

# Bell's theorem and nonlocality

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**146 Philosophy of Physics: Quantum Mechanics**  
Winter 2009

# Post EPR

- Reminder: EPR tried to argue for the incompleteness of QM
- ⇒ Idea that there exists a “hidden reality” behind what is captured in the QM-description
- ⇒ [David Bohm \(1917-1992\)](#), in his *Quantum Theory* (1951), formulated a (nonlocal) hidden variable (HV) theory that was empirically equivalent to QM
- In this work, Bohm extended the EPR thought experiment
- ⇒ ignited the interest of [John S Bell \(1928-1990\)](#)

# John Stewart Bell (1928-1990)



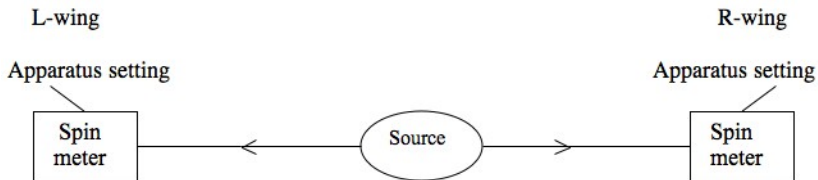
- studied physics at Queen's University Belfast, PhD U Birmingham, CERN
- “On the Einstein-Podolsky-Rosen paradox” (1964): derivation of **Bell's inequality**
- **Bell's theorem**: this inequality, derived from basic assumptions about locality and separability, conflicts with the predictions of QM
- “On the problem of hidden variables in quantum mechanics” (1966): von Neumann's arg against possibility of HV thys does not succeed, and neither do args drawing on Gleason's thm

# Bell's relevance

N. David Mermin, "Is the moon there when nobody looks? Reality and the quantum theory", *Physics Today*, April 1985, 38-47.

- By the mid-60s, almost all physicists just moved on and worked with QM, but didn't reflect its foundations.
- ⇒ many of them didn't notice, and still fail to appreciate the relevance of Bell's thm
- But not all: "Bell's theorem is the most profound discovery of science" (Henry Stapp)
- A bit more nuanced (but only a bit): "Anybody who's not bothered by Bell's theorem has to have rocks in his head" ("a distinguished Princeton physicist")
- Mermin's classification of physicists:
  - Type 1 bothered by EPR and Bell's thm, type 2 (the majority) not bothered
  - Type 2a explain why not, but either miss the point entirely or make assertions that are demonstrably false
  - Type 2b refuse to explain why they are not bothered

# Mermin's version of the EPR-Bohm thought experiment



- Three pieces: two detectors (A and B), and a source (C)
- Each detector has switch with three settings (1, 2, 3), and responds to event by flashing red light (R) or green (G)
- No connections bw pieces  $\Rightarrow$  no signals other than particles
- Switch of each detector is **independently** and **randomly** set to one of its settings, and button is pushed at source to initiate process of creating pair and sending them to opposite wings
- many runs of the experiments are made, data of form (11GG, 23GR, etc) collected

- Note: since there are no connection bw parts of apparatus, the only thing that travels bw them are the particles (this can be tested by sliding walls, etc)

The data has two features:

- 1 For those runs when **settings were the same in A and B**, we find that the light always flashed in same colour. (PERFECT CORRELATION)
- 2 For **all runs regardless of the settings in A and B**, the pattern of flashing is completely random. In particular, half of the time the same colour flashes, half of the time a different one does. (NO CORRELATION)

# How can this data be explained?

- perfect correlation cries out for explanation
- Traditional possibilities: events are really parts of one larger event, or A causes B or vice versa, or they have common cause
- If detectors could communicate, this would be easy. But they don't. And can't.
- Neither can the detectors have been preprogrammed always to flash same colour, since they also need to account for data point 2, and their settings are **random** and **independent**.
- Born offers an explanation (in a letter of May 1948 to Einstein): "objects far apart in space which have a common origin need not be independent... Dirac has based his whole book on this."
- Mermin makes this more concretely on p. 43f, let's look at this

# A local hidden variable explanation

- At core a **common cause expl**: both particles are imparted the same ordered triple of labels as they leave the source (three bits of information, e.g. RRG, GRG, etc), each telling the detector which colour to flash, depending on its setting.
  - Mermin imagines an possibility: particles come in eight different kinds (cubes, spheres, tetrahedra, etc), but this is essentially same idea: each particle carries with it a set of instructions for how to flash for **each** of the three settings, and that in **any** run both particles carry the same set of instructions.
  - instructions must cover **each** of the possible detector settings bc there is no communication bw source and detectors other than the particles
  - this also means that instructions must be carried in **every** run, since one can never know at the source whether the settings are the same
- ⇒ can easily account for data 1

- But despite the naturalness of this type of explanation (arguably the only natural expl), it cannot be true: it's inconsistent with data 2!
- Note that “we are about to show that ‘something one cannot know anything about’—the third entry in an instruction set—cannot exist.” (43) (one can never learn more than two of the entries in the instruction sets imparted on the particles)
- Here's the arg for the inconsistency w/ data 2. Consider a possible instruction set, e.g. RRG.
  - ⇒ detectors will flash same colour for settings 11, 22, 33, 12, 21, and different colour for settings 13, 31, 23, 32
  - since settings are random and independent, each of the nine possibilities are equally probable
  - ⇒ instruction set RRG will result in same colour flashing in 5/9 of the time

- Evidently, the same holds for instruction sets RGR, GRR, GGR, GRG, and RGG (bc arg uses only the fact that one colour appears twice, and the other once).
- Two more instruction sets are left: RRR and GGG, but these both result in the same colours flashing all the time (w/ probability one). But this gives us the famous:

### Theorem (Bell's theorem (baby version))

*If instruction sets exist, the same colours will flash in **at least 5/9** of all the runs, regardless of how the instruction sets are distributed among the runs.*

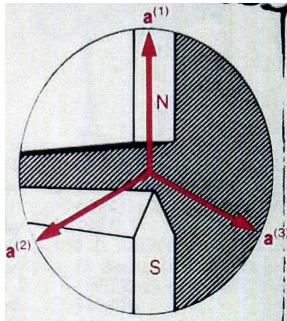
- This is **Bell's inequality (baby version)**: the probability that the same colours flash is larger or equal to 5/9.
  - It's now obvious that data 2 cannot be accounted for: data 2 violates Bell's inequality!
- ⇒ there cannot be a local hidden variable explanation

# The standard QM explanation

- Let the source produce a pair of spin-1/2 particles in a singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle). \quad (1)$$

- Each detector contains a Stern-Gerlach magnet, oriented along three directions  $a^1$ ,  $a^2$ ,  $a^3$  perpendicular to the line of flight, each separated by  $120^\circ$ :



- light on detector A flashes R if particle is deflected north (spin  $\uparrow$ ) and G if deflected south (spin  $\downarrow$ ), detector B uses **opposite** colour conventions
- this allows us to account for the data
- data 1 is accounted by structure of singlet state which ensures that the measurements along the same axis yield opposite spin and thus the same colour
- to get data 2, we need the concept of an **expectation value**

### Definition (Expectation value)

*An expectation value of an observable for a state is the statistical mean of the measured values of that observable for that state for a large number of measurements. More precisely, given a Hermitian operator  $\hat{A}$  defined on a Hilbert space  $\mathcal{H}$ , the **expectation value** of  $\hat{A}$  in the state  $|\psi\rangle \in \mathcal{H}$  is defined as  $\langle\psi|\hat{A}|\psi\rangle$ .*

- Note that the product  $m_1 m_2$  of the two spin measurement results (each of which is  $+1/2$  or  $-1/2$ ), we will get  $-1/4$  when the light flashes are of the same colour and  $+1/4$  when the colours are different.
  - **To be shown:** the product  $m_1 m_2$  vanishes when averaged over all nine distinct pairs of orientations of the two Stern-Gerlach magnets
  - For a given pair of orientations  $\mathbf{a}^i$  and  $\mathbf{a}^j$ , the value of  $m_1 m_2$  is just the expectation value in the state  $|\psi\rangle$  of the corresponding product of commuting (Hermitian) operators  $\mathbf{a}^i \cdot \mathbf{S}^1$  and  $\mathbf{a}^j \cdot \mathbf{S}^2$ .
- ⇒ data 2 thus requires:

$$\sum_{i,j=1}^3 \langle \psi | [\mathbf{a}^i \cdot \mathbf{S}^1] [\mathbf{a}^j \cdot \mathbf{S}^2] | \psi \rangle = 0 \quad (2)$$

- Linearity of  $\mathcal{H}$  (cf. L04, p. 8, first eq.) and fact that we use only linear operators (ibid., p. 11) gives us

$$\begin{aligned}
 \sum_{i,j=1}^3 \langle \psi | [a^i \cdot S^1] [a^j \cdot S^2] | \psi \rangle &= \langle \psi | \sum_{i,j=1}^3 [a^i \cdot S^1] [a^j \cdot S^2] | \psi \rangle \\
 &= \langle \psi | \sum_i [a^i \cdot S^1] \sum_j [a^j \cdot S^2] | \psi \rangle \\
 &= \langle \psi | [(\sum_i a^i) \cdot S^1] [(\sum_j a^j) \cdot S^2] | \psi \rangle
 \end{aligned}$$

But we know that  $\sum_i a^i = \sum_j a^j = 0$ , since the three unit vectors around an equilateral triangle add up to zero.  $\square$

# Comments

- simplified thought experiment exactly captures the relevant features of the EPR-Bohm experiment, except that it introduces runs where the orientations in both wings are not aligned
  - Baby Bell theorem shows why there cannot be local hidden vars, contra EPR who argued that QM was incomplete
  - Bell was the who added the runs with different settings in order to extract from QM the prediction about data 2
  - It was exactly data 2 that showed that a local HV story is incompatible with the predictions of QM.
  - [Alain Aspect](#), Paris 1982; [Nicolas Gisin](#), Geneva 1997: detectors are 10 km apart, settings chosen [after](#) photons left source
- ⇒ experimental falsification of local HV thy

# Bertlmann's socks and Bell's theorem

J.S. Bell, "Bertlmann's socks and the nature of reality", in *Speakable and Unspeakable in QM*, 139-158.



*"Dr. Bertlmann likes to wear two socks of different colours. Which colour he will have on a given foot on a given day is quite unpredictable. But when you see... that the first sock is pink you can already be sure that the second sock will not be pink. Observation of the first, and experience of Bertlmann, gives immediate information about the second. There is no accounting for tastes, but apart from that there is no mystery here. And is not the EPR business just the same?" (139)*

No, since many physicists

*“... came to hold not only that it is difficult to find a [classical explanation of the EPR business] but that it is wrong to look for one—if not actually immoral then certainly unprofessional. Going further still, some asserted that atomic and subatomic particles do not **have** any definite properties in advance of observation... It is as if we had come to deny the reality of Bertlmann's socks, or at least of their colours, when not looked at. And as if a child has asked: How come they always choose different colours when they **are** looked at? How does the second sock know what the first had done?” (142f)*

- Bell goes on to use the example of pairs of socks, of which we want to know what the probabilities are that they survive a thousand washing cycles at a certain temperature. (Sec. 3)
- Using a random sampling hypothesis (148), and the fact that socks are paired à la Bertlmann (ibid.). he derives an inequality (a Bell inequality) which can be shown to be violated in QM. (149)

*“The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting...*

*“But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we **cannot** dismiss intervention on one side as a causal influence on the other.” (149f)*

- Bell then proceeds to generalize the argument in several respects, to show that “certain particular correlations, realizable according to quantum mechanics, are **locally inexplicable**.” (151f) This means that they “cannot be explained... without action at a distance.” (152)

Bell sees at least four different positions that might be taken wrt to the EPRB business:

- 1 QM is wrong in sufficiently critical situations. But that's unconvincing in the light of empirical evidence.
- 2 The detector settings are not independent variables. But this would imply strange conspiracies bw spatially distant apparatuses, or our free will is conspiratorially entangled with them or both.
- 3 Causal influences can go faster than light, perhaps by reintroducing an aether. But this would create formidable challenges...
- 4 Perhaps there is no reality beyond some "classical" "macroscopic" level.

## A closer examination of the assumptions of Bell's theorem

G. Grasshoff, S. Portmann, A. Wüthrich, *Brit. J. Phil. Sci.* **56** (2005): 663-680.

- There are many inequivalent sets of assumptions that are sufficient to derive a Bell-type inequality that is violated by QM and experiment.
- Dialectical situation: try to derive Bell ineq from set of assumptions that is as weak as possible; since we know that Bell ineq is violated, we know that at least one premise must be false
- But which one?!?
- Traditionally, apart from a number of auxiliary assumptions, or assumptions that come directly from QM, what is often called **Bell locality** is assumed.
- Nomenclature:  $L_i, R_j$  are the settings of the apparatus in the left and right wings, respectively ( $i, j = 1, 2, 3$ );  $L_i^a, R_j^b$  are the possible outcomes measured in the left and right wings, given settings as indicated by  $i, j$ , where  $a, b = \uparrow, \downarrow$ ;  $V$  is a common cause variable

**Assumption (Bell locality)**

$$p(L_i^a \wedge R_j^b | V \wedge L_i \wedge R_j) = p(L_i^a | V \wedge L_i) p(R_j^b | V \wedge R_j).$$

This condition can be unpacked into several weaker ones, such as

**Assumption (Separability)**

*The coinciding instances of  $L_i^a$  and  $R_j^b$  are distinct events.*

**Assumption (Locality 1)**

*No  $L_i^a$  or  $R_j^b$  is causally relevant for the other.*

**Assumption (Principle of Common Cause (PCC))**

*If two event types  $A$  and  $B$  are correlated and the correlation cannot be explained by direct causation or by event identity, there there is a common cause variable s.t.  $p(A \wedge B | V) = p(A | V) p(B | V)$ .*

SEPARABILITY, LOCALITY 1, and PCC jointly entail (together with auxiliary assumptions)  $p(L_i^a \wedge R_j^b | V) = p(L_i^a | V) p(R_j^b | V)$ .

### Assumption (Locality 2)

*If  $L_i \wedge R_j \wedge X$  is sufficient for  $L_i^a$ , then  $L_i \wedge X$  is alone sufficient for  $L_i^a$ . Similarly, if  $L_j \wedge R_j \wedge Y$  is sufficient for  $R_j^b$ , then  $R_j \wedge Y$  alone is sufficient for  $R_j^b$ .*

### Assumption (No conspiracy)

*The common cause variable  $V$  is not influenced by the setting or the measurement operations in the two wings:  $p(V | L_i \wedge R_j) = p(V)$ .*

The result on top of this page, together with LOCALITY 2 and NO CONSPIRACY (and auxiliary assumptions) then yields the Bell inequality (although I have glossed over some subtleties).

# Upshot

So Bell locality must be violated. But since the assumption of Bell locality can be unpacked into several weaker assumptions, there are various ways in which it can be violated:

- The measurement events in the two wings are not separate.
- One of the measurement events instantaneously causes the other.
- There is no common cause at the source.
- The settings in one wing have a causal influence on the measurement in the other wing.
- There is backward causation such that the settings in either or both of the wings (which can be set after the particles departed the source) causally influence the common cause at the source event.

Note: one of these must be true.

# Additional considerations on nonlocality

Albert, *Quantum Mechanics and Experience*, Ch. 3.

- EPR thought that the nonlocal character of measurements on non-separable states is a merely disposable artifact of the particular formalism of standard QM.
- The upshot of Bell's thm is that this is **demonstrably wrong**:

*“What Bell has given us is a proof that there is as a matter of fact a genuine nonlocality in the actual workings of nature, **however** we attempt to describe it, period. That nonlocality is... necessarily... a feature of every possible manner of calculating... which produces the same statistical predictions as quantum mechanics does; and those predictions are now experimentally known to be correct.”*  
(70)

# Nonlocality is subtle

- Given the non-separable state  $|\psi\rangle$ , the statistics of outcomes of spin measurements on electron in  $L$ -wing depend nonlocally on the outcomes of spin measurements on electron in  $R$ -wing, and vice versa.
- But do the statistics of the outcomes of spin measurements on an  $L$ -electron, given  $|\psi\rangle$ , depend nonlocally on **whether** a spin measurement is performed on the  $R$ -electron, or vice versa?
- Given  $|\psi\rangle$ , the outcome of a measurement of the colour of the  $L$ -electron is equally likely “white” or “black”, **whether or not** a measurement of the colour of the  $R$ -electron has previously been made. (Why?)
- In fact, since  $|\psi\rangle = 1/\sqrt{2}(|^1\text{hard},^2\text{soft}\rangle - |^1\text{soft},^2\text{hard}\rangle)$ , the outcome of a colour measurement of the  $L$ -electron is equally likely “black” or “white”, regardless of whether a **hardness** measurement is first carried out on the  $R$ -electron. (Why?)

In fact, we have the following general result:

### Theorem

*For any state  $|Q\rangle$  of a quantum system  $S$  consisting of two subsystems  $s_1$  and  $s_2$ , and any observables  $\hat{A}$  of  $s_1$  and  $\hat{B}$  of  $s_2$ , the probabilities of the various possible outcomes of a measurement of  $\hat{A}$  don't depend on whether or not a measurement of  $\hat{B}$  is carried out first. (72)*

- outcomes of measurements sometimes depend nonlocally on outcomes of other (distant) measurement, but they don't depend nonlocally on **whether or not** any other (distant) measurements get carried out
- ⇒ nonlocality cannot be exploited to transmit detectable signals bw distant locations

# Three final comments

Tim Maudlin, *Quantum Non-Locality and Relativity*, Ch. 1.

I will not go over all in the chapter since it duplicates a lot of what I said so far already three times. But you should have a good look at the thought experiment in the section “How do they do it?”.

Three results concerning the “quantum connection”:

- 1 It is **unattenuated**: in contrast to classical (instantaneous) action, the quantum connection is unaffected by distance.
- 2 It is **discriminating**: while gravitational forces affect similarly situated objects in the same way, the quantum connection is a private arrangement bw entangled particles.
- 3 It is **instantaneous**: while Newton's thy of gravity has gravity propagate instantaneously, it need not do so, and GR certainly involves no instantaneous gravitational action; but the quantum connection appears to act essentially instantaneously.