitions which involve the generation of a thing, or failing that, at least its constitution, i.e. a way in which it appears to be either producible or at least possible" (*Phil*., VII, 294); and *A Specimen of Discoveries* (*Phil*., VII, 310).

[57] It is in this manner, for example, that after having defined parallels as "two straight lines situated in the same plane which do not intersect," we see that such lines exist by invoking the theorem: "From a point situated outside a straight line one can draw only one perpendicular to that line," and we give at the same time the means for constructing parallels by erecting perpendiculars on the same line.

construction or in some other way, so that every definition implies or invokes a theorem.[58]

It is again the example of geometers that Leibniz recommended to his nephew Loefler, who had undertaken to demonstrate, among other theological propositions, the dogma of the Trinity.[59] And it is from them that he borrowed the rule that a definition must contain nothing more than what is strictly necessary to demonstrate all the properties of the object defined, and consequently, that it must never contain any properties that could later be deduced from it.[60] He concluded from this that it is useless to define God as a spirit, given that one could demonstrate that God is a spirit simply by defining him as an absolutely necessary Being. We see from this that Leibniz rejects the scholastic rule whereby definitions must be given in terms of the proximate genus and the specific difference (for *spirit* is the proximate genus of God); he substitutes for it a rule that can be formulated mathematically as follows: "A definition must include the conditions which are necessary and sufficient for demonstrating all the properties of the object defined."

9. This entire theory of definition, moreover, proceeds from the principles of his logic, that is, from the combinatory art.[61] What proves this clearly is the problem Leibniz sets himself here and claims to be able to solve. Every definition, by the very fact that it agrees in every way with the defined and only with the defined, expresses a "reciprocal property" or characteristic of it.[62] Now every reciprocal property must exhaust the essence of the object, and consequently we must be able to deduce from it all the other properties of the object, even those which are reciprocal.[63] But not all definitions are equally perfect; not all of them display the possibility of the object defined. From this, there is reason to formulate the following problem: given any definition of a term, deduce

[58] Leibniz to Burnett (not sent, 1699): "It was necessary to give this correct criterion for distinguishing true and false ideas; this is what I did in the meditation cited above, according to what I had learned from geometers" (allusion to the *Meditations* of 1684, *Phil*., III, 257). And indeed he says elsewhere: "Geometers, who are true masters of the art of reasoning, have seen that for the demonstrations one derives from definitions to be sound it is necessary to prove, or at least postulate, that the notion contained in the definition is possible" (*Phil*., IV, 401; cf. 405).

[59] *Two Letters to Loefler on the Trinity, and on Mathematical Definitions Concerning God, Spirit, etc. (*Dutens, I, 17ff.). See the following note by the editor: "It seems appropriate to share with the good reader the formula treating the doctrine of the Trinity mathematically, which Leibniz prescribed for his reflection and which they consider in these letters" (p. 18).

[60] "Since you wanted to write mathematically, it was necessary to reflect on definitions, such as mathematicians demand, in which nothing ought to be posited that can already easily be demonstrated from the definition itself…. Mathematicians customarily conceive of definitions in such a way that nothing belongs to them which harbors doubt or difficulty, while nonetheless everything is in them which suffices for subsequently settling controversies." Letter II, 24 February 1695 (Dutens, I, 22).

[61] At the end of his letter to Tschirnhaus of 1679 (*Math*., IV, 482; *Brief.*, I, 405-6), he recalls the origin of *On the Art of Combinations* and traces the discovery of the "true analysis" to it.

[62] In precise terms, let x be the term to be defined and a one of its properties; we can define the term x by the property a, if every x possesses the property a, and if, *reciprocally*, everything which possesses a is x. Cf. the definition of a *proper attribute* in the *Specimen of a Universal Calculus* (*Phil*., VII, 226).

[63] Leibniz to Tschirnhaus, May 1678 (*Math*., IV 462; *Brief.*, I, 381). Cf. LH IV 7B, 2 Bl. 57 verso: "Every reciprocal property can be a definition…. Any reciprocal property exhausts the entire nature of the subject, or from any reciprocal property everything can be deduced…. If one from among a number of definitions is chosen, the others will be demonstrated from it as properties."

the perfect definition from it.[64] Leibniz says that he is in a position to resolve this problem by a "certain" or determinate analysis: but this analysis is nothing other than the progressive resolution of concepts into their simple elements.[65]

Leibniz still conceives of this analysis of concepts by analogy with the decomposition of numbers into factors, and it is this mathematical analogy which explains his entire theory of definition.[66] Just like a non-prime number, a complex concept can (in general) be decomposed in several ways into a product of factors, but it can only be composed in a single way into a product of prime factors. A notion has as predicates all of its divisors, and as *convertible* predicates every product of these divisors that is equal or equivalent to it. But there is in general a multitude of convertible predicates, as many as there are ways of combining and grouping the prime factors of the concept in question. Each of these convertible predicates expresses a reciprocal property or characteristic of the concept, and can serve to define it.[67]

But this is in general only a *nominal* definition: for there are convertible properties that Leibniz calls *paradoxical*, which suffice to characterize the defined object without revealing its possibility.[68] In order to obtain a *real* definition, it is necessary to decompose the factors of the concept in such a way as to show that they are compatible among themselves, that is, non-contradictory; and for this purpose we can start from any of the nominal definitions, since each of them exhausts the comprehension of the concept and contains its entire essence: whichever one we choose, we will always converge surely

[64] "For I can solve this problem by a certain analysis: *Given reciprocal properties or any such definitions of every term, discover definitions of the optimal kind.*" Leibniz to Tschirnhaus, 1679 (*Math*., IV, 481; *Brief.*, I, 405).

[65] "*Resolution* is the substitution of a definition in place of the defined*.* *Composition* is the substitution of the defined in place of a definition" (LH IV 7B, 2 Bl. 57).

[66] See Chap. II, §§6 and 7; Chap. III, §7. This analogy is so much a part of Leibniz's thinking that he assimilates prime numbers to the highest genera and compound numbers to species that derive from the highest genera by multiplication: Thus 2 will be the genus of multiples of 2, 3 the genus of multiples of 3; the product of the genus 2 and 3 being the species 6, every multiple of 6 will be both a multiple of 2 and a multiple of 3. (*On Universal Synthesis and Analysis*, *Phil*., VII, 292.) Curiously, this idea has reappeared in our time in Dedekind's "theory of modules": a *module* is the set of multiples of the same number; the set of numbers common to two modules (what we may call their logical product) is their smallest common multiple, that is, the module of the smallest multiple of the corresponding numbers. If these numbers are prime, their smallest common multiple is their product. Thus, these analogies of arithmetic and logic are not simple curiosities; they are real and have a useful and fruitful application. See Dedekind, "Sur la théorie des nombres entiers algébriques," §I, in *Bulletin des Sciences mathématiques*, vol. XII (1877), articles collected and published in part by Gauthiers-Villars.

[67] "*There can be many definitions of the same defined*. For let the defined be *a*, and its definition *bcd*, and let *bc* equal *l* and *bd* equal *m* and *cd* equal *n*; then there arise three new definitions of *a* itself, namely: *a* equals *ld*, *a* equals *mc*, and *a* equals *nb*, to which a fourth is added, *a* equals *bcd*. For example, 24 is 2.3.4. Now 2.3 is 6, and 2.4 is 8, and 3.4 is 12. Therefore, it follows that 24 = 6.4, 24 = 8.3, 24 = 12.2, and finally, 24 = 2.3.4" (LH IV 7B, 2 Bl. 57). Cf. *On Universal Synthesis and Analysis*, quoted Chap. II, §7.

[68] Such, for example, is this property of a circle: that a segment is seen according to the same angle from any one of its points. If we made use of this property in order to define it, we could not know *a priori* if such a curve is possible. On the other hand, the ordinary definition of a circle through generation indeed shows its possibility (*Phil*., VII, 294); cf. Leibniz to Foucher, 1686 (*Phil*., I, 385). Thus Leibniz approved of Euclid for having explicitly postulated the possibility of describing a circle (*Phil*. IV, 401).

(though more or less directly) on the same final decomposition.[69] This is the most perfect definition, that which serves as the common foundation of all the others and gives the reason for them, for we can deduce all of them from it by combining in different ways the simple factors that it contains. This is the best of the real definitions, because it is the one which best shows the possibility of the concept by explicitly enumerating all its elements.[70]

10. Thus, the best way of proving that a concept is possible, that is, non-contradictory, is by analyzing it completely. For insofar as the concept is defined by notions which are still complex, there can be a hidden incompatibility among them, in that they conceal contradictory elements. But when it is revolved into its simple elements, the least contradiction would become obvious and would immediately destroy the concept; from this it follows that an adequate concept is necessarily true. However, there is a more profound reason for this: for Leibniz, all simple ideas are compatible among themselves.[71] This undoubtedly results from the fact that all simple ideas are not only different, but, according to his technical expression, *disparate*,[72] that is, they do not possess any common element (otherwise they would not be simple); they therefore cannot be contrary to one another or, as we say, cannot interfere with one another, and consequently a contradiction cannot slip into any of their combinations.[73]

11. It is on the basis of this theory that Leibniz criticizes Descartes's ontological argument: he blames Descartes for not having previously established that the idea of God is possible, that is, non-contradictory; but he agrees with him that once this possibility is

[69] "Whoever wishes to establish a characteristic or universal analysis can initially use any definitions, since in the end through continued resolution they terminate in the same ones." Leibniz to Tschirnhaus, May 1678 (*Math*., IV, 462; *Brief*., I, 381).

[70] "Further, those real definitions are most perfect which are common to all hypotheses or modes of generation and involve a proximate cause, and from which, finally, the possibility of a thing is immediately evident…, that is, when a thing is analyzed into nothing but primitive concepts, understood through themselves. Such knowledge I am accustomed to call *adequate* or *intuitive*; for if there should be any inconsistency anywhere, it would appear at once, since no further analysis can be carried out." *On Universal Synthesis and Analysis* (*Phil*., VII, 295).

[71] See the fragment entitled *That a Most Perfect Being Exists*, in which Leibniz demonstrates that "*all perfections are compatible among themselves*, or can be in the same subject" (*Phil*., VII, 291). The fragment dates from 1676, since Leibniz says that he submitted it to Spinoza at The Hague (*ibid*., 262). Cf. Stein, *Leibniz und Spinoza*, p. 258.

[72] See Chap. VIII, §20.

[73] For the metaphysical importance of this thesis, see the letter to the Duchess Sophie (*ca*. 1680?), in which Leibniz, after having briefly indicated the principle and usefulness of his characteristic, adds that its foundation is the same as that of the demonstration of the existence of God: "For the simple thoughts are elements of the characteristic, and the simple forms are the source of things. But I maintain that all simple forms are compatible among themselves. This is a proposition of which I could not very well give the demonstration without explaining at length the foundations of the characteristic. But if it is accepted, it follows that the nature of God, which includes all the simple forms taken absolutely, is possible. But we have proved above that God exists, provided that he is possible. Therefore, he exists, which is what stood in need of demonstration" (*Phil*., IV, 296). Cf. *Meditations* (1684), in which the "first possibles" or irreducible notions are called "the absolute attributes of God," the "first causes and final reason of things" (*Phil*., IV, 425, quoted n. 83).

established we can reason from the idea of God to his existence.[74] Thus, this famous criticism of the ontological argument, which has such a great importance in Leibniz's metaphysics, proceeds directly from his logical theories.

To this criticism is related another more general criticism, which bears on the very criterion of truth. For Descartes, every clear and distinct idea was true: but, Leibniz objects, how will we recognize a clear and distinct idea? How many false ideas are there that we believe ourselves to conceive clearly and distinctly! How many things of which we speak which we have even defined in intelligible terms and which are impossible, that is, imply a contradiction (for example, the fastest of all mostions)! The criterion of obviousness is therefore insufficient and fallacious; it is necessary to substitute for it, or at least add to it, another criterion: every non-contradictory idea is possible and true, and in order to guarantee that an idea does not contain any contradiction, it suffices to decompose it into its simple elements.[75]

12. This theory of definition has a fundamental importance in Leibniz's philosophy, for it is this which gave rise to his theory of knowledge such as it is revealed in the *Meditations on Knowledge, Truth and Ideas*.[76] He distinguishes there, first, *clear* and *obscure*

---

[74] "But if this demonstration is to be rigorous, possibility must be demonstrated beforehand. Clearly, we cannot safely devise demonstrations about any concept unless we know that it is possible, for of that which is impossible, i.e. involves a contradiction, contradictories also can be demonstrated. This is the *a priori* reason why possibility is required for a real definition." *On Universal Synthesis and Analysis* (*Phil*., IV, 294). Cf. the *Meditations* of 1684: "If God is possible, it follows that he exists; for we cannot safely use definitions for the purpose of drawing a conclusion before we know whether they are real or involve no contradiction" (*Phil*., IV, 424); and Leibniz to Arnauld, July 14, 1686 (*Phil*., II, 63). The first criticism of the ontological argument is found in the letter to Oldenburg of 28 December 1675, in which Leibniz speaks of the mechanical and infallible criterion that the characteristic furnishes him (*Math*., I, 85; *Brief.,* I, 145). There then follows the correspondence with Eckhard and Molanus, (1677), *passim* (*Phil*., I, 212-72); the letter to Conring of 3 January 1678, in which Leibniz makes allusion to Eckhard (*Phil*., I, 188); the letters to Malebranche, 1679 (*Phil*., I, 331-2, 337-9); the letter to the Duchess Sophie (*Phil*., IV, 292ff.); the letter to Foucher (1686), in which Leibniz refers back to his *Meditations* of 1684 (*Phil*., I, 384-5); the letter to Placcius of 10 May 1687 (Dutens, VI.1, 44); *Specimen of Discoveries* (*Phil.,* VII, 310); *Animadversions Against the General Part of Descartes's Principles*, 1692 (*Phil*., IV, 358-9; cf. 402, 405); the letter to Burnet of 20/30 January 1699 (*Phil*., III, 248); finally, the letter to Bierling of 10 November 1710 (*Phil*., VII, 490). For a thorough discussion of this criticism, see Hannequin, "La preuve ontologique cartésienne defendue contre la critique de Leibniz," in *Revue de Métaphysique et de Morale*, vol. IV, pp. 433-58 (July 1896).

[75] "On the difference between inadequate and adequate ideas, or nominal and real definitions, whereby an answer is to be given to the Hobbesian difficulty about the arbitrariness of truth and the Cartesian difficulty about the ideas of those things of which we speak." *Plan for an Encyclopedia* (LH IV 7A Bl. 26 verso). Cf. Leibniz to Oldenburg, 28 December 1675: "It seems that many things can be thought by us (certainly confusedly) which nevertheless imply a contradiction: for example, the number of all numbers…. Nor should these notions be relied on before they are subjected to that criterion which I recognize as my own, and which, as though by a mechanical reason, renders truth fixed and visible and (as it were) irresistible" (*Phil*., VII, 9-10; *Math*., I, 85). We know what Leibniz means by this mechanical and sensible thread: it is the characteristic, which furnishes him with the infallible criterion that Descartes lacked. At issue here is the criterion for the truth of ideas; we will come later to what Leibniz thinks of the Cartesian criterion applied to propositions.

[76] Published in the *Acta Eruditorum* of 1684. The draft originally bore the title *On Truth and Ideas*, later the title *On Knowledge, Truth and Ideas* (LH IV 8 Bl. 37-38). Cf. *Discourse on Metaphysics* (1686), §24 (*Phil*., IV, 449-50); the end of the letter to Arnauld, 14 July 1686 (*Phil*., II, 63); and the plan for an encyclopedia already cited, where we read: "Here therefore [namely, in the *Elements of Eternal Truth*] we

knowledge (an idea is clear when it suffices to allow us to recognize the thing represented), then *distinct* and confused *knowledge* (a *distinct* notion is that of which we can enumerate the elements).[77] There are clear notions which are not distinct, for example, those we have of different colors (because we cannot define them). Nevertheless, we necessarily have distinct knowledge of a notion that is primitive, and consequently indefinable. A notion is *adequate* when we have a distinct knowledge not only of it itself, but of all its elements, that is, when its analysis has been carried to its conclusion. Finally, it is necessary to distinguish *blind* or *symbolic* knowledge from *intuitive* knowledge: the first is present when we substitute for the elements of the notion signs or names suited for recalling them; the second is present when we explicitly think of all the elements of the notion. A distinct primitive notion can only be conceived in an intuitive manner; but complex notions can in general only be conceived in a symbolic manner.[78] Such is the notion we have of a chiliagon, where we certainly cannot think of the thousand sides. And in general arithmetical and geometrical notions are like this, for a number that is somewhat large, a figure that is somewhat complicated, cannot be explicitly represented in the mind; they are replaced by a symbol, by a verbal definition that would allow us to reconstruct them in terms of ideas, if it were necessary.[79]

Let us now bring together this classification of ideas with the theory of definitions. An idea is distinct when we have a nominal definition of it, since the latter is the enumeration of its characteristics or conditions.[80] But when we have a real definition, we have adequate knowledge, which implies the possibility *a priori*: for this means that we can carry the analysis of the notion through to its conclusion without encountering a contradiction.[81] In other words, an idea is adequate when it is completely resolved into simple elements; and then it has to be true, that is, non-contradictory.[82]

Here Leibniz inserts a reservation, or expresses a doubt, which we have already noticed and of which we will soon see the importance. He asks himself whether it is possible for humans to complete the analysis of all ideas and reduce them to the simple notions which he calls the first possibles or the attributes of God; and he does not dare to decide.[83] Elsewhere he even seems to decide in the negative, when he says: “However,

must speak of the nature of truth and of absolutely first truths…, of the matter of truth, or concepts and ideas, and how concepts are known to be genuine and not at all fictitious. Concepts are either obscure or clear, and clear concepts either confused or distinct, and distinct concepts more or less adequate…” (LH IV 7A Bl. 26).

[77] “I call something a *distinct idea*, when I understand its conditions or requisites, in a word, when I have a definition of it, if it has one.” Leibniz to Burnett, 20/30 January 1698, in which Leibniz recalls his *Meditations* (*Phil*., III, 247).

[78] *Meditations* (*Phil*., IV, 422-3).

[79] Cf. Leibniz to the Electress Sophie, 12 June 1700: “It is a blind thought, as in algebra, when one thinks of symbols in place of things” (*Phil*., VII, 555). “Often we have only blind concepts of things, by means of analogy and characters…” (LH IV 7A Bl. 26).

[80] “But when anyone has a clear and distinct concept, then they have a nominal definition, which is nothing other than a collection of signs by which we distinguish one thing from another” (LH IV 7A Bl. 26 verso).

[81] “An adequate distinct concept is a real definition, or a definition such that it is immediately clear from it whether the thing in question is possible, or a definition which consists of all the requisites of a thing…” (LH IV 7A Bl. 26 verso).

[82] *Meditations* (*Phil*., IV, 425).

[83] “Whether a perfect analysis of any notion in fact could be carried out by human beings, or whether they could reduce their thoughts to the *first possibles* and irresolvable notions, or (what comes to the same

we should not imagine that we could always carry the analysis through to its conclusion until we reached the first possibles, thus is it not necessary for science. It is true that in that case it would be complete."[84]

We shall see later the cases in which the complete analysis of concepts is possible and the cases in which it is impossible, that is, infinite. We know, moreover, that this analysis is parallel to the analysis of truths. Nevertheless, Leibniz recognizes that it is not necessary for the special sciences; the latter can, and even must, be based on notions that have not been completely analyzed and on principles that have not been demonstrated (hypotheses or postulates). This is the way in which geometers, whom Leibniz always cites as models, have proceeded: they have taken as the basis of their deductions a small number of axioms admitted as self-evident.[85] And this is the way that every science must proceed in order to be established and developed: it would be a waste of time to persist in demonstrating principles that one is obliged to assume; it is necessary to deduce from them progressively all the consequences, even if this means returning later to the principles in order to reduce them to simpler ones, and, step by step, to identical propositions.

13. It is no less true for this that the complete analysis of truths and notions is the ideal end of science and that it would be its culmination. It is only in this way that we could, on the one hand, realize the unity of science, and on the other hand, ground and rationally justify the special sciences, which until now have been independent and autonomous. It is therefore very useful to demonstrate (non-identical) axioms by reducing them to identical propositions.[86] But the demonstration of axioms has a still more important aim, which is

---

thing) the absolute attributes of God, that is, the first causes and ultimate reason of things, I would certainly not dare to determine now" (*Phil*., IV, 425). This objection is found in a letter from Burnett of 18 February 1699 (*Phil*., III, 254), to which Leibniz responds with the letter we have often cited.

[84] Leibniz to Foucher, 1687 (*Phil*., I, 392). Cf. *Introduction to a Secret Encyclopedia*: "An analysis of concepts by which we are able to arrive at primitive notions, i.e. at those which are conceived through themselves, does not seem to be in the power of man. But the analysis of truths is more in human power, for we can demonstrate many truths absolutely and reduce them to primitive indemonstrable truths…" (LH IV 8 Bl. 2 verso). See the analogous texts quoted in §4.

[85] Leibniz to Foucher, 1687 (*Phil*., I, 381): The only means of advancing and ending disputes is to establish many things on a small number of suppositions. This, for example, is what Archimedes has done (cf. *Phil*., VII, 165). Leibniz to Johann Bernoulli, 23 August 1696: "The ancients also saw this: from which Apollonius (in lost writings) and Proclus and others attempted to demonstrate the axioms assumed by Euclid" (*Math*., III, 321). Finally, in the *Animadversions Against the General Part of Descartes's Principles* (1692), after having cited Apollonius and Proclus, Lebniz says that Euclid would have been able to demonstrate the axiom of a straight line if he had had a good definition (see Chap. IX, §17); but he nevertheless praises the ancient geometers for having gone on and for having been able to derive much from little: "For if they had wanted to postpone the discovery of theorems or problems until all the axioms or postulates had been demonstrated, we would perhaps have no geometry today" (*Phil*., IV, 355). Cf. the letters to Foucher of 1679(?) (*Phil*., I, 372) and January 1692 (*Phil*., I, 402). It is enough to formulate explicitly the principles that one postulates, so that one knows that the consequences one derives from them have only a hypothetical necessity (*Phil*., VII, 299). Cf. *New Essays*, IV.ii.8 and IV.xii.10; and p. 139, note 2.

[86] Leibniz to Johann Bernoulli, 31 July 1696: "It is obvious to me that it is desirous that a demonstration of every axiom should be produced, otherwise knowledge is imperfect" (*Math*., III, 312). Cf. *New Essays*, I.i.1, I.ii. end, and I.iii end: "This is one of my great maxims, that it is good to seek demonstrations of the axioms themselves." See also IV.vii.1, and xii.6.

to complete the analysis of ideas[87] and to allow the discovery of the truly primitive notions that must compose the alphabet of human thoughts.[88] Indeed, every necessary proposition that has not been demonstrated must inevitably contain a term that is not yet defined, that is, resolved into its elements; if, therefore, we attempt to demonstrate the supposed axiom, we will discover in this same way the definition of the term in question.[89] "By means of this method, by not allowing any axiom to pass without proof except definitions and identities, we arrive at the resolution of terms and at the simplest ideas."

14. This is why Leibniz does not cease to insist that it is necessary to demonstrate axioms—what appeared paradoxical and even absurd to those who did not know the principles of his logic and were accustomed to conceive of axioms as propositions indemonstrable by definition.[90] Thus, Johann Bernoulli objected that every truth is either an axiom or a theorem demonstrated by means of axioms; it is therefore indeed necessary to admit some axioms without demonstration, first, because we cannot demonstrate everything, second because otherwise we could not demonstrate anything.[91] Leibniz responded that he had not advanced this maxim without design ("*temere*"), and recalled in this connection his *Meditations* of 1684. He explained his paradox by distinguishing identical axioms, which are indemonstrable, and non-identical axioms, which can and must be demonstrated by means of prior propositions. And since Bernoulli had asked him whether he called into question the axiom "the whole is greater than the part," Leibniz seized this occasion to note the difference between his precept and the Cartesian rule of methodical doubt. It is one thing to doubt a truth, another to demand its demonstration; and one can very easily seek to demonstrate an axiom when one could not place it into doubt.[92] Thus the Cartesian doubt goes both too far and not far enough: Descartes goes too far in calling into question mathematical truths and logical deduction, which rest on

[87] "And I know the great use of the demonstration of axioms for the true analysis or art of invention" (*Phil*., IV, 355). "And in my view this concern for demonstrating axioms is one of the most important points of the art of invention" (*Phil*., VII, 165). Cf. the letters to Foucher of 1679(?) and January 1692 (*Phil*., I, 372, 402).

[88] In 1676 Leibniz said of his combinatory: "In truth it differs not at all from the highest analysis, to whose depths, so far as I can judge, Descartes did not penetrate. For an alphabet of human thoughts is needed in order to invent its alphabet." Leibniz to Oldenburg, 27 August 1676 (*Phil*., I, 11; *Brief.*, I, 199). For the criticisms addressed at Descartes, cf. the texts quoted on p. 94 (notably *Phil*., I, 327; IV, 276, 282, 291).

[89] "Thus it is necessary to try to give this demonstration; and we could not give it without discovering this definition" (LH IV 6, 12f Bl. 23). The sentence following is quoted in the text. Cf. LH IV 7C Bl. 51: "The best method of arriving at the analysis of notions *a posteriori* is to seek demonstrations of the most axiomatic propositions, which seem to be known *per se*, from others."

[90] He highly praised, on every occasion, ancient and modern geometers (Apollonius, Proclus, Roberval) who had believed it necessary to demonstrate the axioms admitted by Euclid as obvious and indubitable (*Phil*., VII, 165). On Roberval in particular, see Leibniz to Foucher (*Phil*., I, 372, 402); *New Essays*, IV.vii.1; LH IV 5, 10f Bl. 54; and LH XXXV 1, 2 (*Demonstration of the Axioms of Euclid*, 2 February 1679).

[91] Johann Bernoulli to Leibniz, 15 August 1696 (*Math*., II, 321).

[92] "If Descartes when he spoke of the need for doubting everything had meant only what I demand, he would be blamed by no judge. But he sinned twice over: by doubting excessively and by learning too easily from doubt." Leibniz to Johann Bernoulli, 23 August 1696 (*Math*., II, 321).

the principle of contradiction; and he escapes too easily from his doubt in taking obviousness, that is, the clarity and distinctness of ideas, as the criterion of truth.[93]

In sum, the Cartesian criterion of truth appears no more valuable to Leibniz for propositions than for ideas and for the same reasons (see §11). We say that everything we conceive clearly and distinctly is true; but how will we recognize a clear and distinct thought? An idea which appears clear to one person appears obscure to another.[94] The false very often has an apparent obviousness, without which we would never take it for the true. In order to apply Descartes's first rule, therefore, we need a criterion for a clear and distinct idea; and this formal and infallible criterion is precisely what is supplied in the Leibnizian theory of ideas and truths by the definition of an adequate idea.[95]

In a very remarkable fragment, Leibniz shows that in general the insufficiency of the Cartesian rules consists in the fact that they are psychological precepts and not logical ones, and that consequently they have a subjective significance rather than an objective one. He adds, with a supreme confidence in the sovereign power of the understanding and reason,[96] that it is useless to discourse at length about prejudices and passions, the psychological causes of our errors, and to advocate methodical doubt in order to destroy and uproot them. The only remedy for our errors, he says, is a good logic, before which they will vanish like phantoms of the night before the rising sun.[97]

---

[93] Leibniz to Foucher, January 1692 (*Phil*., I, 402). Cf. *Phil*., VII, 164-5; IV, 327-8, 356, 403-4; and LH IV 1, 4i Bl. 42: "What Descartes boasts of concerning doubt is either false and pernicious, or it reduces to this: to seek demonstrations of even the most accepted truths; and this is assuredly a completely innocent kind of theoretical doubt. But Descartes, who so earnestly inculcates doubts regarding the principle of philosophizing, lets slip the proofs of axioms, omitting with them the true analysis" (Bodemann, 59). Cf. the fragment: LH IV 1, 4d Bl. 4 (Bodemann, 52-3). See Chap. IV, §7.

[94] Thus Malebranche found obscure the idea of the soul and thought, which were clear ideas for Descartes and his disciples (*Phil*., IV, 328).

[95] See Leibniz to Foucher (*Phil*., I, 384); cf. *Phil*., III, 451-3, notes; IV, 403-4. In the same way, Leibniz blamed Arnauld, Malebranche and Locke for having "followed the example of M. Descartes, who misjudged the definition of known terms that everyone understands in his own way and that we in fact ordinarily define *per æque obscurum*. But [he adds] my method of defining is entirely different, and we commonly only understand these terms in a way that is confused and insufficient for reasoning." Leibniz to Bourguet, 22 March 1714 (*Phil*., III, 569); cf. Leibniz to Coste, 4 July 1706 (*Phil*., III, 384).

[96] "For by all other qualities human beings can be rendered worse; right reason alone can only be healthy (*Phil*., VII, 187).

[97] "I do not think that anyone who could correctly propound the principles of truth would have need, for the sake of convincing men, of certain rules that are by no means taken from reality, or are even ambiguous, with the result that they ought to regard as doubtful anything that is once doubted, that they ought to reject as false those things which they do not yet sufficiently understand, and other things of this sort which disturb the souls of men and offer an occasion for various difficulties. Nor is there need to discuss at length our prejudices, the passions of the soul and the different causes of error, for all the monsters of the night vanish of their own accord with the coming of the dawn; and in truth the one proximate cause of errors is that men do not employ the correctly transmitted elements of truth." (Here follows the passage quoted on p. 100, note 2: "… I therefore demand sensible criterion of truth...".) "[A]nd so I disregard those criteria that have any difficulty at all, as when they say that the true is whatever is clearly and distinctly perceived; for there is a need for perceptible marks of the clear and distinct, since men often disagree about these; in the same way, I do not approve of arguments drawn from ideas, when someone insists that he finds the attribute in question in the idea of the thing, for with someone else claiming the contrary there is no way left of ending their disputes, as when one says, and the other denies, that the idea of body consists in extension" (LH IV 5, 6 Bl. 19; in Bodemann, 82).

15. Thus, in order to know whether one can and must demonstrate a proposition, it is not necessary to ask whether it is obvious and indubitable, or even whether one conceives it clearly and distinctly, but only whether it is *identical*, that is, reducible to the principle of identity. This is what Leibniz shows by the example of this arithmetical proposition: "two and two make four," which certain of his contemporaries considered an axiom. He indicates that it is correctly demonstrated by means of the definitions of the numbers 2, 3 and 4, and the axiom of the substitution of equivalents.[98] He applies the same method to the demonstration of what are, properly speaking, axioms, and notably to the one which Johann Bernoulli had raised as an objection to him: "the whole is greater than the part." In fact, Leibniz defines the *greater* and the *smaller* as follows: "A is greater than B, and B is smaller than A, if B is equal to a part of A."[99] On the other hand, he admits as an (indemonstrable) axiom of identity "A = A," that is, each quantity is equal to itself. With this assumed, the axiom in question is demonstrated by a syllogism of the first figure:

> Whatever is equal to a part of the whole is smaller than the whole (by virtue of *definition*).
> A part of the whole is equal to a part of the whole, that is, to itself (by virtue of the *axiom of identity*)
> Therefore, a part of the whole is smaller than the whole. Q.E.D.[100]

We thus see, Leibniz concludes, that every demonstration rests on two kinds of ultimate principles: definitions and identical propositions.[101]

Such is also the conclusion of a "small essay" on "the true analysis of intelligible truths," which is found in a letter to Burnett of 1699. In it Leibniz demonstrates, by way of example, this axiom from Euclid, "If equal quantities are added to equal quantities, equal sums result," by drawing on the *axiom of identity* already cited and on the *definition* of equality: "Those things are *equal* which can be substituted one for another with magnitude preserved."

By virtue of the axiom, we have:

---

[98] Phil., IV, 403; cf. *Remarks on the Arguments of M. Jaquelot*, 1702 (*Phil*., III, 448); *New Essays*, IV.vii.10.

[99] This definition of inequality, which is already found in *On the Art of Combinations*, 1666 (see Note VI), was borrowed by Leibniz from Hobbes. Indeed, he makes use of it in his *Specimen of Political Demonstrations for Choosing the King of Poland* (1669), prop. LI, coroll. 1, and he adds: "Through the definition of the greater and the smaller given by Hobbes in the *Elements of Body*" (Dutens, IV.3, 575). He, in fact, read in *De Corpore* (part II, chap. 8, §13): "*But one body is greater than another, when a part of the former is equal to the whole of the latter*. It is *smaller* when the whole of the former is equal to a part of the latter. Cf. *Geometrical Characteristic* (10 August 1679), §29 (*Math*., V, 153).

[100] The same demonstration is already indicated in the unpublished fragments: *Demonstration of the Axioms of Euclid*, 22 February 1679 (LH XXXV 1, 2), and LH IV 8 Bl. 6. It is found again in the following passages: *Phil*., VII, 300; *Math*., VII, 20, 274.

[101] Leibniz to Johann Bernoulli, 23 August 1696 (*Math*., III, 322; quoted by Erdmann, 81, note). Bernoulli objects that the principle of the syllogism is involved in the demonstration and that it is no more evident than the axiom that is to be demonstrated (letter of 12 September 1696, *Math*., III, 329-30). Leibniz responds that he knows how to demonstrate the principle of the syllogism, and this independently of the axiom in question, such that there is no vicious circle here (letter of 6 October 1696, *Math*., III, 331). See Chap. VIII, §§14 and 21.

$$a + b = a + b$$

But, by hypothesis:

$$a = c, \quad b = d.$$

By virtue of the definition of equality, we can then substitute, on the right-hand side of the first equality, c for a, and d for b; there results:

$$a + b = c + d \quad \text{Q.E.D.}$$ [102]

16. From all this it emerges that the principles of every demonstration are no longer definitions alone, but definitions and identical axioms.[103] These are also the principles of all necessary propositions that are known *a priori*,[104] or, as Leibniz would say, the elements of eternal truth.[105] Thus all necessary truths are *identical*: some explicitly, the first truths or axioms, others "virtually" or implicitly, the demonstrable theorems. To demonstrate the latter is to reduce them to identical truths by analyzing their terms, that is, by defining them.[106] Every demonstration consists in substituting the definition for the defined, that is, in replacing one (complex) term by a group of (simpler) terms which is equivalent to it. Thus, the essential foundation of deduction is the *principle of the substitution of equivalents*.[107] This is the single supreme principle of logic, and not the

[102] Leibniz to Burnett, 1699 (*Phil.*, III, 258-9). The same demonstration is mentioned in the *Example of an Enlightening Geometry* (*Math.*, VII, 274), and in the *Geometrical Characteristic* of 1679, §39 (*Math.*, V, 156). Leibniz wants to show by means of these demonstrations the utility and fruitfulness of identical propositions, which empiricists regard as empty and sterile tautologies: "…so it appears that identities also have their use, and it seems that no truth, however trivial, is completely sterile; consequently, it will soon appear that the foundations of the rest are contained in these" (*Phil.*, VII, 300). Cf. *New Essays*, IV.ii.1; IV.vii and viii, *passim*.

[103] "The principles of the knowledge of truths that are necessary and independent of experience are (in my opinion) two: definitions and identical axioms (*Phil.*, III, 258).

[104] "From this it is already clear that the ultimate analysis of all necessary truths is into definitions or ideas, and identical truths or agreements of ideas. And all necessary truths are virtual identities" (*Phil.*, III, 259).

[105] "It is manifest that all necessary propositions or eternal truths are virtually identical, for they are those which can be demonstrated from ideas or definitions alone (that is, by the resolution of terms) or reduced to primary truths, such that it appears that the contrary implies a contradiction and conflicts with any identity or primary truth" (*Phil.*, VII, 300; cf. 295-6, quoted n. 106).

[106] "From such ideas or definitions all truths can be demonstrated, with the exception of identical propositions; these, it is evident, are by their very nature indemonstrable and can truly be called axioms. But what are commonly regarded as axioms are reduced to identities, i.e. are demonstrated, by the analysis either of the subject or the predicate, or both.... And so a reason can be given for each truth; or the connection of the predicate with the subject is either self-evident, as in the case of identities, or it has to be displayed, which is done by the analysis of the terms. This is the unique and highest criterion of truth in the case of abstractions, which do not depend on experience: namely, that it should either be an identity or reducible to identities. From this there can also be derived the elements of eternal truth." *On Universal Synthesis and Analysis* (*Phil.*, VII, 295-6). It is in this way that Leibniz explains and justifies the scholastic aphorism that axioms are self-evident once we understand their terms (*Phil.*, VII, 295, 300; cf. III, 258).

[107] Which Stanley Jevons has revived in our day as the foundation of logic: *The Substitution of Similars, the True Principle of Reasoning, Derived from a Modification of Aristotle's Dictum* (London, Macmillan, 1869). It is by the substitution of equivalents that Leibniz claims to demonstrate inferences from the direct to the oblique (see Chap. III, §15): "for, nevertheless, it seems to me that there is no need for any demonstration there other than that which depends on the substitution of equivalents for each other"

principle of the syllogism (Aristotle's "dictum of all and none"), for the latter is not, as is often believed, an identical axiom but a theorem that is demonstrated by means of the preceding principle.[108] Indeed, the principle of the syllogism consists in the ability to substitute for the subject of a proposition a term which is smaller (in extension) or which contains it (in comprehension), or alternatively, for the attribute, a term which is greater (in extension) or which is contained in it (in comprehension). It is therefore naturally subordinated to the principle which allows the substitution of one term for another equivalent term and must be logically derived from it; for every allowable substitution must rest on some identity, complete or partial.[109] In a simple proposition, the predicate is contained in the subject (from the point of view of comprehension) and can be substituted for it, because it is identical to a part of the subject. In a reciprocal or convertible proposition, the subject and predicate are mutually contained; they are logically equivalent and can therefore be substituted one for another indifferently.[110] As a consequence, every non-identical proposition must be demonstrated by an analysis of terms and the substitution of equivalents, in such a way as to make obvious the complete or partial identity of its two terms (their equality or inclusion).[111]

(Leibniz to Placcius, 16 November 16, 1686; Dutens, VI.1, 32). Cf. *Specimen of the Demonstration of Inferences from the Direct to the Oblique, sent by Leibniz to Jo. Vagetius* (ibid., 38), in which Leibniz deals with the following example: "Writing is an art; therefore, whoever learns writing learns an art." We find in the unpublished manuscripts some discussions of the difficulties and sophisms to which oblique inferences give rise, for example: A bishop is a man; therefore, whoever makes a bishop makes a man.

[108] Leibniz to Foucher, 1687 (*Phil*., I, 391), quoted in Chap. VIII, §9; *On Freedom* (Foucher de Careil, B, 181); LH IV 8B, 2 Bl. 62; Leibniz to Vagetius, 10 January 1687: "We can even reduce all predications to equivalences by supplying something to complete the inverse of predication; from this, in return, the proposed equivalence appears"; and 10 May 1687: "I employ a similar method for expressing rational arguments by means of certain imitations of algebraic equations" (Dutens, VI.1, 37, 43). Leibniz had already employed this method of substitution in his *Specimen of Political Demonstrations* of 1669, in order to demonstrate, for example, that "everything shameful is dangerous" (Preface, Dutens, IV.3, 524). For the formal demonstration of the principle of the syllogism and the reduction of propositions to logical equations, see Chap. VIII, §§20 and 21.

[109] "But every substitution arises from some equivalence" (*Phil*., VII, 31). Cf. Chap. VIII, §§9 and 13.

[110] *Analysis of Languages*, 11 September 1678: "Moreover, since all knowledge that consists of demonstrations deals only with equivalences or substitutions of thoughts, for they show that in any necessary proposition the predicate is safely substituted in place of the subject and in every convertible proposition the subject also can also be substituted in place of the predicate…" (LH IV 7C Bl. 9). Cf. LH IV 7B, 4 Bl. 15-18; LH IV 7C Bl. 62-63. We read the following in a table of definitions dating from 1702-1704: "To *infer* is to form one proposition from another through the substitution of equivalent terms" (*Monatsberichte der Akademie der Wissenschaften zu Berlin*, 1861, p. 209). For the algebraic translation of these two fundamental logical relations (equality and inclusion), see Chap. VIII, §8.

[111] "Certainly, just as identical propositions are all primary and incapable of proof, and hence true in themselves, for nothing at all can be discovered that might, like a middle term, connect something else with them, so, consequent truths, namely those which are reduced to identical formulas or expressions through an analysis of terms (if for one term an equivalent or included notion is substituted) are virtually identical" (*Phil*., VII, 300). Cf. *On Freedom*: "For demonstration consists simply in this: by the analysis of the terms of a proposition and by substituting a definition or part of a definition for a defined term, one shows a certain equation or coincidence of the predicate with the subject in a reciprocal proposition; or, on the other hand, at least the inclusion of the predicate in the subject, in such a way that what was latent in the proposition and as it were contained in it virtually is rendered evident and explicit by the demonstration" (Foucher de Careil, B, 181). In order to explain himself, Leibniz employs an arithmetical analogy: every multiple of 12 is a multiple of 2.2.3, and consequently a multiple of 2.3 or 6. Cf. *General Language*, February 1678 (LH IV 7B, 3 Bl. 3); LH IV 7B, 4 Bl. 15-20, etc.

17. We have spoken chiefly of rational and necessary truths, but all that has preceded holds equally for every species of truth (Leibniz says this explicitly), for the very definition of truth in general is that in every true proposition the predicate is contained in the subject.[112] By the very fact that a proposition is true, there must be a real and intelligible connection between its subject and predicate, that is, a relation of logical inclusion that can be discovered through a simple analysis of terms.[113]

This holds even for singular propositions, that is, those in which the subject is individual, as is the case for all historical and factual truths. Leibniz goes so far as to maintain that "the individual notion of each person includes once and for all everything that will ever happen to him," so that "there are seen in it *a priori* proofs or reasons for the truth of each event."[114] This, we know, is the main thesis on which rests his controversy with Arnauld; and the reason for it is that every truth is determined by the logical nature of its terms, which is in some way inscribed in them in advance, and it is enough to analyze them in order to discover it there.[115] Indeed, insofar as it is true that such-and-such an individual now or has been the subject of such-and-such an event, and that the concept of the latter is forever included in the concept of this individual, it is thereby also true that the concept of this individual implies for all eternity the concepts of all those event still in the future of which it must be the subject, and it is certain that these events will occur.[116]

---

[112] "Thus the predicate or consequent is always in the subject or antecedent, and in this consists the nature of truth in general.... But this is so for every truth, universal or particular, necessary or contingent...." (LH IV 8 Bl. 6). Cf. *On Freedom*: "However, I saw that it is common to every true affirmative proposition, universal and particular, necessary or contingent, that the predicate is in the subject, or that the notion of the predicate is in some way involved in the notion of the subject, and that this is the principle of infallibility in every kind of truth for him who knows everything *a priori*" (Foucher de Careil, B, 179).

[113] *Discourse on Metaphysics* (1686), §8: "It is agreed that every true predication has some basis in the nature of things, and when a proposition is not identical, that is, when the predicate is not contained expressly in the subject, it must be contained in it virtually. This is what philosophers call *in-esse*, when they say that the predicate *is in* the subject. The subject term, therefore, must always include the predicate term, in such a way that someone who understood the notion of the subject perfectly would also judge that the predicate belongs to it" (*Phil*., IV, 433).

[114] Leibniz to the Landgrave Ernst von Hesse-Rheinfels, 1/11 February 1686 (*Phil*., II, 12; cf. 15), and *Discourse on Metaphysics* (1686), §13 (*Phil*., IV, 427).

[115] "If any concept is complete, or is such that from it a reason can be drawn for every predicate of the same subject to which this concept can be ascribed, it will be the concept of an individual substance" (LH IV 7C Bl. 62-3). "The complete or perfect concept of a singular substance involves all its predicates past, present, and future. For clearly a future predicate that will be the case is already now true, and so is contained in the concept of a thing" (LH IV 8 Bl. 7). Cf. Leibniz to Arnauld, 14 July 1686: "Since it is certain that I shall take it [this journey], there must indeed be some connection between me, who am the subject, and the accomplishment of the journey, which is the predicate, *for in a true proposition the concept of the predicate is always in the subject*"; and further on: "Finally, I have given a decisive argument which in my view has the force of a demonstration, that always, in every true affirmative proposition, necessary or contingent, universal or particular, the concept of the predicate is in a sense included in that of the subject; or else I do not know what truth is" (*Phil*., II, 52, 56).

[116] "When I say that the individual concept of Adam contains everything that will ever happen to him, I mean nothing other than what all philosophers mean when they say that the predicate is in the subject of a true proposition." Remarks on the Letter of M. Arnauld, 1686 (*Phil*., II, 43). This logical thesis is the foundation of the entire Leibnizian metaphysics. For, "it follows that every individual substance expresses the entire universe after its own manner and according to a certain relationship, or, so to speak, according to the point of view from which it regards the universe; and that its succeeding state is a sequel... of its

In sum, every truth is formally or virtually identical, or, as Kant will say, *analytic*; and as a consequence it must be possible to demonstrate it *a priori* by means of definitions and the principle of identity.[117]

18. Yet this conclusion offends common sense: in fact, there are truths which are not demonstrable, namely all truths of experience; furthermore, we are accustomed to regard these as contingent. Leibniz upholds the distinction between truths of reason and truths of fact, between necessary and contingent truths; nevertheless, he takes them all to be equally analytic. However, we are then presented with a serious objection, which he himself raises: does this not render necessary even truths of fact and thereby destroy human freedom and every species of contingency? The solution to this difficulty was suggested to Leibniz, in his own view, by mathematics.[118] For what is it that prevents the demonstration of truths of fact? It is that their demonstration requires an infinite analysis, because the concept of every concrete thing, of every individual, comprehends an infinity of elements or conditions ("requisites"). In point of fact, our concepts of real things are only imperfect or inadequate, that is, incompletely analyzed, and we do not know how to decompose them into their simple elements; this is why we only know things and their properties by experience.[119] Only an infinite understanding (like that of God) can carry out this infinite analysis and consequently have an adequate idea of individual beings and

preceding state, as though only God and it existed in the world; thus each individual substance... is like a world apart, independent of everything except God." Leibniz to Arnauld, 14 July 1686 (*Phil*., II, 57). The latter part of this letter contains a sketch of the doctrine of preestablished harmony, which follows necessarily from the preceding propositions. Cf. Leibniz to Arnauld, 23 March 1690 (*Phil*., II, 136); Leibniz to Foucher, 1686 (*Phil*., I, 383); *New System For Explaining the Nature of Substances and Their Communication Among Themselves, as well as the Union of the Soul with the Body*, 1695 (*Phil*., IV, 475); finally, the fragment LH IV 8 Bl. 7, in which we read: "*Every individual substance involves the entire universe in its perfect notion*, and everything existing in it, past, present, and future.... *Further, all created individual substances are diverse expressions of the same universe*...." We see that these are all the fundamental theses of the *Monadology*, which Leibniz deduced as early as 1686 from the principles of his logic.

[117] "In general, every true proposition (which is not identical or true *per se*) can be proved *a priori* with the help of axioms or propositions that are true *per se* and definitions or ideas. For whenever a predicate is truly affirmed of a subject, it is judged with certainty that there is some real connection between the predicate and the subject, so that in any proposition A is B (or B is truly predicated of A), undoubtedly B is in A, or its notion is contained in some way in the notion of A" (*Phil*., VII, 300).

[118] "A new and unexpected light finally arose in a quarter where I least hoped for it, namely, out of mathematical considerations of the nature of the infinite. There are two labyrinths of the human mind: one concerns the composition of the continuum, the other the nature of freedom, and both spring from the same source: the infinite." *On Freedom* (Foucher de Careil, B, 179-80). This explains the following passage from a letter to the Electress Sophie of 4 November 1696: "My fundamental thoughts turn on two things, namely, unity and infinity" (*Phil*., VII, 542).

[119] "All our concepts of complete things are imperfect.... The sign of an imperfect concept is if there exist several definitions of the same thing, one of which cannot be demonstrated through the others; likewise, if any truth concerning a thing is established through experience and we cannot give a demonstration of it." *Plan for an Encyclopedia* (LH IV 7A Bl. 26 verso). Cf. *General Investigations Concerning the Analysis of Concepts and Truths* (1686), §74: "All existential propositions, though true, are not necessary, for they cannot be proved unless an infinity of propositions is used, i.e. unless an analysis is carried to infinity. That is, they can be proved only from the complete concept of an individual, which involves infinite existents" (LH IV 7C Bl. 25 verso).

an intuitive knowledge of truths of fact. God alone knows these truths *a priori* and sees in them the "reason," which is always the inclusion of the predicate in the subject.[120]

Here again Leibniz employs an arithmetical analogy that was suggested to him by his system of logical calculus. He conceives of this inclusion of the predicate in the subject on the analogy of the divisibility of numbers; truth is analogous to a relation (or "proportion") whose antecedent (subject) is greater than its consequent (predicate), and consequently contains it. But there are commensurable relations and incommensurable relations. In the first case, we determine the common factor of the two terms by means of the Euclidean algorithm, which furnishes at the same time the expression of the relation in a terminating continuous fraction, which can be reduced to a simple fraction. In the second case, application of the Euclidean algorithm goes on without end, which gives rise to a non-terminating continuous fraction whose successive reductions constitute closer and closer approximations of the incommensurable relation.[121] We see to what extent this mathematical analogy is apt and exact: truths of fact could only be demonstrated by an infinite analysis; likewise incommensurable relations can only be expressed by an infinite series.[122] Elsewhere, Leibniz compares contingent truths to asymptotes, that is, to the lines tangent to a curve at infinity.[123]

But these mathematical analogies immediately suggest an objection that the inventor of the infinitesimal calculus must surely have been the first to pose: we know how to calculate an asymptote, how to sum infinite series, and to carry out the synthesis of an infinity of elements; why couldn't we likewise exhaust the infinite set of conditions of a truth of fact and demonstrate it by a sort of logical integration? Leibniz responds, in a somewhat confused and embarrassed way, that the analogy is not perfect, that there can be certain elements in a truth of fact that are not uncovered by any analysis, no matter how far it is extended, and which alone make for the certainty of that truth.[124] Undoubtedly, we can verify such a proposition with greater and greater accuracy to the extent that we continue the analysis; however we have in this way no more than an indefinitely increasing probability and not the certainty that the total and simultaneous

[120] "But in the case of contingent truths, even though the predicate is in the subject, this can never be demonstrated of it, nor can the proposition ever be reduced to an equation or identity. Instead, the analysis proceeds to infinity, God alone seeing not indeed the end of the analysis, since it has no end, but the connection [of terms] or the inclusion of the predicate in the subject, for he sees whatever is in the series...." "This is also the reason why God alone knows contingent truths *a priori* and sees their infallibility in another way than by experience" (Foucher de Careil, B, 182, 181).

[121] *On Freedom* (Foucher de Careil, B, 183).

[122] "But just as in surd ratios the resolution proceeds to infinity... and some series is obtained, but an unending one, so by the same process contingent truths require an infinite analysis, which God alone could pass over. From this it follows that they are known *a priori* and with certainty by him alone" (*Phil*., VII, 200). Cf. the opening of *A Specimen of Discoveries About Marvelous Secrets of Nature in General* (*Phil*., VII, 309).

[123] *General Investigations*, §135: "So the distinction between necessary and contingent truths is the same as that between lines that meet and asymptotes, or between commensurable and incommensurable numbers" (LH IV 7C Bl. 29). An unpublished fragment is entitled "The Origin of Contingent Truths from Progress into Infinity, according to the Example of the Proportion of Incommensurable Qualities" (LH I 6, 2 Bl. 11; Bodemann, 10). We read in another fragment: "The root of contingency is the infinite. A contingent truth is one that is indemonstrable" (LH IV 8 Bl. 89; Bodemann, 121). This should be understood to mean "indemonstrable" for us human beings; for every truth is, on the contrary, demonstrable in and of itself. (See the texts: *Phil*., VII, 301; LH IV 8 Bl. 6, quoted nn. 129 and 132.)

[124] *General Investigations*, §136 (LH IV 7C Bl. 29).

intuition of elements alone can give.[125] Finally, we can no more obtain the complete demonstration of a contingent truth than we can see the point of contact of an asymptote or examine completely an infinite series.[126] Nevertheless, we can "give a reason" for it, in a way that is more and more accurate, and this indefinite process of approximation takes the place of demonstration for us.[127]

19. Does this mean that truths of fact are in themselves only probable and could never equal the certainty of "eternal" truths? On the contrary, they are only probable for us, because we have only an incomplete and approximate knowledge of them; but they are absolutely certain in themselves, in the same way and to the same degree as truths of reason, for they are likewise analytic or virtually identical. Like truths of reason, they are evident *a priori*, at least to an infinite understanding that can embrace all the conditions integral to them.[128] It is in this that the *principle of reason* properly consists,[129] for it signifies nothing more than what we have just said, namely that we must be able to "give a reason" for every truth, even a contingent one, that is, demonstrate it by a simple analysis of its terms.[130] Such is the exact logical sense of this famous principle, whose ordinary statement, "Nothing is or happens without a reason," is only a vulgar formula borrowed from common sense.[131] This principle is at bottom only a corollary of the very definition of truth.[132] It is not, as one might think at first glance, a consequence of the

[125] *General Investigations*, §134: "A true contingent proposition cannot be reduced to identical propositions; nonetheless it is proved by showing that if the analysis is continued further and further, it indeed approaches without limit identical propositions, though it never reaches them. Therefore, God, who grasps the entire infinite in his mind, alone knows all contingent truths with certainty" (LH IV 7C Bl. 29).

[126] *General Investigations*, §136: "But we can no more give the full reason for contingent things than we can constantly follow asymptotes and run through infinite progressions of numbers" (LH IV 7C Bl. 29).

[127] For example, Leibniz says, in speaking of the betrayal of St. Peter: "it must be demonstrated from the concept of Peter, but the concept of Peter is complete, and so involves infinite things; therefore, one can never arrive at a perfect demonstration, yet one always approaches it more and more, so that the difference is less than any given difference." *General Investigations*, §74 (LH IV 7C Bl. 25 verso). One will notice the completely mathematical way of speaking, borrowed from the infinitesimal method.

[128] *General Investigations*, §134: "In God, only the analysis of his own concepts is required, and in him, the whole of this occurs at once. Hence, he knows even contingent truths, whose complete proof transcends every finite intellect" (LH IV 7C Bl. 29).

[129] "It is therefore established that all truths, even the most contingent, have an *a priori* proof or some reason why they are rather than are not. And this is why it is commonly said that nothing happens without a cause, or that nothing is without a reason" (*Phil.*, VII, 301). *General Investigations* (1686), §136: "To be sure, just as with asymptotes and incommensurables, so with contingent things we can prove things with certainty, from the very principle that every truth must be capable of proof" (LH IV 7C Bl. 29). "To give a reason" means to *demonstrate* (see §4 above; nn. 106 and 134). Thus, the principle of reason is originally called the *principle of giving a reason* (see nn. 133 and 137; and LH IV 8 Bl. 57 verso).

[130] The *principle of reason* is found stated for the first time in the *Theory of Abstract Motion*, 1670 (*Phil.*, IV, 232). It is invoked in order to demonstrate the existence of God in *Conversation with Bishop Steno on Freedom*, 27 November 1677 (see Bodemann, 73).

[131] "*It is necessary that there always be some foundation for the connection of the terms of a proposition, which must be found in their notions*. This is my great principle,... of which one of the corollaries is the common axiom that nothing happens without a reason." Leibniz to Arnauld, 14 July 1686 (*Phil.*, II, 56).

[132] *On Freedom* (Foucher de Careil, B, 179; cf. n. 275); LH IV 7C Bl. 29, 62; LH IV 8 Bl. 2: "And indeed, nothing at all happens without some reason, or there is no proposition except identities in which the connection between the predicate and the subject cannot be distinctly explained"; LH IV 8 Bl. 6 verso: "For it immediately follows from this that there arises the received axiom *nothing is without a reason*, or *no*

principle of identity or contradiction: it complements it; it is its counterpart and even its logical converse, for the principle of identity affirms that every identical proposition is true, whereas the principle of reason affirms, on the contrary, that every true proposition is analytic, that is, virtually identical.[133] This principle will serve precisely to demonstrate contingent truths that we cannot prove directly: it is for us human beings the substitute for that infinite analysis that God alone can complete. It has, moreover, a universal import and is valid for every species of truth, for in the end it signifies nothing more than this: in every true proposition, the notion of the predicate is contained in that of the subject.[134] It therefore allows us to know, we too, truths of fact *a priori*, just as God knows them *a priori*, but in a different way, as we shall soon see. In conjunction with the principle of contradiction, it suffices to demonstrate all truths, of whatever order they may be.

20. Here a difficulty presents itself:[135] it seems that Leibniz's thought varied concerning the manner in which the two rational principles correspond to the different species of truth. Sometimes the principle of reason is applied to all truths, both necessary and contingent.[136] Sometimes the principle of contradiction governs only logical and mathematical truths; and physical, metaphysical, and moral truths are dependent only on the principle of reason.[137] The two theses are stated alternately in nearly contemporary

---

effect apart from a cause. Otherwise there would exist a truth that could not be proved *a priori*, or which would not be resolved into identities; and this is contrary to the nature of truth, which is always either expressly or implicitly identical." Cf. n. 289.

[133] "In every universal affirmative truth, the predicate is in the subject: expressly in the case of primitive or identical truths, which are the only truths known *per se*, but implicitly in the case of all the rest. The implicit inclusion is shown by the analysis of terms, by substituting for one another definitions and what is defined. So there are two first principles of all reasonings: the *principle of contradiction*, to the effect that every identical proposition is true and its contradictory false; and the *principle of giving a reason*, to the effect that every true proposition that is not known *per se* has an *a priori* proof, or that a reason can be given for every truth, or as is commonly said, that nothing happens without a cause." *A Specimen of Discoveries* (*Phil*., VII, 309).

[134] "I use *two principles* in demonstrations, one of which is: the false is that which implies a contradiction; the other is: for every truth... a reason can be given, namely, that the concept of the predicate is always either expressly or implicitly in the concept of its subject, and this... holds no less for contingent truths than for necessary ones" (*Phil*., VII, 199-200). Cf. Leibniz to Foucher (1686): "If, then, we suppose, for example, the principle of contradiction, likewise that in every true proposition the concept of the predicate is contained in that of the subject, and some other axioms of this sort, and if we can indeed prove things from them as demonstratively as geometers do, would you not find that this would be of consequence?" (*Phil*., I, 382). Leibniz to Arnauld, 14 July 1686: "And as for metaphysics, I claim to give it geometrical demonstrations, assuming almost no more than two basic truths, namely, in the first place the principle of contradiction,... and in the second place, that nothing exists without a reason, or that every truth has its *a priori* proof, drawn from the concept of terms, although it is not always in our power to arrive at this analysis" (*Phil*., II, 62).

[135] Signalled by M. Boutroux in his edition of the *Monadology*, p. 75 and p. 159, note 1 (Paris: Delagrave, 1881).

[136] *Theodicy* (1710), "Remarks on the Book Concerning the Origin of Evil [by Mr. King]," §14: "Both principles apply not only to necessary truths, but also to contingent ones." Cf. *Phil*., VII, 199-200, quoted n. 134); *General Investigations* (1686), §136, where Leibniz cites corollaries of the principle of reason that are true in the case of both necessary and contingent propositions (LH IV 7C Bl. 29), and *Monadology* (1714), §§33-36.

[137] *A Specimen of Discoveries*: "Arithmetic and geometry do not need this principle [of giving a reason], but physics and mechanics do" (*Phil*., VII, 309). Cf. Leibniz to Jacob Bernoulli, 2 December 1695

writings; we therefore cannot explain this contrast by a change or evolution of the doctrine. On the other hand, it is hardly probable that until the end of his life Leibniz remained uncertain or doubtful on a point so important for his system. According to the preceding explanation, the principle of reason is essentially valid for all truths; but there is an entire order of truths for the demonstration of which we *do not need* to invoke it. These are the propositions of abstract science that depend on possible essences; whereas we do need it to demonstrate the propositions of natural science that depend on real existences.[138] From this we see that although all truths depend on the principle of contradiction, truths of reason may be considered its proper domain; likewise, although all truths depend on the principle of reason, we regard it as being applied in particular to factual truths, which cannot be justified without it. But in reality these two principles are inseparable and equally valid for all species of truths, for "one can say in a way that these two principles are contained in the definition of the true and the false."[139]

21. Thus, truths of fact are no less certain than truths of reason; they have the same evidence in the eyes of God, who knows them *a priori* in the same way as eternal truths, since they are equally analytic.[140] They are consequently "infallible," as Leibniz ceaselessly proclaims, that is, they cannot fail to be verified.[141]

But then, it seems, they are as necessary as eternal truths and the distinction between necessary and contingent truths disappears. However, Leibniz upholds the distinction with vigor, in part for moral and theological reasons that we need not investigate here.[142]

(*Math*., III, 27); Leibniz to Bourguet, 11 April 1710 (quoted n. 157); *Principles of Nature and of Grace* (1714), §§7, 11 (see n. 159); Second Letter to Clarke, 1715 (*Phil*., VII, 355); and a letter with neither date nor address, in which we read: "the principle of contradiction is that of necessity and the principle of giving a reason is that of contingency" (Bodemann, 115).

[138] "Therefore, whenever anything is not of mathematical necessity (in the manner of logical forms and truths of numbers), it must generally be sought from this" (*Phil*., VII, 301). Cf. LH IV 8 Bl. 56, where Leibniz distinguishes sciences of logical necessity and sciences of physical necessity, according to whether they depend on one or the other principle.

[139] *Theodicy*, loc. cit.

[140] *On Universal Synthesis and Analysis*: "In this way, all things are understood by God *a priori*, as eternal truths; for he does not need experience, and yet all things are known by him adequately. We, on the other hand, know scarcely anything adequately and only a few things *a priori*; most things we know by experience, and in this case other principles and other criteria must be applied" (*Phil*., VII, 296). We will speak later of these principles and criteria of empirical truth (§36).

[141] Foucher de Careil, B, 179-81; *Phil*., VII, 200; LH IV 7C Bl. 29 (see nn. 112, 120, 122, 125).

[142] We know how Leibniz reconciles free will with determinism: by making the former enter, purely and simply, into determinism itself, by denying liberty of indifference, and by defining moral freedom in terms of internal, rational determination (or *intellectual spontaneity*). Thus, contingency is so far from excluding determinism that it instead implies it, for the principle of reason excludes all indetermination. Every event is at once contingent and determined. He shows this very clearly in a fragment that appears quite early and must have been destined for the Encyclopedia: "*All the actions of singular substances are contingent.... Nevertheless, all actions are determined and never indifferent.... Liberty of indifference is impossible.... The more substances are self-determined and removed from indifference, the more perfect they are.... Freedom is greater the more it acts from reason; servitude is greater the more it acts from passions of the soul.*" (The last sentence is entirely Spinozistic.) "We therefore reach the heart of the matter: there is no necessity in things, rather everything is contingent. Nevertheless, in return, there is no indifference in things, rather everything is determined" (*Phil*., VII, 108-9; the same fragment in French, ibid., 109-11). Cf. Leibniz to Burnett, 22 November 1695, in which Leibniz recalls "our discussion on

What pulled him away from Spinozistic fatalism, he says, away from the doctrine of universal necessity, was the consideration of possibles that do not exist and will never exist, whereas Spinoza maintained that every possible exists and that everything that does not exist is impossible.[143] Now truths of fact are undoubtedly certain, but they are not for that reason necessary, for "nothing is necessary whose negation is possible."[144] In fact, we know that for Leibniz there is no other necessity than logical necessity, nor any other impossibility than contradiction. Consequently, that only is necessary whose contrary implies a contradiction; all that is not contradictory in itself is possible.[145] But all possibles cannot be realized at once, for they are not *compossible*, that is, compatible among themselves.[146] From this it follows that the choice among possibles does not depend on God's understanding, that is, on logical laws of eternal truth, but on his will, that is, on his providence and goodness: God is able to do everything (whatever is not

---

freedom and fate" (*Phil*., III, 167-8); Leibniz to Coste, 19 December 1707 (*Phil*., III, 400); and a letter without date or address (LH IV 8 Bl. 23; Bodemann, 115-7).

[143] *On Freedom* (Foucher de Careil, B, 178-9). Leibniz contrasts this consideration with Descartes, who believed that matter takes on all the shapes, configurations and arrangements of which it is capable (*Principles*, III, 47). See the letters to Philippi, the second from January, 1680 (*Phil*., IV, 281, 283). He compares this theory to those of Spinoza and Hobbes, and says that "it is... the first wrong step and the foundation of the atheistic philosophy." Cf. LH IV 8 Bl. 71 (2 December 1676), against Spinoza. He maintains that matter is actually subdivided to infinity, but that it nonetheless does not give rise to every possible division (LH IV 8 Bl. 7 verso).

[144] *Discourse on Metaphysics*, §13 (*Phil*., IV, 438). "This does not mean that events are necessary, but the fact is that they are certain from the time God has made his choice of this possible universe, the concept of which embraces this sequence of things." Remarks on the Letter from M. Arnauld (*Phil*., II, 42). "I agree that the connection between events, though certain, is not necessary, and that I am free to take this journey or not, for although it is contained in my concept that I will take it, it is also contained therein that I will take it freely.... If I do not take this journey, that will not do violence to any eternal or necessary truth. However, since it is certain that I will take it, there must indeed be some connection between me, who am the subject, and the accomplishment of the journey, which is the predicate, *for in a true proposition the concept of the predicate is always present in the subject*." Leibniz to Arnauld, 14 July 1686 (*Phil*., II, 52). Cf. Leibniz to De Volder, 31 December 1700: "The determination of future contingents is truth according to causes, but it is nonetheless the reason why those future contingents should not be judged to be necessary" (*Phil*., II, 221-2); and Leibniz to Coste, 19 December 1702, 8 July 1711 (*Phil*., III, 400, 419).

[145] "Possibles are those things which do not imply a contradiction. Actuals are nothing but the best of possibles (after all have been compared); thus those which are less perfect are not for that reason impossible. We must distinguish between what God can do and what he wants: he can do everything, he wants the best." Leibniz to Johann Bernoulli, 21 February 1699 (*Math*., III, 574).

[146] This thesis is already formulated in an unpublished fragment dated 2 December 1676, obviously directed against Spinoza, with whom Leibniz had conversed a little earlier: "A dissertation on the vacuum of forms would be useful for showing that not all things possible in themselves can exist with all the rest, otherwise many absurdities would follow"; and a little later: "If all possibles should exist, there would be no need for a reason for existing and possibility alone would suffice" (LH IV 8 Bl. 71). Elsewhere, Leibniz employs a theological argument: to maintain that every possible exists would be to destroy providence (*Phil*., IV, 344). As for the incompatibility of all possibles, he asserts that it cannot be explained logically: "Nevertheless, until now men have been ignorant of the origin of the incompatibility of different things, or how it can happen that different essences contradict one another, since all purely positive terms seem to be compatible with each other" (*Phil*., VII, 195). We know in fact that all the primary possibles, being positive, are compatible and united together in the essence of God (see §10). What Leibniz lacks in order to explain the incompatibility of different essences, is the consideration of negation, for that is what introduces into complex concepts the contradiction that cannot exist between simple concepts (for example, in the case of two complex notions, if one contains *a* and the other not-*a*, where *a* is a simple concept).

contradictory), but he wills the best (of the compossible worlds).[147] "But every truth that is founded on these sorts of decrees is contingent, although it is certain, for these decrees do not change the possibility of things."[148] The principle of contradiction is the law of possibles or essences, all of which reside in the divine understanding.[149] However, the principle of reason is the law of existences, such as they result from the creator's choice. Thus, necessary truths rest solely on the principle of contradiction; contingent truths are grounded in the principle of reason or the "principle of the best." They "have reasons for being thus rather than otherwise," in that "they have *a priori* proofs of their truth that render them certain and show that the connection of the subject and the predicate has its foundation in the nature of both of them; but... they do not have necessary demonstrations, since these reasons are only founded on the principle of contingency or of the existence of things, that is, on what is or appears to be the best among many equally possible things."[150] Or at least, if they are necessary, since they are still analytic, they are only so hypothetically or accidentally;[151] that is, once admitted, the subject such as God chose it is necessary (for example, Julius Caesar[152] or Sextus Tarquin[153]), but it is not so absolutely, as if it were impossible for the subject not to have been admitted. In other words, it does not follow from a "metaphysical necessity," that is, logical necessity, but from a type of "moral necessity," which consists in the fact that God could, in his sovereign wisdom and goodness, will and realize only the best of the all possible worlds.[154]

---

[147] Leibniz to Johann Bernoulli, cited above.

[148] *Discourse on Metaphysics* (*Phil*., IV, 438). Cf. the following passage: "Still, however strong this reason (though any may suffice for the greater inclination on one of two sides), although it establishes certainty in the mind, nevertheless it does not impose necessity on the thing, or destroy contingency, since the contrary nonetheless remains possible in itself and implies no contradiction; otherwise what we assume to be contingent would instead be necessary or an eternal truth" (*Phil*., VII, 301).

[149] Leibniz to Arnauld, 1686: "the divine understanding, which is, so to speak, the land of possible realities" (*Phil*., II, 55; cf. 42). In *On the Ultimate Origination of Things* (23 November 1697), God is called the region of ideas, the source of essences and existences (*Phil*., VII, 305). Cf. *Monadology*, §43.

[150] *Discourse on Metaphysics* (*Phil*., IV, 438). Cf. the summary of *Discourse on Metaphysics*, §13: "These truths, though certain, are nonetheless contingent, being based on the free will of God and of creatures. It is true that their choice always has its reasons, but these incline without necessitating (*Phil*., II, 12; cf. 56, and VII, 301). We know, in fact, that the free will of human beings is analogous to that of God and that it obeys, just like his, the principle of the best. The difference depends entirely on the understanding: God in his wisdom knows the best and realizes it infallibly, whereas human beings only realize what *appears to them* the best, and in this they are often mistaken (for to sin is to be mistaken). This explains the following words from the sentence quoted in the text: "what is or appears the best."

[151] *Discourse on Metaphysics* (1686), §13: "I assert that the connection or consequence is of two kinds. The one is absolutely necessary, whose contrary implies a contradiction; this kind of deduction holds in the case of eternal truths, such as those of geometry. The other is only necessary *ex hypothesi* and, so to speak, by accident; it is contingent in itself, since its contrary does not imply a contradiction. The connection is based not on pure ideas alone and on the simple understanding of God, but on his free decrees and on the series of the universe" (*Phil*., IV, 437). Cf. Leibniz to Coste, 19 December 1707 (*Phil*., III, 400).

[152] *Discourse on Metaphysics* (*Phil*., IV, 437).

[153] In the celebrated defense that ends the *Theodicy* (§§409ff).

[154] *On the Ultimate Origination of Things*, 1697: "For although the world is not metaphysically necessary, such that its contrary would imply a contradiction or logical absurdity, nevertheless it is necessary physically, that is, determined in such a way that its contrary would imply imperfection or moral absurdity" (*Phil*., VII, 304). *An Anagogical Essay*: "Geometrical determinations carry an absolute necessity, the contrary of which implies a contradiction; but architectonic determinations carry only a necessity of choice, the contrary of which implies an imperfection" (*Phil*., VII, 278). This is why Leibniz

22. Ultimately, it is God who is the "first" or "final reason of things,"[155] for he is their creator and providence; and thus it is that the principle of reason, purely logical in its origin, assumes a metaphysical and theological character. Likewise, applied to causality, it acquires a cosmological sense: every fact, being contingent, must have its reason for being in its physical cause, that is, in an earlier fact; but the latter is also contingent, and so on to infinity. We thus obtain a regressive infinite series (in the order of causality) of all contingent facts, none of which consequently contains the *reason* for the entire series. It is therefore necessary to seek this reason outside the series in a necessary being who supports and encompasses it as a whole in a simultaneous (or rather atemporal) act of understanding and will.[156]

We readily discover the relation between this theory of *causal* and *temporal* contingency and that of *logical* contingency which we have just expounded. It is that, for Leibniz as for Cartesians, the *cause* of a phenomenon is properly the logical ground of the truth of the proposition which asserts it, such that the relation of cause to effect is at bottom identical with the logical relation of ground to consequence. Consequently, to explain a fact amounts to analyzing the corresponding proposition and seeking the *reason* for it in another proposition of which it is the logical consequence; but when it is a question of a temporal event, this other proposition is the assertion of an earlier fact. The infinity of logical conditions or requisites coincides with the infinity of physical causes, that is, antecedent phenomena. Thus, the search for the cause of a phenomenon reduces to the logical analysis of a contingent truth; and both lead to an infinite regress of causes or logical conditions, whose very infinity takes the place of an explanation, for it results from an eternal law laid down by the creator.

But what we have said of truths of fact, singular propositions, is also true of natural laws, universal truths: these are contingent propositions which depend on the principle of reason and not on the principle of contradiction. They are neither arbitrary decrees of God nor necessary thoughts imposed by his understanding. On this point Leibniz is poised equally between Descartes and Spinoza, and adopts a happy middle position, namely that the laws of nature proceed from the divine will, that is, from the choice of the best.[157] This is why Leibniz maintains that the principles of mechanics (which are the

---

attributes only a "moral" certainty (a play of words) to truths of fact, which he opposes to metaphysical necessity. See Leibniz to Burnett (*Phil.*, III, 193, 259). This distinction has an absolute and objective import, not merely a subjective one: "However, though I believe that everything happens as the result of being determined by reasons, nevertheless I do not impose necessity on events but preserve their right to contingency. And I value highly those things which in fact lie between geometrical and physical truth, not only with respect to us, who are ignorant of causes, but also in the things themselves." Leibniz to Jacob Bernoulli, 2 December 1695 (*Math.*, III, 27).

[155] Leibniz to Bayle, 1687 (*Phil.*, III, 54); *A Specimen of Discoveries* (*Phil.*, VII, 310); *Theodicy*, §7; *Monadology*, §38; *Principles of Nature and of Grace*, §8.

[156] "For although a reason can always be given for a later state from an earlier one, nevertheless a reason can in turn be given for this state; and so we never arrive at the ultimate reason in the series. But this very progress into infinity supplies a reason, which in its own way, outside the series, in God the author of things, could immediately be understood from the beginning, and on which depend both earlier and later states and those which in turn depend on them" (*Phil.*, VII, 200). We may note that this is exactly the reasoning employed by Kant in order to demonstrate the thesis of the fourth antinomy.

[157] "Certain Cartesians think that the laws of nature were established by some mere act of will, for which no reason exists...; and others feel that they can be demonstrated from some geometrical necessity.

first laws of nature) have a metaphysical necessity, and not a geometrical (i.e. logical) necessity,[158] and why he wants to rehabilitate (against Bacon, Descartes and the mechanists) the use of final causes in physics.[159]

23. Before studying the application of the principle of reason to the physical sciences, however, it is appropriate to investigate in what sense it can be called the *principle of fitness* or the *principle of the best*, and in what exactly consists the finality that it causes to rule in nature. For this, it is necessary to return to the origin of this principle and to recall that it is meant to direct the choice God makes among the different possibles that cannot all coexist. In opposition to the Spinozistic principle "All that is possible exists," the principle of reason is formulated as follows: "All that is compossible exists."[160] For there must be a reason why something exists rather than nothing; therefore, existence is preferable to non-existence, all possibles must tend toward existence, and this in

---

Neither view is correct; for they indeed arise from reasons, though not of necessity, but of fitness or the best.... A middle position between these should be held, by distinguishing between necessary and contingent truths. Necessary truths, such as those of arithmetic, geometry and logic, are grounded in the divine intellect independent of will; and such is the necessity of three-dimensionality.... but contingent truths arise not merely from the will of God, but from considerations of the best or fittest, directed by the intellect." Leibniz to Bourguet, 11 April 1710 (*Phil*., III, 550). Cf. *An Anagogical Essay* (*Phil*., VII, 272). It is worth remarking that among the (logically) necessary truths, Leibniz does not hesitate to include the number of spatial dimensions; we know indeed that he took credit for demonstrating this, by appealing to the fact that one cannot draw more than three mutually perpendicular straight lines through a single point, which is to beg the question. Against the Cartesians who claimed that God could have given more or fewer dimensions to space (likewise, that he could have conferred existence on contradictory things), Leibniz maintained that three dimensions are necessary "from a blind or geometrical necessity." Leibniz to Coste, 8 July 1711 (*Phil*., III, 419). Cf. *Theodicy*, §351.

[158] *Antibarbarus Physicus*: "*Everything in nature indeed happens mechanically, but the principles of mechanism are metaphysical*; and the laws of nature and motion are not in fact established by an absolute necessity, but by the will of a wise cause, not by a mere fiat, but on account of the fitness of things" (*Phil*., VII, 343-4). *Discourse on Metaphysics*, 1686 (*Phil*., IV, 444). Leibniz to Arnauld, 14 July 1686: "I reduce all of mechanics to a single proposition of metaphysics" (*Phil*., II, 58, 62). Leibniz to De Volder, 1699: "And these indeed can only be demonstrated from the supreme law of order; for they are not of absolute necessity, such that the contrary implies a contradiction. The system of things could have been constituted in innumerable ways, but that one prevailed which displayed the greatest reason" (*Phil*., II, 147, 169; cf. 195). *Animadversions*, 1692 (*Phil*., IV, 391); Leibniz to Des Billettes, October 1697 (*Phil*. VII, 455). See the letters to Hartsoeker (1711), in which Leibniz employs the principle of reason (and its corollary the principle of continuity) in order to refute atomism (*Phil*., III, 519, 529, 532, etc.). Cf. *Letter Concerning the Question: Whether the Essence of the Body Consists in Extension*, June 1691 (*Phil.* , IV, 466); Leibniz to Father Bouvet, 1697 (Erdmann, 146); Leibniz to Johann Bernoulli, 6 May 1712 (*Math*., III, 884).

[159] "For I have found that it is necessary to have recourse to *final causes*, and that these laws [of motion] do not depend on the *principle of necessity*, as do the truths of logic, arithmetic and geometry, but on the *principle of fitness*, that is, on the choice of wisdom." *Principles of Nature and of Grace*, §11. See Leibniz to Philippi, 1679 (*Phil*., IV, 281; cf. 339, 344, 361, 447); and above all *An Anagogical Essay* (*Phil*., VII, 270-9), which Gerhardt (ibid., p. 252) places between 1690 and 1695, but which could not be earlier than 1696, since in it allusion is made to the problem of brachistochrone.

[160] "But my principle is that whatever can exist and is compatible with other things does exist, because the reason for existing in preference to all possibles ought not to be limited by any reason except that not all things are compatible. And so, there is no other reason for determination than that there exist the more perfect things, which involve the most reality." Fragment of 2 December 1676 (LH IV 8 Bl. 71). Leibniz applies this principle in the demonstration of the immortality of the soul.

proportion to their perfection, that is, to their degree of essence or reality.[161] Thus the essence of each thing is its reason for being, its claim or right to existence; and conversely, the existence of a thing is no more than the exigency of its essence, without which it would be inexplicable and unintelligible, for it would need a reason for being that could only be found in another essence.[162]

With this assumed, each possible must be realized, at least provided it is not incompatible with the other possibles that possess at least as much essence or perfection. The choice God makes among all the possibles is rigorously determined by his wisdom and goodness, and can be calculated mathematically. It can also be represented in a symbolic manner as a sort of "struggle for existence" of all possibles, each being endowed with a force proportional to its reality. However, in virtue of their mutual incompatibility, the elementary possibles form an infinite multitude of different combinations in which some exclude others. From all these individually possible combinations that one is infallibly realized which unites in it the greatest sum of essence; and in the particularly simple case in which all the possibles have the same reality, the combination realized is that which contains the greatest number.[163]

Thus, the struggle of all possibles leads, as it were automatically, to the triumph, that is, the realization of the combination that includes the most *compossibles*.[164] Such is the mathematical form of the principle of reason as the law of existences: the determination of the combination that must prevail is reduced to one of those problems of maximum and minimum for which Leibniz precisely had invented his infinitesimal calculus. It is a problem analogous to certain games in which we are to fill as many places as possible on a board according to a fixed law,[165] or again of covering a given space with tiles of a given shape in such a way as to cover the greatest possible surface, or to fit as many tiles as possible into the same area.[166] But this problem resembles most of all the general problem of the equilibrium of a system of weights; for we know that the stable

---

[161] *On the Ultimate Origination of Things*, 1697: "All things that are possible, or express essence or possible reality, tend by equal right toward existence in proportion to the quantity of essence or reality they include, or in proportion to the degree of perfection that belongs to them; for perfection is nothing but quantity of essence" (*Phil.*, VII, 303). The last proposition shows that Leibniz conceives of perfection less as qualitative than as quantitative, and thus renders it susceptible to mathematical evaluation. Cf. *Monadology*, §54.

[162] *On First Truths*: "Unless there is in the very nature of essence some inclination to exist, nothing would exist" (*Phil.*, VII, 194-5). "If existence were something other than the striving of essence, it would follow that this itself has some essence or that something new has been superadded to things, of which in turn it could be asked whether this essence exists, or why this one exists rather than others" (*Phil.*, VII, 195, note). With this, Leibniz responds in advance to the Kantian critique of the ontological argument.

[163] For example, let A, B, C, D be four equally perfect (therefore equally *possible*) possibles; and let us suppose that A, B, C are compatible with each other, but incompatible with D, whereas D is incompatible with A and B, and compatible with C alone. The combination that will be realized is assuredly ABC. For, if D existed, there could exist only the combination CD, which is less perfect than the combination ABC, being fewer in number (*Phil.*, VII, 194).

[164] "And just as possibility is the principle of essence, so perfection or degree of essence (through which there exists the most compossibles) is the principle of existence" (*Phil.*, VII, 304).

[165] Here Leibniz is probably alluding to the game of inverse solitaire that he had conceived (see §29 and Note XVII).

[166] A sort of jigsaw puzzle. If we have the choice of shapes, we can, as we know, completely cover a plane surface with squares, equilateral triangles, regular hexagons, equal rhombuses, etc. (the tiling problem).

equilibrium position of such a system, the one in which it consequently tends to stabilize itself, is that in which its center of gravity is as low as possible, that is, in which the sum of masses cannot descend any further.[167]

There is therefore a perfect analogy between mechanics and metaphysics; it is even more than an analogy, according to Leibniz, for it was the laws of motion which in his view suggested this theory to him, and they are in his eyes only a particular application of the metaphysical principle of the best. This is, according to him, no longer a mere comparison or an empty metaphor, for he holds that possible essences exist in the divine understanding as truly as created things, and that they there struggle according to laws analogous to those of mechanical equilibrium.[168] This is the "divine mathematics" and "metaphysical mechanism" by which the wisdom (even more than the goodness) of the creator is exercised and made manifest.[169]

24. But in order to know how this divine mathematics is realized in nature and how we can discover it there, it is appropriate to study the special axioms by which the principle of reason is expressed and applied in physics.

The first form in which Leibniz employs the principle of reason, or the first corollary he draws from it, is what we may call the *principle of symmetry*. It is in fact in this form that he makes use of it in the *Theory of Abstract Motion* (1670).[170] In order to show that the principle of reason is necessary from the outset in mechanics, Leibniz cites a particular case of it that makes its role and importance clearly understood: this is the *axiom of equilibrium*, or axiom of the balance, invoked by Archimedes, the author of the book *On Equal Weights*. This axiom consists in allowing that if two weights are similarly positioned in relation to the axis of a balance, with everything else being symmetrical, the latter remains in equilibrium; and this is justified by the fact that there is no reason for the balance to incline to one side rather than the other, everything being symmetrical by hypothesis on both sides of the axis of suspension.[171]

---

[167] "[J]ust as in ordinary mechanics itself, when several heavy bodies are operating against one another, the result is that movement which secures the greatest total descent. For just as all things that are possible tend with equal right toward existence in proportion to their reality, so in the same way all weights tend with equal right toward descent in proportion to their heaviness; and just as in the latter case there results a motion involving the greatest possible descent of heavy bodies, so in the former case there results a world involving the greatest production of possibles" (*Phil*., VII, 304).

[168] See a curious fragment in which Leibniz aims to prove that possibles exist in the divine understanding through the consideration of a compressed liquid that tends to escape on all sides and "chooses" the easiest way: this choice in effect presupposes the presence of all the other possibles in the form of tendencies (Bodemann, 74).

[169] "From this it is now wonderfully clear how in the very origination of things a certain divine mathematics or metaphysical mechanism is employed, and how a determination of the maximum holds good" (*Phil*., VII, 304). We will see later that it is less a question of finding the maximum or minimum than the most *determined*. This is the place to recall, and to complete, the sentence that serves as an epigraph to our work: "When God calculates and exercises thought, he creates the world" (*Phil*., VII, 191, note).

[170] *Phil*., IV, 232. (At issue is the collision of two bodies of equal mass.)

[171] *Phil*., VII, 301; cf. 309, and Second Letter to Clarke (1715), *Phil*., VII, 356; LH IV 7C Bl. 62-3; LH IV 8 Bl. 2 recto, Bl. 6 verso. Here is how Leibniz formulates the principle of symmetry in the *General Investigations* of 1686, §136: "If all things are alike on each side in our hypotheses, there can be no difference in the conclusions" (LH IV 7C Bl. 29). For comparisons to the formula of the principle of continuity, see nn. 194 and 195. The principle of symmetry is invoked at the beginning of *On the Estimation of Uncertainty*, 1678 (LH XXXV 3A Bl. 12), a sketch of a theory of probabilities.

We see why this form of the *principle of reason* is called the *principle of symmetry*: it will be applicable wherever there is a symmetry or parity of form. Thus, this principle finds its application in algebra, or rather in the combinatory or theory of forms; it then gives rise to what Leibniz metaphorically calls the *law of justice* and what modern mathematicians call the principle of symmetry.[172]

The principle of symmetry has a great affinity with the famous *principle of indiscernibles*, which also derives, according to Leibniz, from the principle of reason. The latter affirms that there cannot exist in nature two absolutely similar (concrete and individual) things, which differ only in position, or, as is said, in number: for there must be a reason why they would be different, or why they would be two.[173] This principle itself is related to another, which has not received a special name, stated as follows: There can be no purely extrinsic denominations that have no foundation in the denominated thing; and this because the notion of the subject must include those of all its predicates.[174] Thus we see it is always the logical form of the principle of reason that yields these derived axioms.

25. Yet the preceding are only negative forms of the principle of reason; another more positive and more general, and consequently more fertile, corollary is the *principle of the simplicity of the laws of nature*, or again, the *principle of economy*.[175] "For example, this maxim, that nature acts by the shortest ways, or at least by the most determinate, suffices by itself to explain nearly all of optics, catoptrics and dioptrics, that is, everything which occurs outside of us in the actions of light, as I once showed and as Mr. Molyneux strongly agreed in his *Dioptrics*, which is a very good book."[176]

---

172 *Universal Mathesis*: "the principle of similarity or of the same relation" (*Math*., VII, 66). Leibniz explicitly connects it to the principle of reason in *An Anagogical Essay* (*Phil*., VII, 278), and to the *principle of continuity* in the *Metaphysical Foundations of Mathematics*, 1714 (*Math*., VII, 25). We may note that it is by means of the principle of similarity, or the principle of the same reason, that Leibniz explains the infinite (*New Essays*, II.xvii.4), and conversely, he says of the principle of continuity that "it has its origin in the infinite" (Leibniz to Bayle, 1687; *Phil*., II, 52).

173 "From this it follows also that *there cannot be in nature two individual things that differ in number alone*. For it must always be possible to give a reason why they are diverse, which must be sought from some difference in them" (LH IV 8 Bl. 6 recto). We are familiar with the anecdote related in the *New Essays* (II.xxvii.3): "I recall a great princess of lofty intelligence saying one day while walking in her garden that she believed there were no two leaves perfectly alike. A clever gentleman who was along on the walk believed that it would be easy to find them; however after searching far and wide he was convinced by his eyes that some difference could always be recognized between them." The "great princess" was the Electress Sophie, the "clever gentleman," d'Alvensleben, and the garden that of the palace at Herrenhausen (Leibniz to the Electress Sophie, 31 October 1705, *Phil*., VII, 559). The anecdote is all the more pointed as it is a French garden, modelled on Versailles, which everywhere exhibits regularity and geometrical symmetry. It served as the subject for an engraving in the work by Eberhard, *Characteristik des Freiherrn von Leibnitz* (Leipzig: Pantheon der Deutschen, 1795), p. 150.

174 "It also follows that *there are no purely extrinsic denominations*, which have no foundation in the thing denominated. For the notion of the subject denominated must involve the notion of the predicate" (LH IV 8 Bl. 6 verso).

175 "There is always to be found in things a principle of determination that turns on considerations of greatest and least; namely, that the greatest effect should be produced with (if I may put it so) the least expenditure." *On the Ultimate Origination of Things* (*Phil*., VII, 303).

176 *New Essays*, IV.vii.15.

Leibniz here alludes to the note on optics that he had published in 1682 in the *Acta Eruditorum*,[177] in which he had deduced all the laws of reflection and refraction from this single principle: "light tends from the radiating point to the illuminated point by the easiest of all paths."[178] Thus we can ignore the efficient cause of reflection and refraction, and still calculate exactly all their effects by means of the final cause, that is, by supposing that light always follows the easiest path;[179] and so is it false that the search for final causes is useless in physics, as Descartes had maintained. This example of the laws of optics is consequently Leibniz's favorite argument in his struggle against the Cartesians. We see by this how he understood finality in nature: it is in this sense that God (or nature) always acts "in the easiest and most determinate ways."[180] This finality consists less in goodness or moral perfection (as one might believe from the theological formulations of the principle of reason) than in the *logical determination* of the laws of nature. This is what stands out in the *Anagogical Essay*, which is precisely intended to show the usefulness of the search for final causes in physics:

"What seems to me most beautiful in this regard is that this principle of perfection, instead of being limited only to the general, descends also to particular things and phenomena; and this is almost like the method *of optimal forms*, that is, of the *maximum or minimum in perfections*, which we have introduced in geometry in place of the old method of the *maximum and minimum in quantities*. For this best of all forms or shapes is found not only in the whole, but also in each part, and it would not even be in the whole if this were not so. For example, if in the line of the shortest descent between two given points, we take two other points at will, the part of the line between them is still necessarily the line of shortest descent with respect to them. It is in this way that the least parts of the universe are governed according to the order of greatest perfection; otherwise the whole would not be."[181]

This passage is interesting, first, for what it reveals of the theologico-mathematical origin of Leibnizian optimism; and second, because it shows that this *best* or *perfection* again consists, in the end, in a quantitative maximum or minimum; for the forms we call

[177] *The Unique Principle of Optics, Catoptrics and Dioptrics* (Dutens, III, 145). Cf. *An Anagogical Essay* (*Phil*., VII, 275-8).

[178] We know that the law of refraction (law of sines) was discovered by the Dutchman Snell, and then Fermat and Descartes. Leibniz blames Descartes for not having made use of the consideration of finality, like Snell and Fermat (*Phil*., IV, 318-9, 448; VII, 274; cf. LH XXXV 1, 27 c). Fermat indeed accepted that the speed of light in a medium is inversely proportional to the index of refraction of the medium, from which it followed that the refracted light ray follows the path that is shortest in length; but Descartes could not admit this hypothesis (any more than Newton), because in the theory of emission the speed must be, on the contrary, greater in a denser (more refractive) medium. The law of finality therefore made no sense to him. Leibniz rediscovered this finality by accepting that media offer a *resistance* proportional to their index of refraction, such that the speed of the light is in an inverse ratio to this resistance, and consequently to the index.

[179] *Phil*., IV, 340 (the same reference to Molyneux). Cf. 361, 448; VII, 273; and Leibniz to Foucher, 1693 (*Phil*., I, 414).

[180] *Phil*., IV, 447. Cf. the fragment LH IV 6, 12 f Bl. 15: "Everything in the whole of nature can be demonstrated both through final causes and through efficient causes. Nature does nothing in vain; nature acts via the shortest paths, provided they are regular" (Bodemann, 89). Malebranche had already expressed a rather similar maxim, namely that God always acts by the simplest and shortest ways (*Search after Truth*, Clarification to Chap. 2 of the Second Part on Method). But between this principle and that of Leibniz, there is all the distance that separates a theological aphorism and a mathematical axiom.

[181] *Phil*., VII, 272-3.

"the best" are those which "provide a maximum or a minimum." This is what is indicated by the example of the *principle of least action*, whose discovery was long attributed to Maupertuis, but which had in reality been discovered by Leibniz.[182] In fact, Leibniz had formulated this principle as follows: In a free motion, the action of the moving body is "ordinarily a *maximum* or a *minimum*." Now whether it is a maximum or a minimum, the case is the same from the point of view of mathematics; however, it is hardly a matter of indifference from the point of view of ends and theology, for we could then no longer speak of the wisdom and "economy" of the creator, since he would sometimes dispense the maximum in place of the minimum.[183] But what truly proves that this maximum or minimum has no qualitative or moral character is that Leibniz later invokes "another principle, which succeeds the former,[184] and which holds that in the absence of the least it is necessary to keep to the *most determinate*, which could be the *simplest* when it is the greatest.[185] Already, in a kind of summary which precedes the *Essay*,[186] Leibniz had announced that he would show "how in the way of ends, as in the calculus of differences,[187] we consider not only the greatest or the smallest, but in general the most determinate or the simplest."[188] And what indeed does Leibniz find "most beautiful" in the consideration of final causes or "optimal forms"? It is that these forms are the most determinate, and that they are absolutely determined, not only overall, but down to their smallest parts. The *perfection* that Leibniz attributes to them and that he assigns as an aim to nature or the creator is therefore a purely logical perfection, which is intellectual and not moral: it is in a word *determination*; and this is why he sometimes called his great principle the *principle of determinate reason*. In any case, if this principle does prove the existence of God, as he believes it does,[189] it is as an "intelligent cause" rather than as a benevolent and beneficent cause.

Thus, in certain applications, Leibniz returns almost to the principle of symmetry defined above. For example, "if we suppose the case in which nature was required in general to construct a triangle, and that for this effect only the perimeter or sum of the sides was given and nothing more, it would construct an equilateral triangle."[190] This is in fact the most symmetric form of triangle and it is also the one that has the greatest area for a given perimeter. But it is above all the most determinate, and consequently it will

[182] See Note XVI: "On the Principle of Least Action."

[183] It is precisely here that Maupertuis was later mistaken in affirming that the action is always a minimum, an assertion from which he drew an argument in favor of providence.

[184] That is, which replaces it, or is a *successor* of it.

[185] *Phil.*, VII, 274

[186] This summary was later added to the title, which it encircles in the manuscript; the text of the *Essay* begins with the sentence: "I have noted on several occasions..." (LH XXXV 7, 5; cf. *Phil.*, VII, 270).

[187] We may note this new comparison between the consideration of finality and the differential calculus, which is the true source and explanation of it.

[188] *Phil.*, VII, 270; and some lines earlier: "in the investigation of ends there are cases in which it is necessary to have regard for the simpler or more determined, without distinguishing whether it is the greater or the smaller." The next sentence shows that this key idea derives from the "calculus of differences." We know indeed that it is the same method which gives both maxima and minima, and that it is necessary to take care to distinguish the two, seeing that they are in general "determined" by the same equation and that they have the common characteristic of being, as Leibniz terms it, "unique" or "singular" (*Phil.*, VII, 275).

[189] *Phil.*, VII, 301, 303, 310. *Principles of Nature and of Grace* (1714), §11 (*Phil.*, VI, 603).

[190] *Phil.*, VII, 278; cf. 304.

inevitably be preferred by the wise man, who does nothing without a reason.[191] Moreover, it is not only the wise man, but Buridan's ass who does nothing without a determinate reason, since the liberty of indifference is a chimera.[192]

26. Another corollary of the principle of reason is the celebrated *principle of continuity*, which plays so great a role in Leibniz's metaphysics and in his polemic against the Cartesians. He formulates it for the first time in 1687 as follows:[193]

---

[191] "For this reason, too, determinate things are preferred to indeterminate things, in which no reason for a choice can be discerned. Thus, if a wise man were to decide to mark out three points in some space, and there were no reason for one sort of triangle rather than another, he would choose an equilateral triangle, in which the three points are in similar relations. *A Specimen of Discoveries* (*Phil*., VII, 310 note). Cf. LH IV 4, 3 c Bl. 17 (Bodemann, 73).

[192] *Phil*., VII, 111.

[193] It is interesting to recall the occasion. In 1686 Leibniz had published in the *Acta Eruditorum* of Leipzig his *Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law, According to Which God is Said Always to Conserve the Same Quantity of Motion, a Law Which They Also Misuse in Mechanics*, in which he maintained that what is conserved in the collision of two bodies is not the quantity of motion (mv), but the quantity of living force (mv^2). See the same criticism in the *Discourse on Metaphysics* of 1686, §17 (*Phil*., IV, 442-3). The Abbé Catalan, a Cartesian and friend of Malebranche, translated this note and responded to it in the *Nouvelles de la République des Lettres* (September 1686), under the title: *Brief Remark of M. l'Abbé C., in Which He Shows M. G.G.L. the Fallacy Contained in the Preceding Objection* (*Phil*., III, 40-2). Leibniz responded in the same journal (see Leibniz to Bayle, *Phil*., III, 42-9), while implicating "the celebrated author of the *Search after Truth*" (Malebranche, whom he had known in Paris and with whom he had remained in correspondence), who attempted to correct the Cartesian laws of collision but who had not entirely succeeded. Until this point, he had not mentioned the principle of continuity; the only metaphysical principle invoked by Leibniz was that "*law of nature*" held to be "the most universal and most inviolable, namely that *there is always a perfect equation between the full cause and the entire effect*" (*Phil*., III, 45-6). When Malebranche responded to Leibniz's criticisms in the *Nouvelles* of 1687, the latter perceived that Malebranche's rules, like those of Descartes, sinned against the principle of continuity, as he wrote to Arnauld; and he added: "If I reply to M. Malebranche, it will principally be to make known the aforesaid principle" (letter of 1 August 1687, *Phil*., II, 104-5; cf. Leibniz to Arnauld, 14 January 1688, *Phil*., II, 133-4; and Leibniz to Foucher, January 1692, *Phil*., I, 403). This is in fact what he did in *Letter from M. L. Concerning a General Principle Useful for Explaining the Laws of Nature by the Consideration of Divine Wisdom, for the Purpose of Serving as a Reply to the Response of the Rev. Father Malebranche* (*Phil*., III, 51-5), which we quote here. Malebranche was forced to take account of Leibniz's criticisms in his *Traité des Loix de la Communication des Mouvemens* (1692); but there still remained certain errors in it. Leibniz called attention to them in some remarks that he communicated to Malebranche (see Leibniz to Malebranche, 8 December 1692, *Phil*., I, 343), in which, after having said, "I intend to make here some *a posteriori* remarks by employing *my principle of harmony or fitness* which I explained in the *Nouvelles de la République des Lettres*," he invokes "the great rule of order that holds that *with the given things ordered, the sought things are also ordered and in agreement*" (*Phil*., I, 346-7), and he ends by saying that he has made use of it "in order to prove certain rules or theorems *a posteriori*," from "principles of the *real logic* or from a certain *general analysis* that is independent of algebra" (ibid., 349), which clearly shows that the principle of continuity is related in his thought to his general logic.

In his response to Malebranche, Leibniz indicates that "one could render sensible" the "law of continuity," "by a representation, such as" he has "done in certain remarks on a part of the *Principles* of M. Descartes" (*Phil*., I, 350). And indeed, in the *Animadversions Against the General Part of Descartes's Principles* (1692), on article 53 of Part II, we find under the title "Representation of the Rules of Motion in the Case of Equal Colliding Bodies," two diagrams that truly speak to the eyes and represent the laws of collision: 1) "According to Descartes (Unnatural Representation)"; 2) "According to the Truth (Elegant Representation)"; the latter offers perfectly continuous lines, whereas the former presents bizarre and striking discontinuities (*Phil*., IV, 382-3). These remarks were in the hands of Basnage de Beauval for five

"When the difference between two instances can be diminished below any given magnitude in the given things or in what is assumed, it is necessary that it could also be diminished below any given magnitude in the sought things or in what results from the givens; or, to speak in more familiar terms: When the instances (or what is given) approach and are finally lost in one other, it is necessary that the consequences or outcomes (or what is sought) do so also.[194] And this depends on a still more general principle, namely: *With the given things ordered, the things sought are also ordered*."[195]

Thus Leibniz himself presents the principle of continuity as a simple corollary of what he earlier calls the *principle of general order*, of which he says: "It has its origin in the *infinite*; it is absolutely necessary in geometry,[196] but it also succeeds in physics, for

---

years (1692-7) and in the end remained unpublished (*Phil*., IV, 271-2); but other articles by Leibniz on mechanics (notably the *Specimen of Dynamics for the Purpose of Revealing the Remarkable Laws of Nature Concerning the Forces of Bodies and Their Mutual Action and Reducing Them to Their Causes*, published in the *Acta Eruditorum* of 1695: *Math*., VI, 249ff.; cf. *Phil*., IV, 399, a fragment dating from May 1702) eventually convinced Malebranche, who in 1698 corrected his *Traité des Loix de la Communication des mouvemens* (*Phil*., I, 319).

On the other hand, Leibniz had briefly replied to the Abbé Catelan (*Response of M. L. to the Remark of the Abbé D. C. Contained in Article 1 of this Journal, the Month of June 1687, in Which He Claims to Uphold a Law of Nature Advanced by M. Descartes*, *Phil*., III, 49), and he ended this response by proposing the problem of the *isochronic* curve ("To find a line of descent along which a heavy body descends uniformly and equally approaches the horizon in equal times"), while adding this ironic challenge: "The analysis of the Cartesians will perhaps easily provide it" (*Phil*., III, 51). With this, Leibniz wished to show the superiority of his analysis over that of Descartes (see his letter to Malebranche of 13 January 1679, *Phil*., I, 327-8), and he in fact succeeds; but the principle of continuity was precisely the foundation of the higher analysis that he had invented. Thus is explained the claim, at first sight paradoxical and excessive, to prove the superiority of his philosophy (and above all of his logic) over that of Descartes by the solution of a mathematical problem, and to overcome Cartesianism with the aid of the infinitesimal calculus. "I have learned that the success of my other discoveries has removed from some the desire of making objections to me, since one is obliged to admit that even in mathematics, which was the stronghold of M. Descartes, my method goes far beyond his: this is what the Marquis de l'Hospital has just recognized in a notable work recently published." Leibniz to the Electress Sophie 4 November 1696 (*Phil*., VII, 542). The allusion is to the *Analyse des Infiniment Petits pour l'Intelligence des Lignes Courbes* (Paris, 1696) by his disciple and friend L'Hospital, the defender and propagator of the infinitesimal calculus in France. Cf. Leibniz to Duke Johann Friedrich, 1679 (Klopp, IV, 440); Leibniz to Nicaise, 5 June 1692 (*Phil*., II, 535); Leibniz to Bayle, 1687 (*Phil*., III, 49); Leibniz to Arnauld, 14 July 1686 (*Phil*., II, 62); Leibniz to Burnett, 1697 (*Phil*., III, 195). See *Phil*., IV, 276, 282, 291, 301-2, 347.

[194] We will note the wholly mathematical character of this statement, which rests on the distinction between givens and unknowns familiar in algebraic problems. It is obvious that this principle was suggested to Leibniz by his work on the infinitesimal calculus, whose first postulate is that one employs functions that are *continuous* and have derivatives, such that the increment of the function tends toward zero with the increment of the variable. This formula is analogous to that of the principle of symmetry quoted above: in one case, the difference between the givens is strictly zero; in the other, it is infinitely small (see n. 171).

[195] This last formula is found in an unpublished fragment (LH XXXV, 1, 9 b) entitled *Combinatory*, which shows that Leibniz connects the principle of continuity with the combinatory, that is, the science of order (see Chap. VII, §3). Elsewhere Leibniz gives an exactly similar formulation of the principle of symmetry (or "law of justice"): "so that when things are related in the same way in the givens or suppositions, they are also related in the same way in the things which are sought or produced..., and generally it should be judged that with the given things appearing in an orderly manner, the sought things also appear in an orderly manner"; and soon after he states the *law of continuity* (*Metaphysical Foundations of Mathematics*, *Math*., VII, 25).

[196] Leibniz thus immediately gives a geometrical example of the principle, namely that of the ellipse which tends toward a parabola when one of its foci is moved indefinitely away from the other: the ellipse

the sovereign wisdom who is the source of all things acts as a perfect geometer,[197] according to a harmony to which nothing can be added.[198]

Now this harmony, agreement and perfection consist essentially in an intelligible order that reconciles the simplicity of principles with the richness and variety of consequences:

"God has chosen that one [of the possible worlds] which is the most perfect, that is, which is at the same time the simplest in hypotheses and the richest in phenomena, just as might be a geometrical line whose construction were easy and whose properties and effects were extremely remarkable and of wide reach."[199]

Thus the perfection of the world has (at least in its source) a purely rational and intellectual character: God is conceived less as a providence full of justice and goodness than as the "perfect geometer."[200] Perhaps this wholly intellectual and mathematical origin of Leibnizian optimism is itself capable of explaining how its consequences can offend moral sentiment. It is because Leibniz's God is above all the great calculator and eternal logician.[201]

27. Such are the main forms of the principle of reason that Leibniz sometimes calls the "law of supreme order,"[202] just as he considers the "laws of general order" to be the "primitive free decrees," or again the "principal designs or ends of God."[203] Without a doubt, Leibniz often opposes this metaphysical principle to mathematical principles, and

---

being able to differ from the parabola by as little as one chooses, "all the geometrical theorems which are in general confirmed of the ellipse could also be applied to the parabola, by considering the latter as an ellipse one of whose foci is infinitely far away, or (in order to avoid this expression) as a figure which differs from an ellipse by less than any given difference."

[197] See *A Specimen of Discoveries*, in which after having said that God wills the best and chooses the world which contains the most reality or perfection Leibniz adds: "And God acts like the greatest geometer, who prefers the best constructions of problems" (*Phil*., VII, 310 note); Leibniz to Arnauld, 1 August 1687: "And it is strange to see that nearly all M. Descartes's rules of motion violate this principle, which I hold to be as infallible in physics as in geometry, because the author of things acts as perfect geometer" (*Phil*., II, 105). Cf. *Animadversions Against the General Part of Descartes's Principles* (1692), on Article 45 of Part II, in which "the law of continuity" is presented as a "general criterion" and as a "touchstone" (*lapis Lydius*) appropriate for testing the Cartesian laws of motion (*Phil*., IV, 375-6; cf. 399, and VII, 279) and the end of the *Response to the Reflections Contained in the Second Edition of the Critical Dictionary of M. Bayle, Article Rorarius, on the System of Preestablished Harmony*, 1702), quoted in §27 (*Phil*., IV, 571).

[198] *Phil*., III, 52.

[199] *Discourse on Metaphysics*, 1686, §6 (*Phil*., IV, 431).

[200] *Animadversions*, 1692: "Nature, whose wisest author employs the most perfect geometry..." (*Phil*., IV, 375).

[201] The "architect of nature," May 1702 (*Phil*., IV, 499). At the beginning of *History and Praise of the Characteristic Language*, Leibniz recalls the Biblical verse: "It is an old dictum that God has made all things according to weight, measure and number," while emphasizing the universal significance of number (*Phil*., VII, 184, quoted p. 111, note 5).

[202] At least with regard to the laws of motion, he says: "And indeed they can only be demonstrated from the law of supreme order, for they are not of absolute necessity, such that their contrary implies a contradiction. The system of things could have been constituted in innumerable ways, but that prevailed which depended on the greater reason." Leibniz to De Volder, March-April 1699 (*Phil*., II, 147, 169).

[203] Leibniz to Arnauld, 14 July 1686 (*Phil*., II, 51). Cf. Leibniz to Hartsoeker of 7 December 1711, in which the principle of "sufficient reason," or determining reason, is related to the principle of order and the principle of continuity (*Phil*., III, 529-30).

these “architectonic reasons” to “geometrical determinations.”[204] However, it is necessary to give an exact account of what he means when he judges mathematical and mechanical laws insufficient for explaining the universe. Metaphysical principles are neither opposed to nor combined with mathematical laws: they are superimposed on them. The laws of mechanics suffice to explain all the phenomena of nature; but in order to explain these laws themselves we are obliged to call on metaphysical principles.[205]

Leibniz seems, it is true, to subordinate the geometrical and mathematical conception of the world and to declare it incomplete or insufficient, when he writes: “Bayle is right to say with the ancients that God makes use of geometry, and that mathematics forms part of the intellectual world and is best suited for giving entrance to it. But I myself believe that its interior is something more.” In what follows, however, we read what this “something more” is: “I have suggested elsewhere that there is a more important calculus than those of arithmetic and geometry which depends on the analysis of ideas. This would be a universal characteristic, whose formation seems to me one of the most important things we could undertake.”[206]

At first glance, this sentence would appear to be unconnected with the preceding one and would seem rather to contradict it; but the succession of ideas is easily explained as follows (and cannot, we believe, be explained otherwise). If Leibniz claims that mathematics is insufficient for penetrating to the interior of the intelligible world, this is insofar as it is applicable only to number and magnitude, that is, to objects of the imagination. However, he discovered that the mathematical method can also be applied to objects of the pure understanding, that is, to abstract metaphysical objects: this is the true logic, the calculus he attributes to God himself, which must serve to explain the universe, since it has served to create it. It is in this sense only that the world exceeds the reach of ordinary mathematics but falls within the grasp of a more sublime mathematics, which is precisely the universal characteristic. It is not necessary to believe, therefore, that in this passage Leibniz renounces his intellectualistic convictions; on the contrary, he affirms there more energetically than ever the perfect and absolute intelligibility of the universe.

28. But if creation is reduced, as we have seen, to a problem of mechanics, that is, at bottom, of analysis, then divine mathematics becomes accessible to human beings and we can, imitating the eternal geometer, determine and calculate for ourselves what are the “best” combinations, those that must be realized. For this, it suffices to employ two methods. The first is the combinatory, which will show us how to form all the imaginable combinations of the different possibles and to discern those which are truly possible, that is, composed of compossible elements; for these are the only “useful” combinations, that is, the only ones that must be taken into account.[207] The second is the calculus of probabilities, which will allow us, knowing the probability of the elementary possibles, to

---

[204] *An Anagogical Essay* (*Phil.*, VII, 278-9).

[205] See *Discourse on Metaphysics* (1686), §17 (*Phil.*, IV, 444); *Animadversions*, on Article 64 of Part II of Descartes’s *Principles*, 1692 (*Phil.*, IV, 390-1); *Principles of Nature and of Grace* (1714), §11; Leibniz to Remond, 10 January 1714 (*Phil.*, III, 606). Cf. *Antibarbarus physicus* (passage quoted in n. 158).

[206] *Response to the Reflections... of M. Bayle* (1702), end (*Phil.*, IV, 571).

[207] *On Universal Synthesis and Analysis* (*Phil.*, VII, 293). It is true that this “would often be like drinking the ocean”; but where synthesis, that is, the art of forming combinations, is inadequate, one can employ analysis (to be defined later), which “gives us a thread through the labyrinth” and furnishes some “short-cuts” (*New Essays*, IV.ii.7).

calculate the probability of each combination. With this done, we then will have only to determine (by means of the infinitesimal calculus if necessary) the combination of greatest probability in order to know with certainty that it is the one that will be realized. Such is the plan of the logic of probabilities that Leibniz means to establish. This new logic, analogous to mechanics, will be the science of the real, as the old logic, analogous to geometry, is the science of possibles. Whereas the latter rests on the principle of contradiction, the law of essences, and is the science of eternal and necessary truths (the "method of certainty"), the former will be the science of temporal and contingent truths, for truths of fact can be for us only probable.[208] The new logic will undoubtedly be much more difficult and complicated than the old, but it will also be much more useful, since it will be applied to reality and to practical questions that bear on (moral, political and social) realities, and it will allow us either to preview the future as if we had assisted at God's councils and overheard the secret of creation, or to guide our conduct safely in all circumstances.

The idea of a logic of probabilities had been suggested to Leibniz early on by his legal and theological studies.[209] He often cited with praise the subtle distinctions that jurists had established between "degrees of proofs."[210] He had already sketched a theory of probability in *On Conditions* (1665),[211] and he began a work entitled *Toward the Balance of Law, on the Degrees of Proofs and Probabilities*, of which only a fine unpublished preface remains to us, in which he proposes jurists as models of logic in contingent questions.[212] Later, when he had become a mathematician, his first idea was sharpened and confirmed in his mind by study of the works of Fermat, Pascal,[213] and Huygens[214] on games of chance, and of those of Hudde[215] and Johan de Witt, grand

[208] With this, Leibniz takes credit for explaining how physical necessity arises from metaphysical necessity (*Phil.,* VII, 304), or "how from eternal, essential, or metaphysical truths there arise temporal, contingent or physical truths" (*Phil*., VIII, 303).

[209] On learning that Jacob Bernoulli was preparing his *Ars conjectandi*, he wrote: "I, too, have already some time ago thought of such things, especially in the practice of jurisprudence and politics. I call it the doctrine of the degrees of probability." Leibniz to Johann Bernoulli, 5 March 1697 (*Math*., III, 377). Cf. Leibniz to Gabriel Wagner, 1696 (*Phil*., VII, 521).

[210] See the enumeration of different degrees of juridical proofs and evidence in *New Essays*, IV.xvi.9. Cf. Leibniz to Johann Bernoulli, 6 June 1710: "I have already reflected on this matter from my youth, and especially now since I busy myself with the law and deliberate about conjectures, evidence, presumptions, and degrees of proof that are less than full, half-full and full, and other similar matters. For no one has refined this subject more than lawyers themselves; but they do not refer enough to certain principles or method (*Math*., III, 850). Cf. Leibniz to Burnett, 1697 (*Phil*., III, 193); Leibniz to Koch, 2 September 1708 (*Phil*., VII, 477); Leibniz to Eler, 10 May 1716 (Note XVIII).

[211] See Note V; cf. *Phil*., VII, 198.

[212] "Just as mathematicians have best employed logic, that is, the art of reason, in necessary matters, so lawyers have employed it in contingent matters in advance of other men" (LH IV 6 Bl. 17). Cf. *Phil*., VII, 167; p. 93, note 3, and p. 277, note 1.

[213] Leibniz had learned in Paris that the Chevalier de Méré had led Pascal to occupy himself with these problems (*Phil*., III, 570; *New Essays*, IV.xvi.9). Leibniz's attention seems to have been attracted to them by the Duke of Roannez, friend of Pascal, as several unpublished fragments indicate, notably *On the Number of Throws in Dice*, January 1676; with this note: "proposed to me by the Duke of Roannez" (LH XXXV 3B, 14).

[214] *On Reasoning in the Game of Dice*, 14 pp. (1657). Cf. *Leibnitiana*, §113: "Christian *Huygens's* reasonings *on the game of dice*... are an elegant example of reasoning about degrees of probability" (Dutens, VI.1, 318).

pensionary of Holland,[216] on life annuities.[217] In September 1678, he wrote an (unpublished) note, *On the Estimation of Uncertainty*,[218] in which he laid out the principles of the calculus of probabilities, and he soon applied these principles to various contemporary games of chance.[219] In the plan for a new *On the Art of Combinations* (around 1680), he proposed to treat "of the various kinds of games." He even boasted of having led Jacob Bernoulli to cultivate this science,[220] though in this he was mistaken.[221]

29. Leibniz, moreover, does not seem to have suggested to Bernoulli the idea of applying the calculus of probabilities to games of chance, for the latter informed him that he had already applied his "art of conjecturing" to certain games of chance (like dice) and also to ball games,[222] but not to card games and checkers, which he found too complicated.[223] Leibniz responded to him that whether in games of reason (like chess and checkers) or in games of some chance (like cards), or in games of pure chance (like dice), one must always be able to determine the most favorable course or else calculate exactly the value

---

[215] In an unpublished fragment dating from 1680, Leibniz, in sketching the plan for a new *On the Art of Combinations*, noted the investigations of Hudde on life annuities, based on mortality tables for the city of Amsterdam over 80 years (LH XXXV 1, 27 c). He thus related the logic of probabilities to the combinatory.

[216] *Waerdye van lyfrenten nar proportie van los-renten* (*Evaluation of Life Annuities in Proportion to Ordinary Annuities*), published in 1671, reedited in 1879. See the note by Leibniz *On Incomes for Life* (*Math*., VII, 133-7), which must date from around 1682, for it is announced in *Juridico-mathematical Meditation on Simple Interest*, published in the *Acta Eruditorum* of 1682 (*Math*., VII, 132).

[217] *Rules for Advancing the Sciences* (*Phil*., VII, 167); Leibniz to Arnauld, 14 January 1688 (*Phil*., II, 134); Leibniz to Johann Bernoulli, 6 September 1709 (*Math*., III, 844); Leibniz from Johann Bernoulli, 16 February 1697 (*Math*., III, 367); *Response to the Reflections of M. Bayle*, 1702 (*Phil*., IV, 570); *New Essays*, IV.xvi.8; Leibniz to Bourguet, 22 March 1714 (*Phil*., III, 570).

[218] LH XXXV 3A, 12.

[219] *On the Game of Quinquenove*, October 1678; *On the Game of Bassette*; *On the Game of Ombre* (LH XXXV 3A, 8, 9, 11). Basette was an Italian card game, similar to lansquenet, which had been imported into France in 1678 by a Venetian ambassador. Joseph Sauveur, a French mathematician (1653-1716), formulated a theory of it, as well as of other card games, at the request of the famous courtesan Dangeau (see Fontenelle, *Éloge de Sauveur*). He published an article on bassette in the *Journal des Savants* of 13 February 1679. It is in connection with Sauveur that Leibniz was later led to speak to Johann Bernoulli about the theory of games (Leibniz to Johann Bernoulli, 19 January 29, 5 March 1697; *Math*., III, 363, 377).

[220] Leibniz to Bourguet, 22 March 1714 (*Phil*., III, 570).

[221] Johann Bernoulli in fact responded to Leibniz that his brother Jacob had for some years been preparing an "art of conjecturing," of which he had completed the greater part and was lacking only the applications to moral, political, and economic problems, constituting the fourth and final chapter (Leibniz from Johann Bernoulli, 16 February 1697; Math., III, 367). Jacob Bernoulli in fact worked for more than 20 years on his *Ars conjectandi*, which he left unfinished at his death (1705), and which was only published in 1713 by his nephew Nicolas, who had himself composed a dissertation *De usu artis conjectandi in jure* [*On the Use of the Art of Conjecturing in the Law*]. See the correspondence with Jacob Bernoulli, April 1703, 3 October 1703; *Math*., III, 71, 77) and with Johann Bernoulli, 15 April, 2 July, 6 September, 1 October 1709; 26 April, 6 June 1710; 9 September, 6 December 1713; 23 May 1714 (*Math*., III, 842, 844, 845, 846; 847, 850; 922, 925; 931). Cf. Cantor, II, 327. Jacob Bernoulli repeatedly asked Leibniz to procure for him Johan de Witt's essay on life annuities and his own dissertation *On Conditions* (*Math*., III, 78, 89, 91, 93).

[222] And, in fact, Nicolas Bernoulli published the *Ars conjectandi* together with a letter from his uncle under the title: *Epistola Gallice scripta de Ludo pilæ reticularis*.

[223] Leibniz from Jacob Bernoulli, 2 August 1704 (*Math*., III, 91).

of its probability, since in any case this is what clever players do by experience and instinct.[224]

In general, Leibniz strongly desired the formulation of a mathematical theory of all games, which, he said, "would be of great use for perfecting the art of invention, the human mind appearing better in games than in more serious matters.[225] He even sketched the plan for such a work in responding to Remond de Montmort, who had sent him his *Essay d'Analyse sur les Jeux de Hasard*:[226]

"I wish you had dealt with all the games that depend on numbers.... After games that depend solely on numbers come games in which the position also enters, as in backgammon, checkers, and above all chess. The game called 'solitaire' rather pleases me….[227] But what is it good for, one will ask. I reply: for perfecting the art of invention, for it is necessary to have methods for reaching the conclusion of all that can be discovered by reason. After games in which only number and position enter will come games in which motion enters, like billiards and tennis.... Finally, it remains to wish that we had a complete manual of games, treated mathematically...."[228]

30. But if Leibniz did not propose to his mathematician friends any novel idea concerning the calculus of probabilities, if he even did not contribute to the progress of this science through any notable discovery,[229] he at least had a completely personal conception of it which owed nothing to his predecessors, for he already possessed the essential elements of it prior to becoming acquainted with mathematicians or even mathematics. The fundamental ideas of his theory are found, first, in *On Conditions* of 1665;[230] then, in *Specimen of a Political Demonstration for Selecting the King of Poland* (1669), composed at the request of Baron Boineburg, which he later recalled with pleasure,

---

[224] Leibniz to Jacob Bernoulli, 28 November 1704 (*Math*., III, 94).

[225] *New Essays*, IV.xvi.9, end. Cf. Leibniz to Johann Bernoulli, 29 January 1697: "It cannot be said how many thing useful for the art of invention are hidden in games. The reason for this is that men are usually more natural in games than in serious matters, since those things go better for us that we carry out with pleasure" (*Math*., III, 363). *Response to the Reflections of Mr. Bayle*, 1702 (*Phil*., IV, 570); Leibniz to Hermann, 10 March 1705 (*Math*., IV, 270); Leibniz to Burnett, 14 December 1705, in connection with the count of Sunderland, who had composed a Latin book on the game of chess, of which he was a past master (*Phil*., III, 304); Leibniz to Baron Spanheim, 13 December 1705 (Klopp, IX, 185); Leibniz to Nicholas Remond, July 1714 (*Phil*., III, 621); Leibniz to Remond de Montmort, 17 January 1716 (*Phil*., III, 667).

[226] Pierre Remond de Montmort, brother of Nicholas Remond (head of the cabinet of the Duc d'Orleans and a Platonist, who was a correspondent and admirer of Leibniz), had sent Leibniz the first edition of his book (1708), which was lost with his letter, and later the second edition (1713). (See his letter to Leibniz, 10 February 1714; *Phil*., III, 666.) Cf. Leibniz to Johann Bernoulli, 27 June 1708 (*Math*., III, 836; cf. 837); Leibniz from Johann Bernoulli, 15 April 1709 (ibid., 842); 1 October 1709, 26 April 1710 (ibid., 846, 847); and the letter from Nicolas Bernoulli, who had spent some weeks at the home of Remond de Montmort, at Montmort in Champagne (7 April 1713; *Math*., III, 982), and Leibniz to Nicolas Bernoulli, 28 June 1713; ibid., 987); Leibniz from Remond, 5 May 1714 (*Phil*., III, 618; cf. 621).

[227] In Leibniz's unpublished manuscripts there is in fact a note on *The Game of Solitaire* (LH XXXV 3A, 16), which probably dates from 1678, in which Leibniz imagines playing this game backwards, as in his article from the *Miscellanea Berolinensia* (see Note XVII: "On the Mathematical Theory of Games").

[228] Leibniz to Remond de Montmort, 17 January 1716 (*Phil*., III, 667-9). One will note that Leibniz by turns classifies games according to two different principles: sometimes according to the sciences on which they depend, that is, the categories to which they are related; sometimes according to the share of skill and chance, that is, according to the degree of probability of the expectations to which they give rise.

[229] Cantor, III, 342.

[230] See Note V.

undoubtedly because it proved the age and originality of these ideas;[231] finally, in his first letter to Arnauld (1671 or 1672) and in the *Definition of Universal Justice*, which obviously dates from the same period.[232] He had from his youth the plan of composing a treatise on "degrees of probability," to which he often makes allusion.[233]

The first of these ideas is that probabilities are to certainty like parts to a whole, or like proper fractions to a unity.[234] And, in fact, with the probability of an event defined, the relation of the number of favorable cases to the total number of possible cases can only be a proper fraction, and when the latter is equal to one the probability becomes a certainty.[235] For this, it is necessary that all the cases thus enumerated be equally possible or feasible (*faciles*).[236] But there are problems in which the various alternatives are not equally probable, and it is necessary then to evaluate at the outset the respective probability of each of them. To do this, one must resolve them into a certain number of simple cases which must all, by hypothesis, be equally possible; since each of these cases is, as it were, the common measure of all these alternatives, the probability of each of them will be measured by the number of corresponding cases.[237] Leibniz explains this with the example of dice: given two dice, the probability of rolling 7 is to the probability of rolling 9 as 3 is to 2, given that there are three combinations (throws) that produce a total of 7,[238] and only two that produce a total of 9,[239] assuming that all the combinations of the faces of the two dice are equally probable.[240]

More generally, when an event has different probabilities under different conditions (in other respects equally probable), its probability is the mean of these different

---

[231] "It was nearly thirty years ago that I made these remarks publicly," he said in recalling the principles of his theory; these are referred to in the following lines from the same letter: "And in a small book printed without my name in the year 1669, concerning the election of the king of Poland... I myself showed that there is a type of mathematics involved in the weighing of reasons; and sometimes it is necessary to add them together, sometimes to multiply them, in order to obtain the sum. This has not been recognized by logicians." Leibniz to Burnett, 1/11 February 1697 (*Phil.*, III, 194, 190). See n. 245, and Note VIII.

[232] See Note IX.

[233] "I once had in mind to compose something on estimating the degrees of probability, and I was astonished to see neglected that part of logic which is most practical and most flexible in use." Leibniz to Placcius, January 1687 (Dutens, VI.1, 36).

[234] "But in imitation of mathematicians, I regard certainty or truth as the whole and probabilities as parts, so that probabilities stand to the truth like acute angles to a right angle" (ibid.). Cf. *Phil.*, IV, 363; Mollat, 81.

[235] See *On the Estimation of Uncertainty* (September 1678), Rules 1 and 2 (LH XXXV 3A, 12).

[236] This is what the following sentence from the *Rules for Advancing the Sciences* implies: "It [likelihood] is given as a discounting of the hypotheses; however, in order to assess it, it is necessary that the hypotheses themselves be evaluated and that they be limited to a comparison of homogeneous cases" (*Phil.*, VII, 167).

[237] "But when the hypotheses are unequal, one compares them with each other." *New Essays*, IV.xvi.9.

[238] Namely: 1 and 6, 2 and 5, 3 and 4.

[239] Namely: 3 and 6, 4 and 5.

[240] *On the Number of Throws in Dice*, January 1676 (LH XXXV 3B, 14); *New Essays*, IV.xvi. 9; Leibniz to Bourguet, 22 March 1714 (*Phil.*, III, 569-70). Leibniz makes a rather paradoxical application of this rule to jurisprudence: if two litigants A and B claim to have a right to the same sum of money, and if the right of A is twice as probable as that of B, one must divide the sum between A and B in the proportion of 2 to 1 (Leibniz to Placcius, cited above n. 233).

probabilities. This is what Leibniz calls the *prosthapheresis*,[241] that is, simply the arithmetical mean.[242] In short, this is the rule for *total probability*, by virtue of which the latter is equal to the sum of the simple probabilities.[243]

But there are other more complex questions in which reasons must not only be counted, but weighed,[244] or more exactly, in which they are compounded among themselves, not by addition, but by multiplication.[245] In fact, when the reasons are independent and heterogeneous, each reinforces and, so to speak, multiplies each part of the other, such that the total effect is proportional not to their sum but to their product. It is for this reason, for example, that happiness will be measured by the product of well-being (that is to say, its intensity) and its duration.[246]

To return to probabilities, the advantage or expectation of a risky profit is measured by the product of the size of the profit and the probability of its being obtained, insofar as it is proportional at once and separately to the size of the profit and to its probability.[247]

---

[241] *Prosthaphaeresis* (a barbarism formed from the Greek words πρόσθεσις, addition, and ἀφαίρεσις, subtraction) is an operation invented by Johann Werner (1468-1528) to replace multiplication by addition and subtraction, using two trigonometric formulas:

$$2 \sin \alpha . \sin \beta = \cos (\alpha - \beta) - \cos (\alpha + \beta)$$
$$2 \cos \alpha . \cos \beta = \cos (\alpha - \beta) + \cos (\alpha + \beta)$$

(Cantor, II, 454, 597).

[242] Leibniz elsewhere calls this the "discounting of hypotheses" (*Phil*., VII, 167). He cites a rather curious application of this rule in use in his time: in order to evaluate a piece of property, one formed three groups of assessors, each of which gave its estimate independently of the others, and one took the mean of the three estimates as the probable value of the property (*New Essays*, IV.xvi. 9). Cf. *Essay on Some New Arguments Concerning Human Life and the Number of Human Beings*: "Rule for finding the mean appearances, for which it is necessary to stop in uncertainty" (Klopp, V, 327).

[243] Cf. *On the Estimation of Uncertainty* (September 1678), the rule: "If all events are equally likely..." (LH XXXV 3A, 12 Bl. 4).

[244] "We often say with justice that reasons must not be counted but weighed; however, no one has yet given us the scale that must serve to weigh the force of reasons." Leibniz to Burnett, 1/11 February 1697 (*Phil*., III, 194). "Everyone says that arguments should be evaluated by weight rather than by number; but who has given the scale by which mutually contradictory arguments and judgments might be weighed, such that we may choose, on the basis of the givens, which is most probable?" Leibniz to Placcius, cited above n. 233. The same idea is found in *Elements of the General Science* (Erdmann, 85b), in *History and Praise of the Characteristic Language* (*Phil*., VII, 188), and in the letter to Gabriel Wagner, 1696 (*Phil*., VII, 521). In the last text, Leibniz foresees an application of probabilities to medicine: he compares the reasons for and against to the *indications* and *contraindications* of doctors (cf. *New Essays*, IV.xvi.9). The metaphor of a scale is found very often in Leibniz (*Phil*., VII, 125-6, 188, 201). Cf. the title of the unpublished opuscule: *Concerning the Scale of Justice*... (LH IV 6 Bl. 17).

[245] "I often pointed out [to Jacob Bernoulli] that we were missing that part of logic which deals with degrees of probability; but I insist that these should be estimated from degrees of possibility, or from a set of equal possibilities. I once showed in a certain political essay, produced at the command of a prince, that some estimates are arrived at through addition, others through multiplication." Leibniz to Johann Bernoulli, 6 September 1709, P.S. (*Math*., III, 845; cf. Leibniz to Burnett, quoted n. 231). This is again an allusion to the *Specimen of Political Demonstrations* of 1669. (See Note VIII.)

[246] *Phil*., VII, 115. (See Notes VIII and IX.)

[247] "Goods or evils should be evaluated separately, both in terms of their magnitude and in terms of their probability through precedents. And if they are equal, they will exist in a ratio of probabilities; if they are equally probable, they will exist in a ratio of magnitudes. And if they are unequal and unequally probable, they will exist in a compound ratio of magnitudes and probabilities" (Mollat, 92, with an explanatory figure). These are the two moral factors that Leibniz elsewhere calls, following the moralists,

From this Leibniz deduced a more complicated rule for the case in which unequal profits (or unequal probabilities of the same gain) correspond to different, unequally probable hypotheses. This rule is a combination of the rules of addition and multiplication: one must first multiply each possible gain by its probability, and then take the sum of all these products, in order to have the total advantage, that is, the total expected profit.[248]

In sum, Leibniz had discovered by himself the rules of *total probability* and *compound probability*, which form part of the principles of the calculus of probabilities;[249] and he had noted very early on that these two ways of combining probabilities are analogous to arithmetical addition and multiplication, and are explained by them.[250]

31. But if Leibniz owed nothing to contemporary mathematicians who had discovered and cultivated the calculus of probabilities, we can still ask why he seemed to ignore their works in continuing to call for the establishment of a logic of probabilities and deploring its absence.[251] The reason is that for mathematicians, probability theory was only an occasion for posing and solving purely mathematical problems,[252] whereas for Leibniz it was truly a "part of logic," until then ignored or overlooked, which had its own rules and principles, and which must also have its special symbolism and algorithm. He regretted that this logic, the most important and most useful, was neglected by logicians who, following Aristotle, were only aware of the logic of the necessary (the method of

---

"the magnitude of the consequence" and "the magnitude of the consequent"; and he compares them to the two dimensions of a rectangle (*New Essays*, II.xxi.66).

[248] *On the Estimation of Uncertainty* (September 1678), rule: "If from all events..." (LH XXXV 3A, 12 Bl. 5). Leibniz later wrote: "Several probable arguments joined together sometimes produce moral certainty, and sometimes not." Letter to Burnett, 1/11 February 1697 (*Phil*., III, 194). In fact, if their probabilities are added, their sum can be equal to unity (the measure of certainty); but if they are multiplied together, their product must be less than each of them (since they are proper fractions) and therefore cannot be equal to 1. See n. 22.

[249] Here is how we state them to day: "The total probability of an event that must occur under several mutually exclusive and independent hypotheses is the *sum* of the probabilities of all the hypotheses favoring the event." "The probability of an event composed of several mutually independent events is the *product* of their probabilities." Principles I and II of Laplace, *Théorie analytique des Probabilités*, 1812, Introduction: *Essai philosophique sur les probabilités*); Principles I and IV of Cournot, *Exposition de la théorie des chances et des probabilités* (Paris, Hachette, 1843). The general rule formulated by Leibniz in September 1678 is a combination of these two rules, and conversely each of the latter can be deduced from it as a particular case.

[250] This observation is all the more interesting as the calculus of probabilities has a close relationship with the logical calculus: indeed, the total probability corresponds to a *disjunction* of events, and the compound probability to a *conjunction* of events; from this follows the analogy between arithmetical addition and multiplication and the logical operations of the same name.

[251] See Leibniz to Kestner, 30 January 1711: "In truth, that part of logic by which degrees of probability and weights of arguments are determined has until now nowhere been found propounded. As a youth, I once attacked the matter, but distracted by a variety of things, it remained for me no more than a desire. In my opinion, Aristotle's *Topics* does not answer to it. It collects rules which can furnish an occasion for thinking of arguments, but which cannot show us how much weight to assign to any argument or judgment (*Dutens*, IV.3, 264). Cf. *Phil*., IV, 363; Kern, §III; and Leibniz to Eler, 10 May 1716 (Note XVIII).

[252] Likewise, the celebrated treatise of Laplace (apart from the *Essai philosophique*, which serves as its introduction) is little more than a collection of problems in the higher analysis.

certainty),[253] and that it existed only implicitly and semi-unconsciously in the theories of jurists. It was to be, according to him, the true topics or dialectic,[254] that is, the logic of the probable; he opposed it to Aristotle's topics (the theory of common places), which at most serves only for discovering rhetorical arguments in order to plead any cause, but not at all for testing its value and measuring its probative force or "weight," that is, its probability.[255] At the same time, Leibniz was equally opposed to the *probabilism* of theologians and casuists (particularly the Jesuits).[256] He blamed both theories for relying solely on authority and for only taking account of subjective opinions rather than objective and intrinsic reasons that render a judgment more or less truthful or probable.[257]

32. The logic of probabilities was the natural complement for Leibniz of the logic of certainly, especially in the domain of the art of invention. Indeed, he conceives every "invention," every question as analogous to a problem of algebra or geometry. But in every problem of this sort, two cases can occur: either the givens are sufficient to determine the solution, and then the latter is a necessary consequence of the former and can be deduced from them by a certain analysis;[258] or the givens are insufficient, and then they do not completely determine the solution but only limit its indeterminacy, such that there are an infinity of possible solutions. It remains then to know if they are equally possible, or if some are more probable than others. In the latter case, there are grounds for seeking which are most probable; and it must always be possible to discover them, provided that their probability is determined by the same givens that one already possesses.[259]

[253] *Animadversions Against... Descartes*, Part I, Art. 75: "the things which Aristotle prescribes in his logic, although they do not suffice for invention, nonetheless nearly suffice for judgment, where necessary consequences are at least dealt with; the important thing is that the inferences of the human mind have a stability like certain mathematical rules" (*Phil*., IV, 366). Cf. *New Essays*, IV.ii.14; *Theodicy*, "Preliminary Discourse," §§28 and 31.

[254] In the *New Method* of 1667, and in the *Plan for an Encyclopedia* of June 1679 (LH IV 5, 7), topics is identified with the art of invention, whereas in the *Theodicy* (1710), §31, it appears as distinct from it. This slight discrepancy is explained by the interval of time between them. See n. 334.

[255] Leibniz to Kestner, cited above; *Discourse Concerning the Method of Certainty*, end (*Phil*., VII, 183); *New Essays*, IV.ii.14; xvi.9; Leibniz to Koch, 2 September 1708 (*Phil*., VII, 477).

[256] The preface to *Concerning the Scale of Justice* contains a forceful critique of the lax morals of the Jesuits, which appears to be an echo of the *Provinciales* (LH IV 6 Bl. 17).

[257] "I do not speak here of the probability of the casuists, which is founded on the number and reputation of the Doctors [of the Church], but of that which is drawn from the nature of things in proportion to what we know of them and what we can call their likelihood (*Phil*., VII, 167). Cf. Leibniz to Burnett, 1699 (*Phil*., III, 259); Leibniz to Koch (*Phil*., VII, 477); *New Essays*, II.xxi.66 and IV.ii.14.

[258] See the definition of "sufficient givens" in a fragment on the general science (*Phil*., VII, 60-1).

[259] "And when a conclusion or solution is not supported by the data, it must at least be possible to determine the degree of probability from the data." Leibniz to Baron Bodenhausen (*Math*., VII, 355). "Even when it is only a question of probabilities, we can always determine which is the most likely from the givens.... Thus, when we do not have enough given conditions to demonstrate certainty, the matter being only probable, we can always at least give demonstrations concerning the probability itself" (*Phil*., VII, 167); for, as Leibniz often remarks, the laws of the calculus of probabilities are certain by virtue of a mathematical necessity: "One could say with Cardan that the logic of probables has different results than the logic of necessary truths; but the very probability of these results must be demonstrated by results of the logic of necessary truths" (*New Essays*, IV.xvii.6). Cf. Leibniz to Burnett, 1699: "The rules of moral certainty, and consequently also those of simple probability, can themselves be demonstrated with a geometrical or metaphysical rigor" (*Phil*., III, 259). "For even probabilities are subject to calculation and

This is what Leibniz demonstrates *a priori* by invoking the principle of determining reason.[260] All is determined, in nature as in the mind, and by the same laws, which are those of the "real" and "divine" logic;[261] the physical relation of causes to effects is only the concrete expression of the logical relation of consequences to principles.[262] But every logical relation must be intelligible at least to an infinite understanding.[263] If a categorical proposition is true, the conditions (requisites) of the predicate must be contained in those of the subject; if a hypothetical proposition is true, the conditions of the effect (consequent) must be contained in those of the cause (antecedent), such that they could be verified by a simple analysis, even if it would have to be infinite.[264] In a word, every determination is intelligible and every truth is analytic. However, in cases in which the analysis of the truth conditions is infinite (what occurs for all truths of fact), there arise two alternatives:[265] if the givens are sufficient, the analysis can be pursued indefinitely, and for us this indefinite analysis takes the place of a demonstration; if the givens are insufficient, the truth is not entirely determined *for us*, and we can only know it with probability (yet it is certain in itself and for God, who sees all its conditions).[266]

33. In order to understand the essential role that the calculus of probabilities plays in the art of invention, it is important to make precise and to complete the analogy drawn from algebra; for the latter offers all the cases and all the degrees of determinacy and indeterminacy that can occur in problems of every type, and furnishes definitions and precise examples of them. When one has as many (independent) equations as unknowns to discover, the problem is *determinate* and admits of one or more solutions. When one

---

demonstration, since it can always be estimated from given circumstances how probable a future event will be" (*Phil*., VII, 188). Cf. Leibniz to Conring, 3 January 1678 (*Phil*., I, 187); Leibniz to Arnauld, 14 January 1688 (*Phil*., II, 134); Leibniz to the Duke of Hanover, ca. 1690 (*Phil*., VII, 26); *Foundations and Illustrations of the General Science* (*Phil*., VII, 125).

[260] *Phil*., VII, 61-2.

[261] "It is agreed that not only are all truths determined in the nature of things and in the mind of the author GOD, knower of all things, but also that *that is determined which can be inferred by us from the evidence we already have*, whether with absolute certainty or with the greatest probability that can be had from the givens" (LH IV 7C Bl. 87). Cf. the fragment LH IV 6, 12 f, Bl. 14 (in Bodemann, 88).

[262] *On the Method of Arriving at the True Analysis of Bodies and the Causes of Natural Things* (May 1677): "Before all things, I hold it for certain that *all things happen through certain intelligible causes, namely those which could be understood by us if some angel were to reveal them to us*." From this principle Leibniz immediately derives the mechanistic conception of nature (*Phil*., VII, 265).

[263] "I say, therefore, that if any truth or any theory can even be demonstrated to us by an angel from those principles which we already have, we could have discovered the same thing by ourselves by means of this general science..." (*Phil*., VII, 62). Cf. Leibniz to Clüver, August 1680 (*Phil*., VII, 19), quoted p. 100, note 4.

[264] "The brief reason for this is that nothing can be demonstrated for us about any thing, not even by an angel, except insofar as we understand its requisites. Already in every truth, all the requisites of the predicate are contained in the requisites of the subject, and the requisites of an effect which is sought contain the means necessary for producing it" (*Phil*., VII, 62). Cf. LH IV 7C Bl. 73. We know that the relation between cause and effect is expressed through a hypothetical judgement. The terms *cause* and *effect* are still used in the calculus of probabilities to signify *premise* and *conclusion*, as with the Cartesians.

[265] This disjunction of an *indefinite approximation* or a *determinate probability* is very clearly indicated in *On the Universal Science* (*Phil*., VII, 201).

[266] "For every necessary truth whose necessity we understand can be reduced in a demonstration indistinguishable from those of mathematics or the other sciences; and if it is only probable, it is good to demonstrate this too and to estimate in some manner the degree of likelihood" (*Phil*., IV, 345).

has fewer equations than unknowns, the problem becomes indeterminate and allows of an infinity of solutions. Finally, when there are more equations than unknowns, these equations must be compatible among themselves, or else the problem becomes *impossible*; and if it is possible, one can resolve it in several different ways which must lead to the same solution and consequently can be used to check each other.[267]

It is much the same in geometry, as Leibniz shows by means of a simple example.[268] In order to determine a circumference, *three* of its points (not in a straight line) are necessary and sufficient. We know how to construct a circumference when we know *three* of its points, *A*, *B*, *C*: one draws a perpendicular to the midpoint of *AB*, a perpendicular to the midpoint of *AC*; and these two perpendiculars intersect by virtue of the hypothesis) in a point *O*, which is the center of the desired circumference (Fig. 28). If we know only *two* points, *A*, *B* on the circumference, its center is partly determined and partly undetermined; we know only that it lies on the perpendicular at the midpoint of *AB*. But if we know *four* points, *A*, *B*, *C*, *D*, on the circumference, we could determine its center either by means of *A*, *B*, *C*, or by means of *A*, *B*, *D*, or by means of *A*, *C*, *D*, or by means of *B*, *C*, *D*; and the four points thus obtained must coincide, if it is true that *A*, *B*, *C*, *D* belong to the same circumference. This is a condition for the possibility of the problem, which at the same time constitutes a relation among the four given points *A*, *B*, *C*, *D*; and this relation represents precisely the superfluous given or the extra equation.[269] This case is in every way analogous to that in which we have *four* equations for *three* unknowns: in order to solve such a system of equations, it is necessary and sufficient that the coefficients confirm an equation that is obtained by eliminating the three unknowns of the system. This remarkable analogy between underdetermined, determined, and overdetermined problems in different rational sciences must have vividly struck Leibniz and suggested to him early on the idea of a common logic for all these sciences.

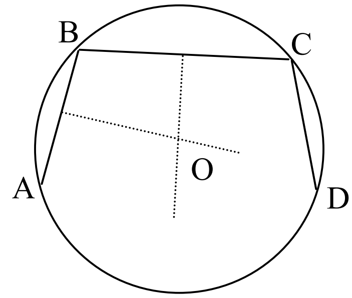

Fig. 28

34. The same distinction is found again in a problem that essentially involves the art of invention, namely, the deciphering of a cryptogram.[270] If the cryptogram is very short, it will not contain enough givens (letters or numerals) to determine the key, with the result that it admits of a (finite or infinite) number of keys and, consequently, of different (more or less probable) interpretations. If it is of a certain length (which depends on the length of the key or the complexity of the numeral), it will furnish exactly the givens that are necessary and sufficient to determine the key. Finally, if it exceeds this minimum length, the givens will be superfluous (more than enough), and they will allow us to determine the key in several different ways that will serve to mutually confirm each other (like *proofs* in arithmetic). The fact alone that the key determined by means of a part of the

[267] Let n be the number of unknowns and $n + k$ the number of equations. Every combination of n equations provides a determinate solution; but the number of these combinations is $(n + k)! / n!\, k!$. This is the number of distinct solutions that must coincide.

[268] *Phil.*, VII, 61.

[269] This condition is expressed geometrically by the fact that the fourth point must be found on the circumference that is already determined by the first three points; or rather, that the point *O* constructed by means of *A*, *B*, *C* must also be on the perpendicular at the midpoint of *AD* (or *BD*, or *CD*).

[270] *Phil.*, VII, 61.

cryptogram furnishes a plausible translation of the rest is already a very probable, and generally sufficient, confirmation.

Thus the art of cryptography is not only a part of the art of invention, already very interesting in itself:[271] it is the exact and complete symbol of it. Leibniz does not content himself with assimilating the solution of algebraic equations to the discovery of the key of a cryptogram. He sees it as only a particular case of the discovery of a concealed thing, that is, something implicitly given in complex combinations or relations, which needs to be extracted and isolated in an explicit form.[272] We understand, therefore, the importance he attached to the progress of this apparently secondary art, whose significance seems entirely practical, if not frivolous,[273] and the interest he took in Wallis's investigations in this area.[274]

35. Thus, the logic of probabilities serves already in the rational and mathematical sciences. Nevertheless, it is chiefly in the natural and experimental sciences that it finds its application; as we shall see, it is even their own distinctive method. Before that, however, it is necessary to show the exact role that experience plays in the general science; for it would seem to have no place in this entirely rational, deductive and *a priori* method.

The natural sciences have for their object truths of fact, and we know that the laws of nature themselves are contingent truths. But we have two ways of knowing contingent truths of fact. One is *a priori*: deduction founded on the principle of reason, as we have earlier seen; the other is *a posteriori*, and this is experience or a distinct perception of the facts.[275] And this second way is in reality the first and easiest: we first establish the facts by experience and then seek to explain them deductively by discovering the "reason" for them.

Experience replaces for us human beings the interminable analysis that it would be necessary to complete in order to "give the reason" for the least truth of fact, and delivers to us all at once and in a brute form the result of an infinite synthesis, or an infinite logical integration, that God alone can carry out. Nature is the product of a divine logic, of the immense calculation which is creation; it is for us an admirable calculating

[271] Leibniz to Tschirnhaus, end of May 1678: "Cryptography is also a part of this science.... For what the root is in algebra, the key is in divine cryptography" (*Math*., IV, 469). Cf. *On the Origin, Progress and Nature of Algebra*, in which the art of deciphering, along with the "art of playing brigand," is related to the combinatory (*Math*., VII, 206); *On Universal Synthesis and Analysis*, end (*Phil*., VII, 298).

[272] "The resolution of an equation is only a species of the art of discovering the key to an obscure matter." Plan for *On the Art of Combinations*, 1680 (LH XXXV 1, 27 d).

[273] In connection with the *Ars conjectandi* of Jacob Bernoulli, Leibniz asked if the work also dealt with the art of deciphering (Leibniz to Johann Bernoulli, 5 March 1697; *Math*., III, 377).

[274] He endlessly implored Wallis to publish a treatise on the art of deciphering cryptograms (the "cryptolytic art") (*Math*., IV, 14, 18, 27, 39, 42, 44, 55, 60, 65, 73, 76, 82). He alluded to these entreaties in his letter to Burnett of 1/11 February 1697, and added: "I have been told that there is another person in England who excels even more in deciphering. I would like to know his name and his circumstances; for it is a matter that is still half mathematical (*Phil*., III, 190).

[275] "However, there have been left to us two ways of knowing contingent truths: one is the way of experience, the other the way of reason. The way of experience is when we perceive a thing clearly enough by our senses; the way of reason is derived from the general principle that nothing happens without a reason, *or that the predicate is always in some way in the subject*" (emphasis added). *On Freedom* (Foucher de Careil, B, 182).

machine, for it furnishes us, fully completed, the results of calculations that surpass the scope of our understanding. Or, if we can to some degree supply or reconstitute this calculation by means of the principle of reason, nature gives us the means of confirming our rational deductions, just as the casting out of 9s allows us to confirm numerical calculations.[276] Thus, experience precedes our imperfect reason; it confirms it, regulates it and guides it. Between it and reason there exists a perfect accord, a true "preestablished harmony," for experience still belongs to reason: a reason latent and confused, but infinite. It is an immanent logic of facts, which renders them explicit and discursive to human reason in advance of intelligibles.[277]

Thus is explained how experience could prove *a posteriori* the possibility of an idea. Undoubtedly, if a thing exists it must be possible. But why is this simple reason decisive? It is because nature is penetrated by logic, or better, that it is a living logic, such that it can realize nothing that is either contradictory or unintelligible. When, therefore, we cannot assure ourselves *a priori* of the possibility of an idea by analyzing it completely, we can rely on the criterion of experience: the syntheses of nature are subordinated to the principle of contradiction and take the place for us of the infinite analysis that we cannot carry out.[278]

36. For this reason, Leibniz does not hesitate to admit, in addition to *absolutely first truths*, which are *a priori* rational principles, *truths first for us*, which are basic experiences, the primitive givens of consciousness; and this without making the least concession to empiricism, simply because empirical truths must have their reasons and be able to be demonstrated *a priori*, at least by an infinite understanding.[279]

This theory, which seems destined to correct and complete that of Descartes, appears at first glance to be irreconcilable with the theory of the two rational principles (of contradiction and reason) that suffice to account for all truths. There are two primary experiences: first, "I think" (that is, Descartes's *cogito*); second, "I think of different things."[280] From the first, Leibniz concludes (like Descartes) that "I am"; from the second, which he considers to be just as primitive and fundamental, and reproaches

[276] "For experience is to reason what proofs (like those of the casting out of 9s) are to arithmetical operations" (*Phil*., VII, 173; quoted p. 156, n. 2).

[277] We know that for Leibniz sensible perception envelops a veritable implicit and unconscious calculus: "Music is a secret exercise of arithmetic unknown to the soul that reckons." Leibniz to Goldbach, 17 April 1712 (*Dutens*, III, 437). Cf. *Principles of Nature and of Grace*, §17; *Response to the Reflections of M. Bayle*, 1702 (*Phil*., IV, 550-1). Thus music itself is a science subordinate to arithmetic (*Phil*., VII, 170). Nothing better shows to what extent Leibniz, in the words of Kant, "intellectualized" sensation.

[278] See *Meditations on Knowledge, Truth and Ideas*, 1684 (*Phil*., I V, 425); Leibniz to Arnauld, 14 July 1686 (*Phil*., II, 63; cf. 43); Leibniz to Burnett, 1699 (*Phil*., III, 257).

[279] "Therefore, of the two types of propositions certain in themselves, some are established by reason or are manifest from their terms, and these I call known through themselves or also identities; other propositions are produced and made known to us in indubitable experiences, and such are the testimonies of immediate consciousness. But although those which are produced also have their reasons, and therefore by their nature can be resolved, they still could not be known by us *a priori* through their causes, unless the entire series of things were known, which exceeds the force of the human intellect; thus they are learned *a posteriori* in experiences." *Preliminaries to the Encyclopedia* (*Phil*., VII, 44).

[280] *On Universal Synthesis and Analysis* (*Phil*., VII, 296). Cf. *Animadversions Against... Descartes's Principles* (1692), Part I, Art. 7 (*Phil*., IV, 357), and the fragment LH IV 6, 12 f Bl. 19: "*Concerning principles.* The two first principles: one of reason, the other of experience, since various things are perceived by me..." (Bodemann, 89).

Descartes for having neglected, he concludes that there exist other beings than me, and that these produce in me the variety of thoughts or sensations; in short, that the external world is real.[281] But as factual truths are always only probable, the reality of the external world itself has only a "moral certainty," that is, a very high probability, and not a metaphysical certainty, or a logical necessity.[282] It in fact rests on the agreement of phenomena among themselves, an agreement that cannot be fortuitous but must have a cause.[283] It is also this agreement of the phenomena that allows us, according to Leibniz, to affirm the truth of sensations, to distinguish waking from dreaming; it is again this which serves as the foundation of empirical induction, as well as of the testimony of men and of authority.[284] But all these conclusions drawn from the agreement of phenomena are always only probable, for it is not impossible that this agreement is fortuitous and without cause: only the probability of this is infinitely small, so that the contrary is "morally" certain. It is by these probabilistic arguments that Leibniz claims to refute skeptics who call into question the existence of the external world, historical truths and human testimony, under the pretext that these truths are not certain or metaphysically necessary, that is, such that the contrary implies a contradiction.[285]

But what of truths whose contrary is logically possible? These are contingent truths which depend on the principle of reason. And it is in fact the principle of reason which at bottom accounts for the evidential value of this agreement of phenomena: it is by virtue of this principle that the agreement cannot be due to pure chance, but must have a "cause," or rather a reason for being, which consists precisely in the existence of objects corresponding to these phenomena—objects which are *analogous*, if not *similar*, to them.[286] This, in paticular, is what allows us to infer the existence of an external world from the order and variety of our perceptions. This theory has well-known consequences

[281] Leibniz to Foucher (1679?): "Thus, there are two truths that are absolutely general, that is, which speak of the actual existence of things. One is that we think, the other that there is a great variety in our thoughts. From the first it follows that we are, from the second that there isomething other than us" (*Phil.*, I, 370). Cf. a fragment against Descartes (1690?): "Our primary experiences... are not only that I am someone who thinks, but also that there is a *variety* in my thoughts and I judge these two experiences to be independent of each other and equally primitive).... Just as internal experiences are the foundation of all truths of fact, so the principle of contradiction is the principle of all truths of reason, and without it all reasoning is undermined" (*Phil.*, IV, 327, 329). Cf. *New Essays*, IV.ii.1 end; IV.ix.2; this text proves that this is not a youthful theory that Leibniz abandoned for another theory (that which rests on the principle of reason). By an analogous deduction, M. Hannequin, in a work inspired by Leibniz, has tried to demonstrate the existence of the external world (*Essai Critique sur l'Hypothèse des Atomes*, Part. II, Chap. 2, §2; Paris: Alcan, 1895). Is this not, moreover, the reason why Kant believed it necessary to admit the existence of things in themselves as transcendent causes of our perceptions?

[282] *On the Method of Distinguishing Real from Imaginary Phenomena* (*Phil.*, VII, 320). Leibniz opposes this method to the Cartesian argument which grounds the reality of the external world in God's veracity, an argument he considers to be worthless (*Phil.*, VII, 321).

[283] "For in such things that are not of metaphysical necessity, we must take as the truth the agreement of phenomena amongst themselves, which will not occur by chance but will have a cause." *On Universal Synthesis and Analysis* (*Phil.*, VII, 296). Cf. *New Essays*, IV.iv.5.

[284] *Phil.*, VII, 296, 320 (see p. 158, n. 4). The rational probabilism of Cournot is founded on analogous considerations.

[285] Phil., VII, 296, 320; cf. LH IV 8 Bl. 3, plan for *Plus Ultra*: "chap. 3. *Elements of truth* against the skeptics." LH IV 8 Bl. 2 verso: "Here we must argue against the skeptics." It is in this context that he recalled the *De utilitate credendi* of St. Augustine (*Phil.*, VII, 296; Leibniz to Burnett, 1/11 February 1697, *Phil.*, III, 193; *New Essays*, IV.xx.17).

[286] See Chap. IV, §11

for both the theory of knowledge ("Our perceptions are well-connected dreams") and metaphysics ("Bodies are only phenomena, but well-founded phenomena").[287]

The principle of reason is thus the foundation of those empirical principles that Leibniz (provisionally) takes as first truths.[288] The *immediate internal experiences* are *first* truths only *for us*, human beings; but *absolutely first truths* are, on the one hand, identical propositions (which are reduced to the principle of contradiction), and on the other, the principle of reason, "by which we can demonstrate all experiences *a priori*," which Leibniz formulates as follows: "Every possible demands existence."[289]

Once again, experience is only a surrogate or substitute for reason, and empirical principles only replacements for the principle of reason. Nevertheless, Leibniz never fails to appreciate the usefulness of experience in the natural sciences; he even assigns it a role, not only provisional and secondary, but essential and permanent, given that experience takes the place of the infinite syntheses that we cannot and could never complete.[290]

---

[287] See *Antibarbarus Physicus* (*Phil.*, VII, 344). Cf. LH IV 8, Bl. 7 verso: "Extension, motion, and bodies themselves... are not substances, but true phenomena, like rainbows and parhelia."

[288] The most complete table of these principles is found at the end of a very interesting unpublished fragment: *Introduction to a Secret Encyclopedia, or Foundations and Specimens of the General Science*:

"Principles of metaphysical certainty.

First principles a priori:

Nothing can at the same time be and not be, but everything either is or is not.
Nothing is without a reason.

First principles of a posteriori knowledge, or of logical certainty:

Every perception of my present thinking is true.

Principle of moral certainty:

Everything which is confirmed by many indications, which can hardly concur except in the truth, is morally certain, or incomparably more probable than its opposite.

Principle of physical certainty:

Everything which men have experienced always and in many ways will still happen: e.g. that iron sinks in water.

Principles of topical knowledge:

Everything is presumed to remain in the state in which it is.
The more probable is that which has fewer requisites, or which is easier" (LH IV 8, Bl. 2 verso).

[289] "*Absolutely first truths* are, among truths of reason, *identities*, and among truths of fact, those from which all experiences could be demonstrated *a priori*, e.g. *Every possible seeks to exist.... First truths according to us* are experiences…. Every truth either can be demonstrated from absolutely first truths..., or is itself an absolutely first truth. And this is why it is usual to say that nothing should be asserted without a reason, and even that nothing happens without a reason" (*Phil.*, VII, 194-5).

[290] Leibniz to Cluver, August 1680: "It is (or at least will have been!) ridiculous to expect universal knowledge [*pansophia*] from any characteristic, just as from any analysis, for many things are known only through experience" (*Phil.*, VII, 19). In his letter to Oldenburg of 28 December 1675, after having spoken of his characteristic as the general method of the rational sciences, Leibniz wrote: "when these studies are

37. Experience serves first as a basis for induction; but this is not, according to Leibniz, the proper and essential work of science. Generalization from observation and experience is not a "scientific" procedure (in the proper sense of the word), because it has no logical value. Leibniz distinguishes and opposes, in effect, empirical successions, which are common to us and animals, and rational consequences, that is, deductive reasoning.[291] The first are explained mechanically by the association of ideas and give rise to inductions that sometimes succeed but are often mistaken. Science, however, consists in rational and deductive knowledge of the reasons for phenomena, reasons that are universal and necessary; but no induction can ground a universal and necessary proposition. In short, Leibniz categorically rejects induction, such as the empiricists understand it, as insufficient and even deceptive.[292]

He explains this with an example borrowed from mathematics; for induction and empirical generalization also have their place in the rational sciences. Consider the following series of natural numbers:

1 2 3 4 5 6 7 8 9 10

and the series of their squares:

1 4 9 16 25 36 49 64 81 100

---

finished... men will return to the investigation of nature alone, which will never be completely controlled; for in experiments luck is mixed with ingenuity and hard work. Once men follow our method [i.e. the characteristic] through to the end, then they will always philosophize in the manner of Boyle ["the manner of Boyle" being the experimental method], except insofar as the very nature of things, to the extent that it is known, can be subject to a calculus and, when new qualities are discovered and reduced to a mechanism, it will give new opportunities for applying geometry (*Phil*., VII, 10; *Math*., I, 86; *Corres.*, I, 145). Thus, Leibniz had, at an early date, conceived very clearly the relations between mathematics and physics, and the conditions governing the application of the former to the latter: it would be necessary to reduce sensible qualities to mechanical ones, in order to be able to subject them to a calculus. Nature becomes intelligible through mechanism, which provides ever new materials for mathematical deductions. Cf. *Plan for a New Encyclopedia*, June 1679 (LH IV 5, 7, Bl. 5 recto); LH IV 6, 12 f, Bl. 26 (Bodemann, 90); and LH XXXV 2, 21; LH XXXV 1, 5, b (texts quoted Chap. IX, §1).

[291] "It should be known that there are two completely different types of consequences: empirical and rational. Empirical consequences are common to us and beasts..." (*Phil*., VII, 331). Cf. Leibniz to Tolomei, 6 January 1705: "And men themselves, insofar as they are no more than empiricists, proceed only in the manner of animals. But eternal and necessary truths, which alone are completely universal, render us certain; thus reasons and knowledge also do not belong to beasts" (*Phil*., VII, 464). See p. 103, note 2.

[292] "But man, insofar as he acts rationally rather than empirically, does not rely on experience alone, or on *a posteriori* induction from particulars, but instead proceeds *a priori* through reasons... such is the difference between the empirical and the rational, between the inferences of beasts and human reasoning.... And so beasts... do not know the universality of propositions, because they do not know the reason for the necessity. And although empiricists are sometimes led to truly universal propositions through induction, it nevertheless happens only accidentally and not through the force of the inference" (*Phil*., VII, 331-2). Cf. *Preface to Nizolius*, 1670 (*Phil*., VI, 161-2) and the letter to Princess Sophie, 12 June 1700: "When we have learned some truth through experience..., we will never be assured of the necessity of the thing without calling on the help of demonstrative reasoning, founded on the internal light that is independent of the senses." And Leibniz adds this remark, which clearly reveals the origin of his entire logic: "This is what few people recognize, even among philosophers, because one is rarely a philosopher and a mathematician at the same time, and demonstrations are almost only seen in mathematics" (*Phil*., VII, 553).

If we form the series of the differences of consecutive squares:

3 5 7 9 11 13 15 17 19

we observe that this is the series of odd numbers.

To generalize this observation is to perform an induction: but such an induction has no probative value, hence no certainty. In order to establish the truth of the presumed law, it is necessary to demonstrate it *a priori* by means of universal and necessary reasons.[293]

This is not, moreover, a simple analogy: for even an empirical induction must assume a mathematical form, according to Leibniz. It consists in extending a series of numerical givens, by conjecturing from the first terms their general law of formation. It also consists in completing the series, filling in its gaps by interpolation, as we say today, and this by virtue of the principle of continuity, which allows us to assume that the hypothetical law, (approximately) confirmed by a series of discontinuous phenomena, is also confirmed by all the intermediate phenomena. Induction, so understood, finds a nearly indispensable aid in the numerical tables that summarize a series of experiences in a synoptic form: for the comparison of experiential givens brings out their "analogies" and "harmonies," and suggests the law or mathematical function that unites them.[294]

38. But induction, even in this mathematical form, serves only to suggest the law and to make it "presumed"; it remains to demonstrate it, and this is the work of deduction. Insofar as it is not demonstrated, the empirically inferred law is called simply an "observation" or "experience," undoubtedly because it only expresses or summarizes the

---

[293] Leibniz to Princess Sophie (*Phil*., VII, 553). Cf. *Plan for a New Encyclopedia*, June 1679 (LH IV 5, 7, Bl. 1 verso); Leibniz to Gabriel Wagner, 1696 (*Phil*., VII, 524); and Leibniz to Queen Sophie Charlotte of Prussia, ca. 1702 (*Phil*., VI, 490, 495, 504).

[294] In *On the Art of Discovering Theorems* (7 September 1674), after having spoken of analogy as a method for discovering new theorems, Leibniz discusses induction, and immediately relates it to analogy: "and in this consists the entire art of experimenting," for we seek new experiments "by means of already known experiments, through analogy. But analogy is founded on this: that we believe that those things that agree or disagree in many ways, also agree or disagree in given ways similar to the former" (LH IV 6, 12d, Bl. 2). We see that it is the principle of continuity that is the foundation of reasoning by analogy. See *Plan for a New Encyclopedia* (June 1679): "For just as in progressions of numbers, when a table of any length has been established, there usually appears a way of continuing it without any effort" (there follows the mathematical example cited above). "In the same way, with discoveries in any type of matter properly ordered as in a table, a way of extending the discoveries, that is, of discovering new things, will be obvious" (LH IV 5, 7, Bl. 1 verso). "From this there emerges many new things, of which we would not otherwise have thought; and certain harmonic series will appear, by following the thread of which a way to greater things will become clear" (ibid., Bl. 2 verso). Likewise, in *On the Art of Discovering Theorems*, Leibniz proposes forming "tables" of experiments, then "comparisons of tables," "for the sake of confirming certain harmonies or analogies" (loc. cit.). Cf. the plans for a new *On the Art of Combinations*, in which we read: "On the art of observing something novel from given tables" (LH XXXV 1, 27 b, c). "On tables constructed from known things in such a way that by the interpretation and continuation of the series unknowns can be conjectured." Leibniz compares these inductive tables to the mortality tables of Hudde and to the tables of magnetic declinations that he wanted to see established, "from which any man endowed with intelligence may construct any hypothesis" (LH XXXV 1, 27 c; probable date, 1680). This is indeed the inductive procedure of the physical sciences: see Bouasse, "De l'applicationes sciences mathematiques aux sciences experimentales" (*Revue de Metaphysique et de Morale*, vol. 7, pp. 1-25), and Cournot, *passim*.

phenomena and has no further significance.[295] But in what does the demonstration of a factual truth, or empirical law, consist? It consists in deducing it from a more general hypothetical law that could serve as a principle for other empirical laws, and in this way progressively moving back from laws to more and more general laws, in such a way as to make all empirical laws depend on the smallest possible number of principles or hypotheses.[296]

Thus the natural sciences must be constituted according to the same deductive model as the rational sciences;[297] moreover they differ from them much less than it seems, for the latter are also founded on undemonstrated general hypotheses that we call axioms or postulates,[298] and they also employ experience, that is, the observation of mathematical *facts* of which we seek the laws.[299] Leibniz therefore assimilates the method of the physical sciences to that of mathematics, for both are essentially deductive. Their difference consists solely in the fact that the one is progressive or synthetic, and the other regressive or analytic. Still, it is necessary to note that the latter is what we employ to solve mathematical problems, so that the search for the laws of nature is carried out according to the same method as the search for the solution to a geometrical problem.[300]

39. We are familiar with this method, which Pappus called *analysis*, that is, inverse solution. We suppose the problem solved and from the hypothetical solution we deduce all the necessary consequences, until we arrive at an already known truth or at a

---

[295] "*Phenomena* are propositions that are proved through experience.... *Observations* are produced solely by induction from phenomena." *Plan for a New Encyclopedia* (LH IV 5, 7, Bl. 2 recto). Cf. p. 158, note 3.

[296] "*Hypotheses* are propositions that are very useful and successful, and they are confirmed by the agreement of conclusions, known from elsewhere, which depend on them; nevertheless, we cannot yet demonstrate them exactly enough, and so in the meantime they are assumed" (ibid.)

[297] Leibniz wrote to Johann Bernoulli (15 October 1710): "Physics written in the manner of mathematics is a great and desirable thing" (*Math*., III, 856); and he urged him to apply the mathematical method to medicine in order to explain as many phenomena as possible mechanically (Leibniz to Johann Bernoulli, 6 May 1712, *Math*., III, 884). Cf. Leibniz to Conring, 24 August 1677: "I wish, therefore, that a man skilled in the art of demonstration would delineate some foundations for medicine, in which the certain was distinguished from the uncertain, and at least those things that can be claimed with certainty were demonstrated" (*Phil*., I, 182).

[298] *New Essays*, IV.xii.10: "I agree that the whole of physics will never be a perfect science for us.... We must not hope to account for all phenomena, for even geometers have not yet proved all their axioms; but just as they content themselves with deducing a large number of theorems from a small number of rational principles, so it is enough also that physicists, by means of certain principles of experience, account for a great many phenomena and are even able to predict them in practice." Cf. §12 of this chapter.

[299] "There are some experiments, better called observations, that need only to be inspected and not produced. Such are experiments that occur in the context of examining numbers; likewise, celestial observations and observations of winds and tides...." *On the Art of Discovering Theorems*, 7 September 1674 (LH IV 6, 12 d, Bl. 2).

[300] "It follows from this that it will be easy for us to derive the innermost nature of these bodies from just a few experiments. For if this nature is simple, the experiments must easily follow from it; and if the experiments easily follow from it, it in turn must also easily follow through analysis from a sufficient number of experiments. Such analysis occurs in algebra; and in everything else it could occur by means of some sort of mathematical calculus, if only men would hold to the true art of reasoning. But we look in vain for the true art of reasoning in difficult and somewhat abstruse matters, such as physics, as long as we lack the characteristic art or rational language, which wonderfully abridges the operations of the mind and alone can offer in physics what algebra does in mathematics." *The Method of Physics*, May 1676 (LH IV 5, 6 c; Foucher de Careil, VII, 203).

construction that we know how to carry out. With this done, we pass over the chain of reasoning in the opposite direction, so as to deduce the new truth from the known truth or the desired solution from the known construction. In brief, Pappus's analysis is an inverse deduction, which returns from the consequence to the principle. The conditions under which this process is valid were studied and refined by Leibniz. In order that the deductive chain could thus be traversed in two directions, it is necessary that all the propositions that compose it be reciprocal or convertible, and consequently that the subject and attribute of each of them have the same extension. But it is also necessary, adds Leibniz, that they have the same extension from one proposition to another, otherwise the deduction would only be possible in one direction: for we can only infer from a narrow term to a broader term. In other words, it is necessary to employ only logical *equations*, that is, convertible propositions (of which the reciprocal is true) and to only pass from one to the other by replacing a term with a term equal in extension (by virtue of the principle of the substitution of equivalents).[301]

40. Such is the method that physics must employ in order to demonstrate the laws found by induction; for, as in a geometrical problem, it is a question of moving backwards from a known conclusion to an unknown principle, from the (logical) *effect* to the (logical) *cause*.[302] This cause is a hypothetical law from which we could deduce the observed law, along with many others, for it will ordinarily be more general. But we know the condition under which the hypothetical law will be *true*: namely, when it is the only possible principle of explanation; we could then, inversely, derive it from its consequence, such that they both mutually imply each other (they are logically equivalent). It is only under this condition that a hypothesis is demonstrated in geometry; but "in astronomical and physical hypotheses, the reverse does not hold… thus success does not demonstrate the

[301] It is in this way that Leibniz justified analysis against the criticisms of Conring, who did not grasp that one can validly proceed upwards from the consequence to the principle: "When we arrive at last at already known truths by starting from an assumption of whose truth we are uncertain, we cannot conclude from this that the assumption is true... unless we make use in our reasoning of pure equations or propositions that are convertible or whose subject and predicate are equally inclusive. We must take care, that is, not merely that in each proposition the predicate is as inclusive as the subject and vice versa (which is true in reciprocal propositions), but also that the subject and predicate in one proposition are as inclusive as the subject and predicate in all the other propositions occurring in the same demonstration." Leibniz to Conring, 19 March 1678 (*Phil*., I, 195). And Leibniz adds: "Moreover, equations of this kind occur not only in mathematics but in all other reasoning, that is, wherever definitions occur." And, in fact, a definition is a logical equality in which we suppose one term to be equivalent to a combination of other terms that constitutes its *formula* (to use Leibniz's own word). Thus, from 1678, he had a very clear idea of what we call a *logical equality*, thanks to the logical calculus he was thinking about during this period. Long after, he recalled this discussion with Conring in the *New Essays*: "But I subsequently showed him that analysis employs definitions and other reciprocal propositions, which offer the means of proceeding in reverse and discovering synthetic propositions" (IV.xii.6). "It is necessary that propositions be reciprocal, so that a synthetic demonstration can retrace in reverse the steps of the analysis" (IV.xvii.6).

[302] It is always in this sense that Leibniz employs the terms 'cause' and 'effect'. And this sense is the only one that agrees with the practice of the experimental sciences, which never search for *causes*, in the strict sense, but only for phenomenal *laws*.

truth of the hypothesis,"[303] for, as Conring objected, from false hypotheses we can deduce a true conclusion.[304]

Furthermore, in the absence of this inverse deduction we can always admit that other hypotheses could serve just as well as explanatory principles; and, in fact, we often find that the same phenomenon (or empirical law) can equally be deduced from many different hypotheses. Such a hypothesis therefore will never be certain, but it will be probable; and if there are other admissible hypotheses, they too will be more or less probable, since they cannot all be true at once.

Now, under what conditions will a hypothesis be probable, and how will we measure its probability? According to Leibniz, a hypothesis is more probable to the degree: 1) that it is simpler; 2) that it explains a greater number of phenomena by a smaller number of assumptions; 3) that it allows us to predict new phenomena and explain new experiences. In the last case especially the hypothesis will be equivalent to the "truth": it will have a "physical" or "moral" certainty, that is, the highest probability, just like a presumed key that allows us to completely decipher a long cryptogram by giving it an intelligible and sustained sense.[305]

This comparison of the method of the physical sciences to the art of deciphering is neither accidental nor paradoxical: it rests on the real analogy between the two methods.[306] Just as a short cryptogram can agree with many keys, whereas a longer cryptogram admits of no more than one, so a small number of phenomena can be explained by a multitude of "causes," that is, can be deduced from many different hypotheses; but more numerous, and above all more varied, phenomena will restrict the choice among the different hypotheses, though perhaps (unlike cryptograms) without our ever succeeding in eliminating them all except one (which would then be certain), because we will never possess enough givens to be able to effect the "reverse," that is, to deduce from the facts their law.

This relationship, moreover, is in agreement with the essential Leibnizian idea of harmony, of the correlation or "conspiracy" of all things. We know that for Leibniz truth does not consist in the conformity of ideas to things, but in their *analogy* or proportion; likewise, physical laws are true *when everything occurs as if* nature obeys them. A physical law is a mathematical function, therefore, a symbolic expression of natural processes; and Leibniz is well aware that the same process is susceptible to a multitude of

[303] *New Essays*, IV.xvii.6. Cf. Leibniz to Conring, 19 March 1678: "But those who deduce known phenomena from some physical hypothesis that has been assumed without demonstration cannot in this way demonstrate that their hypothesis is true, unless they observe the condition just stated" (*Phil*., I, 195).

[304] The induction of the empiricists has no logical value, not so much because it infers from the particular to the general, from *some* cases to *all* (which the principle of continuity allows us to do), but because it infers from the truth of the consequence to that of the principle, which is a sophism pure and simple.

[305] "Let it must be admitted that a hypothesis becomes more probable to the extent that it is simpler to understand and wider in force and power, that is, the greater the number of phenomena that can be explained using the fewest assumptions…. But those hypotheses deserve the highest praise (next to truth), by whose aid predications can be made even about phenomena or experiments that have not yet been attempted; for a hypothesis of this kind can then be applied in practice in place of truth. And it can happen that a certain hypothesis can be taken as physically certain when it completely satisfies all the phenomena that occur, just like the key to cryptograms." Leibniz to Conring, 19 March 1678 (*Phil*., I, 195-6).

[306] "The art of discovering the causes of phenomena, or true hypotheses, is like the art of deciphering, in which an ingenious conjecture often greatly shortens the way." *New Essays*, IV.xii.13.

different, but equivalent, expressions;[307] it suffices to know one of them (the simplest possible)[308] in order to have the *key* to the phenomena and to be in a position to calculate and predict them, which is the aim of science. The world of phenomena is an immense cryptogram, whose *keys* are the laws of nature; the more a key allows us to decipher a large number of words and phrases, the more it acquires a high probability. This metaphor is all the more apt as the phenomenal world is in fact only the image or symbol of the intelligible world (of monads), such that the laws of efficient and mechanical causality express and represent the metaphysical laws of the finality and activity of minds.

41. This analogy of the art of cryptography suggests and leads to another art, namely, that of algebra, since the discovery of a key is analogous to the discovery of a root. And indeed, in what do the givens furnished by experience consist? Each experience (or series of experiences summarized in a table) furnishes a new relation among the physical magnitudes whose law one seeks, that is, a new *equation* for the problem, a new logical connection among the unknowns.[309] But the more independent equations we have, the more the problem becomes determined, the more the collection of solutions is restricted. Thus, for Leibniz, algebra is not only an image, but a fragment and an application of the art of invention.[310]

All these analogies make us understand better the mathematical form under which Leibniz conceives of his art of invention. There are two sorts of operations in mathematics: one synthetic (addition, multiplication, raising to powers), the other analytic, the inverses of the first (subtraction, division, extraction of roots and logarithms). Leibniz generalizes this distinction: the most general synthetic operation consists in the construction of a series or table by means of a known formula or law of formation; and, given such a table or series, the most general analytic operation consists, inversely, in discovering the *key*, the origin, or the construction, that is, the law of formation (for example, the *reason* of a progression).[311] However, the art of discovering theorems (or of discovering the laws of nature—it is all one) is nothing but this second method, for it consists equally in finding the key or law of a table of numerical givens

---

[307] See his theory of the "equivalence of hypotheses" in mechanics, in his *Dynamica* (*Math*., VI, 484, 507, etc.).

[308] This explains and justifies the rule that recommends that we prefer the simplest hypothesis, not because it is the truest, but because it is the most convenient and most intelligible. Understood in this way, the principle of the simplicity of the laws of nature has a genuine scientific value, albeit a subjective rather than an objective one; it is not constitutive but regulative (in the language of Kant).

[309] From the point of view of logic, an experiment is expressed by a hypothetical judgment that asserts a correlation between givens and unknowns, and not by a categorical judgment. Consequently, no experiment allows us to determine separately and absolutely anyone of the unknowns; but all the experiments together combine to determine them through a system of simultaneous equations.

[310] Moreover, algebra is the embodiment of the analytic method: "for algebra is the method of deducing knowns from unknowns, such that when, with the knowns given, a system of equations is established among the unknowns, even the unknowns become known." *Method of Physics*, May 1676 (*Foucher de Careil*, VII, 103).

[311] *New Foundations of the Universal Mathesis* (LH IV 7B 6, Bl. 11). These two processes include, respectively, the summation and differentiation of a series; we recognize in this the basic methods of the differential and integral calculus.

(furnished by experience). In this way, the art of forming hypotheses is merged with the art of deciphering cryptograms and the art of conjecture.[312]

Thus, the experimental sciences have the same methods as the rational sciences: namely, synthesis and analysis; in a word: deduction, direct or inverse. Leibniz was too conscious of the unity of the human mind and the unity of science to separate and oppose, in the manner of empiricist logicians, the deductive and inductive sciences, as if there were two distinct and contrary methods for discovering and demonstrating the truth.[313] There is in the end only one method, because there is only one sense in which we could legitimately infer one truth from another. Induction, or rather the discovery of natural laws, is reduced to analysis, that is, to inverse deduction; but we always reason only by deduction.[314] Physics has the same method as the mathematical sciences; or rather its method consists in the application of mathematics to nature, in which reason and experience, the one marching before the other, meet, unite and collaborate in the search for truth.[315] In sum, abstract mathematics is the true logic of the natural sciences; and we can say without paradox that the only experimental method is deduction.[316]

42. The experimental sciences indeed employ deduction in two forms: the logic of the certain and the logic of the probable; or, as each must be translated into symbols and algorithms, the characteristic and the calculus of probabilities. The characteristic, first, will allow us to deduce from the givens of experience all the logical consequences which follow *necessarily* from them. In this way, it will save on experiments, and consequently the time and the effort of the experimenter. But next, and above all, it will guide him in his further research, by showing exactly which givens he is missing and by suggesting the experiments that could provide them.[317] It will even allow him to predict to a certain

[312] *On the Art of Discovering Theorems* (Paris, 7 September 1674): "The art of forming hypotheses, or the art of conjecturing, is of a different sort" (Leibniz has just spoken of induction and analogy; see n. 294) "to this is related the art of deciphering cryptograms, which must be regarded as the greatest example of the pure art of conjecturing..." (LH IV 6, 12 d, Bl. 8).

[313] At bottom, empiricist logicians only acknowledge deduction for appearance's sake, or on account of their respect for tradition; in reality they deny it any scientific value, regarding it as a tautology or vicious circle, or reducing it to induction, which according to them is the only useful and fertile method, since it is synthetic. Thus the charge of "duplicity" is addressed less to empiricism than to the bastard eclecticism that still rules in our textbooks and courses of philosophy.

[314] "It is always to draw inferences" (*New Essays*, IV.xvii.6). "Everything we know with certainty is established either through *demonstrations* or *experiments*. And in both cases reason is master. For the *art of preparing experiments* itself relies on definite reasons or the apparatus and other things of this sort, insofar as it obviously does not depend on chance or luck" (*Phil*., VII, 198).

[315] Cf. Chap. V, §19.

[316] We may add that such in fact is the true method of the experimental sciences, not the method of empirical induction of Bacon and Mill, with their unduly famous *tables*, which have never been used in a laboratory and have never led to the discovery of anything.

[317] By means of the characteristic "one would derive *from given experiments* all that can be derived from them, just as in algebra." Leibniz to the Duchess Sophie, ca. 1680 (*Phil*., IV, 296). Cf. Leibniz to Galloys, December 1678: "However, we will by this route draw near, insofar as it is possible *from the given experiments or the existing things in our control*. We will often even judge which experiments are still necessary in order to fill the gap" (*Phil*., VII, 23; *Math*., I, 187); *Foundations and Illustrations of the General Science*: "On the method of preparing experiments, so that they may serve to supply what is missing among the givens" (*Phil*., VII, 57; cf. 63, 201); Leibniz to Gabriel Wagner (1696), in which Leibniz cites his infinitesimal calculus as an example of the perfect logic that allows us to draw from the

degree the result of experiments, by indicating in which direction one must look for and find the unknown, and by restricting as much as possible the field of indetermination in which the latter moves.[318] This method is properly the art of invention or rather the art of inquiry; it is applied not only in the search for the laws of nature, but to every investigation or "question." It includes the *art of interrogation*, in order to obtain testimonies in a methodical way and to test them against each other, which is useful not only in the law but also in history and even in travelling;[319] the art of guessing riddles by cleverly posing questions, combining the useful answers, and excluding the fallacious or superfluous answers given only to mislead; finally, we have seen, the art of deciphering cryptograms and also inscriptions, whether in an unknown language or in a known language that is fragmentary and mutilated. Furthermore, the art of experimenting is itself only the art of interrogating nature, and, according to the words of Bacon, of questioning it in order to extract its secrets.[320] Thus, the art of invention gives rise to that "general critical art" that Leibniz called for, and this must include the criticism of testimony, texts and documents of every sort.[321]

43. On the other hand, the art of invention necessarily appeals to the calculus of probabilities: for analysis, we have seen, consists in going backwards from effects to causes, that is, from conclusions to principles, from observed phenomena to hypothetical laws. But we know that this regression, the inverse of the direct deductive order, is only probable, when "the reverse does not occur"; it is then a question of evaluating the degree of probability of the hypothesis thus established, and this is precisely the role of the *inverse* calculus of probabilities, which consists in estimating what we call the *probability of the causes* that can produce a given effect known through experience.[322] Just as the characteristic embodies the deductive method, so the true inductive method is the calculus of probabilities.

In the direct calculus of probabilities, the givens, which are the probabilities of causes, can be known and calculated *a priori*; but in the inverse calculus the givens are

---

givens all possible consequences. The former had in fact furnished him with solutions to problems which, though determined in themselves, had surpassed the scope of the algebra of Descartes (*Phil*., VII, 526).

[318] "The characteristic art shows not only how experiments are to be used, but also which experiments need to be undertaken and which are sufficient for determining the nature of the thing in question. Just as in those common tricks in which it is customary to guess at a number that someone has silently proposed to himself, it can easily be judged by someone trained in algebra whether those things which anyone says to himself about the hidden number are sufficient for arriving at it." *Method of Physics*, May 1676 (*Foucher de Careil*, VII, 103). We again note the constant assimilation of experimental problems to problems of algebra. Arithmetical problems were very much in fashion at this time; numerous examples are found in Bachet De Méziriac, *Problèmes plaisans et délectables qui se font par les nombres* (1st ed., 1612; 4th ed., Paris, Gauthiers-Villars, 1879).

[319] Leibniz to Gabriel Wagner, 1696 (*Phil*., VII, 518).

[320] *Foundations and Illustrations of the General Science*: "To the art of invention belongs knowing how to pose questions, or what amounts to the same thing, knowing how to organize observations and conduct experiments" (*Phil*., VII, 126).

[321] See Chap. V, §20.

[322] The direct calculus of probabilities shows how to evaluate the probability of an *effect* (or event), given the probabilities of its causes or conditions. The inverse calculus (which is more difficult) shows how to evaluate the probability of the *cause* (or principle), given that the effect has actually occurred (or that the consequence is true). It is worth noting that the Cartesian (purely logical) sense of the word *cause* and *effect* is traditionally preserved in the calculus of probabilities.

the probabilities of effects, and as these cannot be derived from the probabilities of causes (which are precisely the unknowns), they can be known only *a posteriori*. Leibniz only acknowledged this idea quite late. Indeed, in his *Essay on Certain New Arguments Concerning Human Life and the Number of Human Beings*,[323] composed shortly after 1682,[324] in which he recalled his earlier work on life annuities,[325] he starts from this "fundamental supposition, that 81 newly born infants will die uniformly, that is, one per year during the next 81 years"; or, in other words, that every year of human life is equally fatal. From this he derives the "precise conclusion that the average length of human life is 40 years"; the "rule for finding the presumptive average life that a person of a certain age probably still has to live"; the "proportion of human beings who die at each age"; the "reasonable proportion of the number of people living at each age"; finally, the "conclusion that about as many people of one age die as of another." However, the assumption which served as the principle for all these deductions was a totally gratuitous, if not false, hypothesis, and all the *a priori* probabilities that are derived from it have no more value than it. They would have had much more value if they had been calculated *a posteriori*, according to the mortality tables of a country or city that show the actual number of people of each age who die each year. Nevertheless, Leibniz did not scorn the teachings of statistics, as is proved by a list of 56 questions, most of them concerning what we now call *demographics*, which probably was to serve as the basis for the empirical evaluation of the probabilities relating to human life.[326]

Be this as it may, when Jacob Bernoulli explained his theory of *a posteriori* probabilities, that is, those determined by experience and drawn from statistics, to him, Leibniz made certain objections to him based on the essential contingency of the empirical data.[327] Bernoulli responded that the probability of an empirical law grows with the number of instances, and that we can calculate the number of instances necessary so that the law has whatever degree of probability we want.[328] He completely assimilated *a posteriori* probabilities to *a priori* ones and compared the approximate values obtained for them from statistics to the indefinite approximation of which Ludolph's number (the number π) is susceptible.[329] What is ultimately in question in this discussion is knowing whether contingency precludes determinism, or whether the law of large numbers subordinates individually contingent phenomena to an apparent determinism. Leibniz had even less reason not to adopt this latter thesis as he himself had claimed that contingency in no way precludes determinism, and even implies it. In any case, he appears to have been converted to the idea of *a posteriori* probabilities, for he later expressed it on his

[323] Klopp, V, 326-37.

[324] In connection with the *Essay on Political Arithmetik, concerning the growth of the city of London, with the measures, periods, causes and consequences thereof*, by Sir William Petty, F.R.S. (1682) (Klopp, V, p. xxxviii).

[325] *On Incomes for Life* (*Math*., VII, 133-7).

[326] *Investigations of a Political Calculus Concerning Human Life, and Related Matters* (Klopp, V, 337-40).

[327] Leibniz to Jacob Bernoulli, 3 December 1703 (*Math*., III, 83-4).

[328] He added that he had submitted the proof of this theorem to his brother Johann a dozen years earlier; Jacob Bernoulli to Leibniz, 20 April 1704 (*Math*., III, 88).

[329] Jacob Bernoulli to Leibniz, 2 August 1704 (*Math*., III, 91).

own behalf.[330] On this point, he borrowed much more from Jacob Bernoulli than he lent him.[331]

44. We have seen the use of the art of invention in the experimental theoretical sciences. In the applied sciences, the art of invention must further serve to resolve technical problems, for example, to construct a machine or apparatus capable of producing a certain desired effect;[332] in a word, it will serve for what are properly speaking scientific and industrial "inventions." It will show how to find, by a regular and infallible method that leads straight to the goal, everything that is susceptible of rational determination, and it will allow us confidently to bring to a successful conclusion all the investigations that until now have relied only on guesswork or groping, on chance or instinct.

Finally, the art of invention will be very useful in the so-called moral sciences: not in the theoretical part of these sciences, which proceed rationally and *a priori*, but in their application to practice, which depends on the givens of experience.[333] Here again it is necessary to employ analysis rather than synthesis. Moral theorems are established by the deductive method, starting from *a priori* principles; but moral problems are posed in the form of particular cases and in terms of experience. We must solve them, as we do mathematical problems, by reducing them to known theorems and by deducing from the latter the sought or presumed solution. It is therefore still the same method that we must employ, and here as elsewhere the calculus of probabilities will generally play a role.[334] In fact, it is very rare that a practical question of any complexity could be decided by a unique and absolute principle; most often it depends on several different, if not opposing, principles, which give rise to as many different solutions. Only the calculus of probabilities could save us from uncertainty by determining the most probable solution, that is, the most reasonable,[335] the most just or the most advantageous, according to the case.

In the solution of practical problems, on the other hand, we often have to take account of an infinity of reasons or motives, because the reality that gives rise to them everywhere

---

[330] Leibniz to Bourguet, 22 March 1714 (*Phil.*, III, 570).

[331] See §28.

[332] *Foundations of the General Science* (Erdmann, 86a).

[333] See Leibniz to Burnett, 17/27 July 1696: "I agree with you that morals and politics could be established in a solid and incontestable manner; but in order to apply them in practice, we would need a new species of logic completely different from those which we have now. This is what is principally missing in these practical sciences" (*Phil.*, III, 183). As early as 1677 Leibniz wrote to Galloys: "If we had characters such as I conceive them in metaphysics and morals, and what depends on them, we could form some very certain and important propositions in these fields. When it is a question of deliberation, we could show the advantages and disadvantages directly from the facts, and we could estimate the degrees of probability, almost like the angles of a triangle. But it is almost impossible to succeed without this characteristic" (*Phil.*, VII, 22; *Math.*, I, 181).

[334] "For philosophy has two parts: the theoretical and the practical. Theoretical philosophy is founded on the true analysis, of which mathematicians give some examples, but which also must be applied to metaphysics snd natural philosophy by giving good definitions and solid axioms. But practical philosophy is founded on a true topics or dialectic, that is, on the art of estimating degrees of proofs, which is not yet observed in writers on logic, but of which jurists have given some examples that are not to be scorned and can serve from the start to create a science of proofs suitable for verifying historical facts and giving the meanings of texts." Leibniz to Burnett, 1/11 February 1697 (*Phil.* , III, 193-4). Later, Leibniz calls the logic of probabilities a "moral dialectic" or a "natural jurisprudence."

[335] Cf. *New Essays*, II.xxi.66.

involves infinity and the continuous. This is why these sorts of questions generate interminable discussions in which the opposed parties can invoke in turn a multitude of more or less specious arguments, whose enumeration would be endless and whose recapitulation appears impossible. Thus, Leibniz compares those who argue in this manner to merchants who, in order to balance their accounts, would each recall their innumerable debts at random, without ever coming to a total, nor consequently to the final comparison that alone would settle the debate.[336] But how can we come to the end of a summation of infinitely many, infinitely small elements? The integral calculus will furnish the means and will allow us to evaluate the total probability on each side by taking account of all the small motives that militate for or against it while varying in continuous degrees. Thus, the infinitesimal calculus applied to probabilities will replace the vague and confused deliberations and discussions in which sentiment reigns and rhetoric triumphs,[337] and it will dictate the decisions to take with a mathematical rigor and precision.

There is more: this same calculus will allow us to determine exactly not only the most probable side but also the most secure, that is, that which offers the least risk or uncertainty;[338] and this in the same manner in which it allows us to determine in geometry not only the maximum and minimum forms but *the best* forms with respect to a certain relation or in view of a certain end,[339] that is, by taking into account not only the absolute quantity but also such-and-such a quality considered as desirable or preferable. One could calculate which alternative offers the greatest sum of some quality or some advantage as easily as one calculates the brachistochronic curve.[340] We thus clearly see how many important and varied applications such a calculus would find in moral casuistry, judicial practice, politics, diplomacy, military arts, and, finally, in every case in which we weigh testimonies or opinions and in which we are obliged to decide on the basis of reasons that are more or less probable, but never necessary and decisive.

45. Such must be the art of invention that would constitute, in Leibniz's eyes, his greatest and most precious discovery, since it was the infallible means of making an infinity of other discoveries.[341] This art, he says, he had cultivated since childhood, and we know in

---

[336] *History and Praise of the Characteristic Language* (*Phil*., VII, 188). Leibniz also compares the evaluation of probabilities to debit and credit bookkeeping (*ibid*.; and *New Essays*, II.xxi.67).

[337] *Discourse Concerning the Method of Certainty*, end (*Phil*., VII, 183).

[338] "And this we will judge because it is not only more probable but also more *secure*, to the extent that it is appropriate to purchase hope at the price of danger." *On the Universal Science* (*Phil*., VII, 201); cf. *New Essays*, IV.ii14. This distinction was borrowed from the casuists, who professed either *probabilism* or *prudentialism*. See Leibniz to Duke Johann Friedrich, 1679 (Klopp, IV, 427).

[339] *An Anagogical Essay* (*Phil*., VII, 272). See §25.

[340] It is considerations of this sort that Daniel Bernoulli introduced into the calculus of probabilities by defining *moral hope* and *moral luck* (of a player, for example). See *Commentarii Academi Petropolitan* (1730 and 1731); and Laplace, *Essai philosophique sur les probabilités*, Principles IX and X.

[341] Leibniz to Duke Ernst August: "I do not attach much importance to particular discoveries; what I most desire is to perfect the art of invention, and instead to give methods for solving problems, since a single method includes an infinity of solutions" (*Phil*., VII, 25). Leibniz to Oldenburg, 27 August 1676: "But I am so constituted that often, with general methods discovered, I am satisfied to have the matter in hand and willingly leave the rest to others. For all these things are not to be especially valued, except that they perfect the art of invention and cultivate the mind" (*Math*., I, 119). Cf. the letter to Conring (1678), in which Leibniz says that he has been occupied since childhood with the art of invention, which he reckons to be more useful and more valuable than anything else (*Phil*., I, 203); Leibniz to Placcius, 1678 (Dutens,

fact that he identified his combinatory with the art of invention. It was for the sake of penetrating its secrets that he studied mathematics, because these sciences were until then the only ones in which this art was known and applied;[342] and it was by seeking to perfect it that he made all his mathematical discoveries.[343] Thus is explained, on the one hand, the intimate connection between Leibniz's logic and mathematics; and, on the other hand, the analogy between, and near perfect identity of, his metaphysics and his "real" logic.[344]

We understand, finally, why Leibniz sought to give to metaphysics, and more generally to philosophy (conceived as the totality of theoretical sciences),[345] a mathematical form, as the only demonstrative form.[346] The geometrical method always appeared to him the ideal universal method, the best guarantee of logical correctness; and he blamed Descartes and Spinoza, not for having employed it outside of mathematics, but

---

VI.1, 22); Leibniz to Burnett, 1/11 February 1697 (*Phil*., III, 194, 196); and, above all, *On the Solutions to the Problem of the Chain or Rope, and Others Proposed by the Esteemed Jacob Bernoulli in the Acta of June 1691*, *Acta Eruditorum*, 1692, in which Leibniz says of his infinitesimal calculus: "I published its elements some years ago, being mindful of public usefulness rather than my glory, which I perhaps could have pursued more zealously with the method suppressed. But it pleases me to see the fruits that have grown, also in the gardens of others, from the few seeds I have scattered. For it was not completely up to me to cultivate these things satisfactorily, nor were other matters wanting in which I uncovered new approaches, because I always judged the prize and valued methods rather than particulars, though the latter are commonly thought more praiseworthy" (*Math*., V, 258). Cf. Leibniz to the editor of the *Journal des Savants* (*Math*., V, 258), cited p. 295, n. 2.

342 See the important letter to Duke Johann Friedrich and a curious autobiographical fragment, 1679 (Klopp, IV, 444, 454), quoted p. 165, n. 2.

343 He wrote to the Landgrave, while imploring him ro communicate to Arnauld his discoveries in geometry: "I admit very freely, however, that these sorts of curiosities have no better use than that of perfecting the art of inventing and reasoning well." Leibniz to the Landgrave Ernst von Hesse-Rheinfels, 4/14 August 1683 (*Phil*., II, 6).

344 Leibniz to the Duchess Sophie: "But for my part I have only pursued mathematics because I have found in it traces of the *art of invention in general*.... I come now to metaphysics, and I can say that it is for the love of it that I have passed through all these stages; for I have recognized that the true metaphysics is no different than the true logic, that is, the art of invention in general (*Phil*., IV, 291, 292). It is a fact that Leibniz considered logic a *real*, and not simply a *formal*, science: "the true logic is not only an instrument, but also contains in some way the principles and true ground of philosophizing, since it treats of those general rules on the basis of which the true and the false can be distinguished." *Preface to Nizolius*, 1670 (*Phil*., IV, 137). We may recall that Leibniz calls *real logic* that "general analysis" from which he borrows the principle of continuity (*Phil*., I, 349). See n. 193.

345 "Philosophy is the complex of universal doctrines; it is opposed to history, which is concerned with particulars" (LH IV 8, Bl. 56).

346 "I never write anything in philosophy that I do not treat by definitions and axioms, though I do not always give it that mathematical appearance, which puts people off." Leibniz to Burnett, 14 December 1705 (*Phil*., III, 302). "If anyone wanted to write like a mathematician in metaphysics or morals, nothing would prevent him from doing so with rigor" (*New Essays*, II.xxix.12). "Metaphysics is no less evident than mathematics, if treated correctly" (Leibniz to Jacob Bernoulli, 3 December1703; *Math*., III, 83). Cf. Leibniz to Arnauld, 14 July 1686: "And as for metaphysics, I claim to give geometrical demonstrations in it" (*Phil*., II, 62); *History and Praise of the Characteristic Language*, in which he says that his characteristic would render rational philosophy as clear, certain and irrefutable as arithmetic (*Phil*., VII, 187, above and below); Leibniz to Foucher, 1686 (*Phil*., I, 381); Leibniz to Burnett, 1/11 February 1690 (*Phil*., III, 190); Leibniz to Tolomei, 6 January 1705, end of P.S., and 17 December 1705 (*Phil*., VII, 466, 468); Leibniz to Des Bosses, 30 June 1715 (*Phil*., II, 499); Leibniz to Dangicourt, 1716: "I am delighted that a mind as mathematical as yours also applies itself to philosophical investigations. This will aid my plan of rendering philosophy demonstrative" (Erdmann, 745). Cf. Chap. IV, §5.

for having employed it badly.[347] We know that he flattered himself on having succeeded in this better than them, thanks to his universal characteristic, which would give to all the sciences the rigor and precision of mathematics. This is why he went so far as to say: "My metaphysics is entirely mathematical, as it were, or it could become so."[348] But in order to extend the mathematical method to all the sciences, it was necessary that he have an original conception of its value and importance, and that he generalize the very idea of mathematics. It is this conception of a universal mathematics that we must now study, before moving on to the special applications and techniques of the characteristic.

---

[347] *New Essays*, II.xxix.12 (continuation of the text quoted in the preceding note): "Some have made claims to do so, and have promised us mathematical demonstrations outside mathematics; but it is very rare that anyone has succeeded at it." *Remarks on the Sixth Philosophical Letter Printed at Trevoux*, July 1712: "It is praiseworthy to want to apply the method of geometers to metaphysical matters, but it is necessary to admit that until now hardly anyone has succeeded at it; and M. Descartes himself, despite all that very great cleverness that one cannot refuse to him, perhaps never had less success than when he undertook to do so in one of his replies to objections (*Phil*., VI, 349, note). The allusion is to the attempted geometrical demonstration by means of which Descartes had summarized his *Meditations* at the end of the *Replies to the Second Objections* (see p. 94). Cf. Leibniz to Conring, 24 August 1677 (*Phil*., I, 188); Leibniz to Malebranche, 1679 (*Phil*., I, 337); Leibniz to Burnett, 1699 (*Phil*., III, 259); and *Phil*., IV, 320, 326, 469; VII, 64, 125, 324. On Spinoza, see Leibniz to Galloys, 1677: "It is not as easy as one thinks to give genuine demonstrations in metaphysics. However, they exist and are very beautiful. One could not have them without having established good definitions, which are rare (*Math*., I, 179); Leibniz to Arnauld, 14 January 1688: "This Spinoza is full of rather puzzling dreams, and his supposed demonstrations "concerning God" lack even the appearance of demonstrations (*Phil*., II, 133). Cf. Leibniz's critical notes on the *Ethics* (*Phil*., I, 139-52) and on Spinoza's letters to Oldenburg and Schuller (*Phil*., I, 123-38, notes). See Stein, *Leibniz und Spinoza* (Berlin, 1890) and the unpublished pieces contained in that book, notably the correspondence of Leibniz and Schuller, 1677-8 (Appendix III). One finds a similar critique of Descartes and Spinoza in the fragment LH IV 6, 12f Bl. 27, in which Leibniz reviews (for the preface of the *Elements of Eternal Truth*) the authors who have tried to apply the mathematical method outside mathematics, especially to philosophy (cf. the fragment LH IV 1, 19c Bl. 13). Among these authors, he often cites Abdias Treu, professor of mathematics at the University of Altdorf (near Nuremburg), who had edited the *Physics* of Aristotle in the form of Euclid's *Elements*, in order to compete with the Cartesians: "lest the Cartesians alone should boast about the mathematical method." Leibniz to Thomasius, 19/26 December 1670 (*Phil*., I, 34; cf. 21); Leibniz to Conring, 3 January 1678 (*Phil*., I, 187); Leibniz to Honoré Fabri, 1676 (*Phil*., IV, 247; *Math*., VI, 84). He cites at the same time as him Thomas Barton, author of a *Metaphysical Euclid* (see Note I) and Father Honoré Fabri: "Father Fabri claimed to transform all of philosophy into geometry" (*Phil*., VII, 166). Cf. Leibniz to Johann Bernoulli, 15 October 1710 (*Math*., III, 856). Once again, what Leibniz criticizes in these authors is not the principle of their method but the bad application they make of it. We have a proof of this, or a counter-proof, in the judgment he rendered on Locke. We know that he did not value him at all as a logician: "In my view, Locke's opinion is in this respect uninspiring, for although he is clever enough, he is not sufficiently solid or profound." Leibniz to Koch, 2 September 1708 (*Phil*., VII 478, note). In connection with classical logic, he says: "Locke and others who scorn it do not understand it." Leibniz to Koch, 15 July 1715, P.S. (*Phil*., VII, 481). Finally, he says very brusquely of Locke: "The art of demonstration was not his strong point." Leibniz to Burnett, 26 May 1706 (*Phil*., III, 307). Now what is the reason for these severe judgements? Leibniz indicates it elsewhere: "M. Locke had subtlety and skill, and some sort of superficial metaphysics that he knew how to promote, but he was ignorant of the method of mathematicians." Leibniz to Remond, 14 March 1714 (*Phil*., III, 612).

[348] Leibniz to the Marquis de l'Hospital, 27 December 1694 (*Math*., II, 258). Leibniz speaks immediately after of his *characteristica situs* (see Chap. IX, §3).